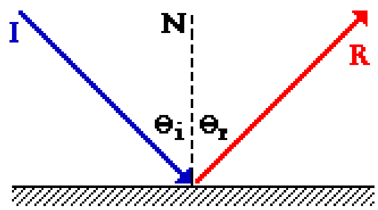
FEBRUARY 2009 PROBLEMS

Please send your solutions or questions to Janet Vassilev (jvassil@math.unm.edu) or Dimiter Vassilev (vassilev@math.unm.edu). We are looking forward to hearing from you.

1) Let $\{a_n\}_{n=0}^{\infty}$, be a sequence of real numbers. Suppose $a_0 = -1$ and $a_1 = 11$ and $a_{n+2} = 8a_{n+1} + 2009a_n$ for $n \geq 0$.
   (a) Determine a_2 and a_3.
   (b) Find a formula for a_n.

2) Let A be a point inside a circle distinct from its center. Can one always direct a ray of light from A so that it returns to A after at least three reflections by the circle? You should use that the angle of incidence is equal to the angle of reflection, see the following figure where N is the line perpendicular to the tangent of the circle at a point of reflection.



3) Let S be a set and $f : S \times S \to S$ be a function.
   (a) Suppose $f(a, f(a, b)) = a$ for all $b \in S$. Show that $f(a, a) = a$.
   (b) If $f(a, f(b, c)) = f(f(a, b), c)$ and $f(a, f(a, b)) = a$ for all $b \in S$, show that $f(a, b) = a$ for all $b \in S$.

4) Let O be a point of intersection of two circles c and c', not necessarily of the same radius. Suppose we draw two lines l and l' which are tangent to each of the circles. Let B and B' be the points of contact of the line l_1, correspondingly, with c and c'. Similarly, let C and C' be the points of contact of the line l_2, correspondingly, with c and c'. Show that the circles circumscribed around the triangles $\triangle OBC$ and $\triangle OB'C'$ are tangent to each other.