SEPTEMEBR 2010 PROBLEMS

Please send your solutions or questions to Janet Vassilev (jvassil@math.unm.edu) or Dimiter Vassilev (vassilev@math.unm.edu). We are looking forward to hearing from you.

1. 7 boys and 4 girls are seated around a round table. How many ways can they be seated if:
   (a) There are no restrictions on seating.
   (b) No two girls can be seated next to each other.
   (c) All girls form a single block.

2. A gardener plants two apple trees, three cherry trees, four peach trees and five walnut trees in a row. He plants them in random order, each arrangement being equally likely. Let $\dfrac{m}{n}$ be the probability in lowest terms that no two walnut trees are next to each other. Find $m + n$.

3. Find the number of non-negative solutions to
$$rx_1 + x_2 + \cdots + x_n = kr$$
where k, r, n are all counting numbers.

4. Find the number of non-negative solutions to
$$(x_1 + x_2 + x_3)(y_1 + y_2 + y_3 + y_4) = 35$$

5. How many ways can you fill 3 identical Smith's bags with 10 different grocery items assuming at least one item is in each bag?

6. Let $S = \{1, 2, \ldots, n\}$. Find the number of subsets A of S satisfying the following conditions:
   - $A = \{a, a + d, \ldots, a + kd\}$ for some positive integers a, k and d.
   - Find the number of 8 digit natural numbers where every digit appearing in the number appears at least 3 times.