Stat 345 Solutions - Section 5.1 (3^{rd} ed.)

$\underline{\text{Problem } 5.1}$

Check definition 5-1, pg. 143

- (1) $f_{XY}(x, y) \ge 0$ for all x, y yes
- (2) $\sum_{x} \sum_{y} f_{XY}(x, y) = \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + \frac{1}{8} = 1$
- (3) $f_{XY}(x,y) = P(X = x, Y = y)$ yes

Problem 5.2

(a)
$$P(X < 2.5, Y < 3) = P(X = 1, Y = 1) + P(X = 1.5, Y = 2) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

(b) $P(X < 2.5) = P(X = 1, Y = 1) + P(X = 1.5, Y = 2) + P(X = 1.5, Y = 3) = \frac{1}{4} + \frac{1}{8} + \frac{1}{4} = \frac{5}{8}$
(c) $P(Y < 3) = P(X = 1, Y = 1) + P(X = 1.5, Y = 2) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$
(d) $P(X > 1.8, Y > 4.7) = P(X = 3, Y = 5) = \frac{1}{8}$

<u>Problem 5.3</u>

We first need to find the pmfs. Let's start with X:

$$f_X(x) = \sum_y f_{XY}(x, y)$$

Then,

$$E(X) = \sum_{x} f_X(x) = (1)\frac{1}{4} + (1.5)\frac{3}{8} + (2.5)\frac{1}{4} + (3)\frac{1}{8} = 1.81$$

Similarly, the pmf for Y is

Then,

$$E(Y) = \sum_{y} f_Y(y) = (1)\frac{1}{4} + (2)\frac{1}{8} + (3)\frac{1}{4} + (4)\frac{1}{4} + (5)\frac{1}{8} = 2.875$$

Problem 5.4

(a) See Problem 5.3 above

(b) To find the conditional pmf, we find $P(Y = y | X = 1.5) = \frac{P(X=1.5, Y=y)}{P(X=1.5)}$ for all values of Y.

Thus we have, for example,

$$P(Y = 2|X = 1.5) = \frac{P(X = 1.5, Y = 2)}{P(X = 1.5)} = \frac{1/8}{3/8} = \frac{1}{3}$$

The pmf is then

and we can check that this sums to 1.

(c) We have

$$P(X = 1.5|Y = 2) = \frac{P(X = 1.5, Y = 2)}{P(Y = 2)} = \frac{1/8}{1/8} = 1$$

and the rest of the probabilities are all 0.

(d) $E(Y|X = 1.5) = \sum_{y} yP(Y = y|X = 1.5) = (2)\frac{1}{3} + (3)\frac{2}{3} = \frac{8}{3}$ (using the distribution from part (b))

Problem 5.5

We are given that $f_{XY}(x, y) = c(x + y)$ for x = 1, 2, 3 and y = 1, 2, 3 and we need to find c. We know that the $\sum_x \sum_y f_{XY}(x, y) = 1$.

$$\sum_{x} \sum_{y} f_{XY}x, y = c \sum_{x} \sum_{y} (x+y)$$

= $c[\sum_{x} x+1+x+2+x+3]$
= $c[(1+1+1+2+1+3) + (2+1+2+2+3) + (3+1+3+2+3+3)]$
= $36c$

Then setting 36c = 1, we have $c = \frac{1}{36}$.

Problem 5.6

(a)
$$P(X = 1, Y < 4) = P(X = 1, Y = 1) + P(X = 1, Y = 2) + P(X = 1, Y = 3) = \frac{2}{36} + \frac{3}{36} + \frac{4}{36} = \frac{1}{4}$$

- (b) P(X = 1) is the same as (a), since Y must be less than 4.
- (C) $P(Y = 2) = P(X = 1, Y = 2) + P(X = 2, Y = 2) + P(X = 3, Y = 2) = \frac{3}{36} + \frac{4}{36} + \frac{5}{36} = \frac{1}{3}$ (d) $P(X < 2, Y < 2) = P(X = 1, Y = 1) = \frac{2}{36}$.