January 2010 Problems

Please send your solutions or questions to Janet Vassilev (jvassil@math.unm.edu) or Dimiter Vassilev (vassilev@math.unm.edu). We are looking forward to hearing from you.

1) Show that given n stones we cannot determine with absolute certainty the one with minimal weight using a balance less than $n-1$ times.

2) What is the minimal number of comparisons in the worst case for an algorithm that finds the stone with minimal weight among n given stones using a balance?

3) Prove that it is possible using a balance to find the apples with minimal and maximal weights among $2n+1$ apples using $3n$ comparisons.

4) There are n stones that look identical, but in fact, some of them have different weights. There is a device that can be applied to two stones and tells whether they are different or not (but it does not say which one is heavier). It is known in advance that most of the stones (more than 50%) are identical. Find one of those identical stones making no more than n comparisons.

   Hint. If two stones are different, they may be discarded, because one of them does not belong to the majority and the majority remains.

5) Let a, b, c be positive numbers with $a+b+c=1$, prove that

$$\left(\frac{1}{a}-1\right)\left(\frac{1}{b}-1\right)\left(\frac{1}{c}-1\right) \geq 1.$$

6) Show that if $\triangle ABC$ is a triangle with all angles less than or equal to 120^o, there is a unique point P such that $\angle APB = \angle BPC = \angle CPA = 120^o$. The point P is known as the Fermat point of the triangle.