Stat 345 Solutions - Section 4.2

Problem 4-1

(a) $P(X > 1) = \int_1^\infty e^{-x}dx = -e^{-x}|_1^\infty = 0 + e^{-1} = 0.3679$
(b) $P(1 < X < 2.5) = \int_1^{2.5} e^{-x}dx = -e^{-x}|_1^{2.5} = -0.0821 + 0.3679 = 0.2858$
(c) $P(X = 3) = 0$
(d) $P(X < 4) = \int_0^4 e^{-x}dx = -e^{-x}|_0^4 = -0.0183 + 1 = 0.9817$
(e) $P(X \geq 3) = \int_3^\infty e^{-x}dx = -e^{-x}|_3^\infty = 0 + 0.0498 = 0.0498$

Problem 4-2

(a) We want to find x such that $P(X > x) = 0.10$.

$P(X > x) = \int_x^\infty e^{-x}dx = -e^{-x}|_x^\infty = 0 + e^{-x}$.

Thus, we have $e^{-x} = 0.1$ and so $x = 2.3$.

(b) We want to find x such that $P(X \leq x) = 0.10$.

$P(X \leq x) = \int_0^x e^x dx = -e^{-x}|_0^x = -e^{-x} + 1$.
Thus, we have $1 - e^{-x} = 0.1$ and so $e^{-x} = 0.9$, which gives $x = 0.1054$.

Problem 4-5

(a) $P(X > 0) = \int_0^1 1.5x^2dx = 1.5\frac{x^3}{3}|_0^1 = \frac{3}{2}(\frac{1}{3}) = 0.5$. Alternatively, you can just notice that the density is symmetric around 0, and so the probability of being greater than 0 is $\frac{1}{2}$.

(b) $P(X > 0.5) = \int_{0.5}^1 1.5x^2dx = 1.5\frac{x^3}{3}|_{0.5}^1 = \frac{1}{2}(1 - \frac{1}{8}) = 0.4375$

(c) $P(-0.5 \leq X \leq 0.5) = \int_{-0.5}^{0.5} 1.5x^2dx = 1.5\frac{x^3}{3}|_{-0.5}^{0.5} = \frac{1}{2}(\frac{1}{8} + \frac{1}{8}) = \frac{1}{8} = 0.125$

(d) $P(X < -2) = 0$

(e) $P(X < 0 \texttt{ or } X > -0.5) = 1$, since this includes all values between -1 and 1. Alternatively, $P(X < 0 \texttt{ or } X > -0.5) = P(X < 0) + P(X > -0.5) - P(X < 0 \texttt{ and } X > -0.5) = 0.5 + \int_{-0.5}^\infty 1.5x^2dx - \int_{-0.5}^0 1.5x^2dx = 0.5 + (0.5 - \frac{-0.5^3}{2}) - (0 - \frac{-0.5^3}{2}) = 1$

(f) $P(x < X) = \int_x^1 1.5w^2dw = .5w^3|_x^1 = .5(1 - x^3), -1 < x < 1$, so $P(x < X) = 0.05 \Leftrightarrow .5(1 - x^3) = 0.05 \Leftrightarrow x^3 = 0.9 \Leftrightarrow x = 0.9^{1/3} = 0.9655$