Stat 345 Solutions - Section 4.9

Problem 4-73

$X \sim exp(\lambda = \frac{1}{10} = 0.1)$. Thus the cdf is

$$F(x) = \begin{cases} 0, x < 0 \\ 1 - e^{-0.1x}, x \geq 0 \end{cases}$$

(a) $P(X > 10) = 1 - P(X \leq 10) = 1 - [1 - e^{-0.1(10)}] = e^{-1} = 0.3679$

(b) $P(X > 20) = 1 - P(X \leq 20) = 1 - [1 - e^{-0.1(20)}] = e^{-2} = 0.1353$

(c) $P(X > 30) = 1 - P(X \leq 30) = 1 - [1 - e^{-0.1(30)}] = e^{-3} = 0.0498$

(d)

$$\begin{aligned} P(X < x) = 1 - e^{-0.1x} &= 0.95 \\ e^{-0.1x} &= 0.05 \\ -0.1x &= \ln 0.05 \\ x &= 29.96 \end{aligned}$$

Problem 4-75

Log-ons to a computer follow a Poisson process with mean $\lambda = 3$ per minute. Let X be the time between counts. Then $X \sim \exp(\lambda = 3)$.

(a) $E(X) = \frac{1}{3} = 0.33$ min

(b) $SD(X) = \sqrt{\frac{1}{9}} = 0.33$ min

(c) Find x such that $P(X < x) = 0.95$.

$$\begin{aligned} P(X < x) = 1 - e^{-3x} &= 0.95 \\ e^{-3x} &= 0.05 \\ x &= 0.9986 \end{aligned}$$

So $x \approx 1$ min.

Problem 4-81

Let X be the time between arrivals of taxis in minutes. Then $X \sim \exp(\lambda = \frac{1}{10})$.

(a)

$$\begin{aligned} P(X > 60) &= 1 - P(X \leq 60) \\ &= 1 - [1 - e^{-0.1(60)}] \\ &= e^{-6} \\ &= 0.0025 \end{aligned}$$

Alternatively, define Y to be the time between arrivals of taxis in hours. Then $Y \sim \exp(\lambda = 6)$.

$$\begin{aligned} P(Y > 1) &= 1 - P(Y \leq 1) \\ &= 1 - [1 - e^{-6(1)}] \\ &= e^{-6} \\ &= 0.0025 \end{aligned}$$

(b) Using the lack of memory property,

$$\begin{aligned} P(X < 10 + 60 | X > 60) &= P(X < 10) \\ &= 1 - e^{-0.1(10)} \\ &= 0.6321 \end{aligned}$$

Problem 4-85

Let X be the lifetime in hours. Then $X \sim \exp(\lambda = \frac{1}{400})$.

(a) $P(X < 100) = 1 - e^{-\frac{1}{400}(100)} = 0.2212$

(b) $P(X > 500) = 1 - P(X \leq 500) = 1 - [1 - e^{-\frac{1}{400}(500)}] = 0.2865$

(c) Using the lack of memory property, $P(X < 100 + 400 | X > 400) = P(X < 100) = 0.2212$ from (a) .