September 2009 Problems

Please send your solutions or questions to Janet Vassilev (jvassil@math.unm.edu) or Dimiter Vassilev (vassilev@math.unm.edu). We are looking forward to hearing from you.

1) The point P lies on the arc AB of the circle circumscribing the equilateral triangle $\triangle ABC$. Show that the distance of P to C equals the distance of P to A plus the distance of P to B, i..e, $PC = PA + PB$.
2) Is it possible to write the integers 1, 2, 3, 4, 5, 6, 7, 8 at the vertices of a regular octagon so that the sum of the integers in any three consecutive vertices is greater than a) 13 b) 11 12?
3) Suppose x and y be two integers satisfying $3x^2 + x = 4y^2 + y$. Show that $x - y$ is a perfect square. Are there integers x and y satisfying $3x^2 + x = 4y^2 + y$?
4) We call the set of all points in the plane with both coordinates integer numbers the *integer lattice points*. Prove that there is a point A in the plane so that every two different integer lattice points are at a different distance to A.
5) Prove that given a natural number n, there is a circle in the plane which contains exactly n integer lattice points. Maybe you can use a point as A from the last example to put some order among the lattice points, which can help you solve the problem.