Stat 345 Solutions - Section 5.7 (3^{rd} edition)

Problem 5-87

(a)
$$E(2X + 3Y) = 2E(X) + 3E(Y) = 2(0) + 3(10) = 30$$

- (b) $Var(2X + 3Y) = 2^{2}Var(X) + 3^{2}Var(Y) = 4(4) + 9(9) = 97$
- (c) Let W = 2X + 3Y. Then $W \sim N(30, 97)$. Thus,

$$P(W < 30) = P(\frac{W - 30}{\sqrt{97}} < \frac{30 - 30}{\sqrt{97}})$$

= $P(Z < 0)$
= 0.5

(d)

$$P(W < 40) = P(\frac{W - 30}{\sqrt{97}} < \frac{40 - 30}{\sqrt{97}})$$

= $P(Z < 1.02)$
= 0.8461

Problem 5-89

Let X_1 be the thickness of the first half and let X_2 be the thickness of the second half. Note that X_1 and X_2 are both $N(2, \sigma^2 = (0.1)^2)$ and that they are independent.

(a) Let Y be the total thickness, $Y = X_1 + X_2$. Then,

$$E(Y) = E(X_1 + X_2) = E(X_1) + E(X_2) = 2 + 2 = 4$$
, and
 $Var(Y) = Var(X_1 + X_2) = Var(X_1) + Var(X_2) = 0.01 + 0.01 = 0.02.$
Thus, $SD(Y) = \sqrt{0.02} = 0.1414.$

(b) We want to find P(Y > 4.3). We know that $Y \sim N(4, 0.02)$.

$$P(Y > 4.3) = P(\frac{Y-4}{0.1414} > \frac{4.3-4}{0.1414})$$

= $P(Z > 2.12)$
= $1 - P(Z < 2.12)$
= $1 - 0.9830$
= 0.017