Problem 5-67

First, we find the marginal distributions. The marginal distribution for X is

x	1	2	4
$f_X(x)$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

The marginal distribution for Y is

y	3	4	5	6
$f_Y(y)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$

The covariance is given by

$$cov(X,Y) = E(XY) - E(X)E(Y)$$

First, we find $E(XY)$:

$$
\begin{aligned}
E(XY) &= \sum_x \sum_y xy f_{XY}(x,y) \\
&= (1)(3)\frac{1}{8} + (1)(4)\frac{1}{4} + (2)(5)\frac{1}{2} + (4)(6)\frac{1}{8} \\
&= 9.375
\end{aligned}
$$

Now we find $E(X)$ and $E(Y)$:

$E(X) = (1)\frac{3}{8} + (2)\frac{1}{2} + (4)\frac{1}{8} = 1.875$

$E(Y) = (3)\frac{1}{8} + (4)\frac{1}{4} + (5)\frac{1}{2} + (6)\frac{1}{8} = 4.625$

Thus, $cov(X,Y) = 9.375 - (1.875)(4.625) = 0.7031$.

The correlation is given by

$$corr(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(x)VarY}}$$

So we need to find the variances:

$Var(X) = E(X^2) - E(X)^2 = (1)\frac{3}{8} + (4)\frac{1}{2} + (16)\frac{1}{8} - 1.875^2 = 0.8594$

$Var(Y) = E(Y^2) - E(Y)^2 = (9)\frac{1}{8} + (16)\frac{1}{4} + (25)\frac{1}{2} + (36)\frac{1}{8} - 4.625^2 = 0.7344$

Thus the correlation is

$$corr(X, Y) = \frac{0.7031}{\sqrt{0.8594}0.7344} = 0.8850$$