Stat 345 Solutions - Section 7.5

Problem 7-35

Let X_i be the tensile strength of one fiber specimen. We are given that $X_i \sim N(75.5, \sigma^2 = (3.5)^2)$. Let $\bar{X}$ be the mean tensile strength of the six specimens in the sample. Then, $\bar{X} \sim N(75.5, \frac{3.5^2}{6})$ (note that the distribution of $\bar{X}$ is exactly normal, since the original X_i are normal).

We want to find $P(\bar{X} > 75.75)$.

$$\begin{aligned} P(\bar{X} > 75.75) &= P(\frac{\bar{X} - 75.5}{1.4289} > \frac{75.75 - 75.5}{1.4289}) \\ &= P(Z > 0.17) \\ &= 1 - P(Z < 0.17) \\ &= 1 - 0.5675 \\ &= 0.4325 \end{aligned}$$

Problem 7-37

Let X_i be the compressive strength of one concrete specimen. We are given that $X_i \sim N(2500, \sigma^2 = (50)^2)$. Let $\bar{X}$ be the mean compressive strength of the five specimens in the sample. Then, $\bar{X} \sim N(2500, \frac{50^2}{5})$ (note that the distribution of $\bar{X}$ is exactly normal, since the original X_i are normal).

We want to find $P(2499 \leq \bar{X} \leq 2510)$.

$$\begin{aligned} P(2499 \leq \bar{X} \leq 2510) &= P(\frac{2499 - 2500}{50/\sqrt{5}} \leq \frac{\bar{X} - 2500}{50/\sqrt{5}} \leq \frac{2510 - 2500}{50/\sqrt{5}}) \\ &= P(0.04 \leq Z \leq 0.45) \\ &= P(Z \leq 0.45) - P(Z \leq 0.04) \\ &= 0.6736 - 0.5160 \\ &= 0.1576 \end{aligned}$$

Problem 7-39

The standard error of the sample average $\bar{X}$ is $\frac{\sigma}{\sqrt{n}}$. Setting the standard error equal to 1.5, we have

$$\begin{aligned}
\frac{\sigma}{\sqrt{n}} &= 1.5 \\
\frac{5}{\sqrt{n}} &= 1.5 \\
\sqrt{n} &= \frac{5}{1.5} \\
n &= (\frac{5}{1.5})^2 \\
n &= 11.11
\end{aligned}$$

And thus we would have to sample 12 items in order for the standard error to be 1.5.

Problem 7-42

Let X_i be the amount of time that customer i waits. We are given that X_i has mean 8.2 minutes and standard deviation 1.5 minutes. Let $\bar{X}$ be the average time waiting in line for a sample of $n = 49$ customers. Then, using the CLT, $\bar{X} \approx N(8.2, \frac{1.5^2}{49})$.

(a)

$$\begin{aligned}
P(\bar{X} < 10) &= P(\frac{\bar{X} - 8.2}{1.5/7} < \frac{10 - 8.2}{1.5/7}) \\
&= P(Z < 8.4) \\
&= \sim 1
\end{aligned}$$

(b)

$$\begin{aligned}
P(5 \leq \bar{X} \leq 10) &= P(-14.9 \leq Z \leq 8.4) \\
&= P(Z \leq 8.4) - P(Z \leq -14.9) \\
&= 1 - 0 = 1
\end{aligned}$$

(c)

$$P(\bar{X} < 6) = P(Z < -10.26) = 0$$