*Please send your solutions or questions to Janet Vassilev (jvassil@math.unm.edu) or Dimiter Vassilev (vassilev@math.unm.edu). We are looking forward to hearing from you.*

### UNM - PNM STATEWIDE MATHEMATICS CONTEST XLII
November 7, 2009      First Round

1) Let $f(n)$ be the sum of $n$ and its digits. Find a number $n$ such that $f(n) = 2009$.
2) What is the largest number of points that you can place on a circle of radius 1 so that the distance between any two points is strictly greater than $\sqrt{2}$?
3) At a roadside stand, you can buy apples in bags of 6 or in bags of 11. What is the smallest number $n$ such that you can buy exactly $N$ apples for any $N \geq n$?
4) David, Bill and George are three thieves. One of them committed a robbery. During the interrogation they made the following statements:
   - David: Bill is not the robber. I am not the robber.
   - Bill: David is innocent. George is the robber.
   - George: I am innocent. David is the robber.
   It was determined that one of them lied twice, one told the truth twice, and one lied once and told the truth once. Who is the robber?
5) Suppose $f(x) = ax + b$. If $f(f(f(x))) = 64x + 63$, what is $a + b$?
6) Suppose a group of people have a code between themselves on how they can send messages to others in the group.
   - If $A$ can send a message to $B$ and $B$ can send a message to $C$, then $C$ can send a message to $A$.
   - For each pair of distinct people $A$ and $B$ in the group, either $A$ can send a message to $B$ or $B$ can send a message to $A$ but not both.
   What is the largest number of people in the group?
7) For which $k$ does the system $x^2 - y^2 = 0$, $(x - k)^2 + y^2 = 2$ have exactly two real solutions?
8) A library is open every day except Sunday. One day three girls, Adele, Bonnie and Clara visit the library together for the first time. Thereafter they visit the library many times. Adele makes her next visit two days after the previous visit unless the library is closed, in which she goes after three days. Bonnie makes her next visit three days after the previous visit unless the library is closed, in which she goes after four days. Clara makes her next visit four days after the previous visit unless the library is closed, in which she goes after five days. If their next meeting in the library was Friday. What day of the week was their first visit?
9) How many pairs of positive integers have greatest common divisor 3! and least common multiple 18!?
10) (a) Consider a right triangle and a square inscribed in a given circle of radius $R$. For which of the two figures is the sum of squares of its sides the largest?
    (b) Among all triangles inscribed in a given circle of radius $R$, which triangle has the largest sum of squares of its sides?
11) Suppose you are planning an expedition the goal of which is to take a letter to a remote destination that is nine days from your location. There is no food and water along the way so the team has to carry everything they need to survive. Each member of the team can carry enough food and water for six days for one person. What is the minimum

number of people that you have to put on the team in order for at least one of the them to reach the destination, while the remaining members either reach the destination or return safely home with enough supplies to survive the trip back? Assume also that food and water cannot be left anywhere without someone staying at this location.