

APRIL 2010 PROBLEMS

Please send your solutions or questions to Janet Vassilev ([jvassil@math.unm.edu](mailto:jvassil@math.unm.edu)) or Dimiter Vassilev ([vassilev@math.unm.edu](mailto:vassilev@math.unm.edu)). We are looking forward to hearing from you.

1. Let  $n$  be a natural number, and  $f$  be a real-valued function on the plane such that for every regular  $n$ -gon in the plane, the average value of  $f$  at the vertices is the same constant  $k$  (The average value of  $f$  at points  $A_1, \dots, A_n$  is  $\frac{f(A_1) + \dots + f(A_n)}{n}$ ).
  - (a) Show that  $f$  is a constant function.
  - (b) Find this constant.
2. The sequence  $1, 82, 163, 244, \dots$  is an arithmetic progression. Show that this arithmetic progression contains infinitely many powers of 100.
3. A circle path has 18 trees planted along the path. At a certain time of the day in each of the trees is sitting a robin. Suppose that the robins move between the trees according to the following rule: a robin can fly to either of the trees directly next to his current tree, at which time another robin flies in the opposite direction along the path to his neighboring tree. Thus, if a robin moves one tree to the left then another robin moves one tree to the right at the same time and these are the only two robins moving at this time. Is it possible for all the robins to end in the same tree if they fly in the described manner?