Here, $A(S)$ denotes the group of automorphisms of S (or $\mathfrak{s}$) that preserve the left-invariant metric on S. The spaces constructed in this manner became known as Damek-Ricci spaces, see [22] for more details. It is worth recalling [57] where it is shown that the just described solvable extension of H-type groups, which are not of Iwasawa type, provide noncompact counterexamples to a conjecture of Lichnerowicz, which asserted that harmonic Riemannian spaces must be rank one symmetric spaces.

The Heisenberg type groups allowed for the generalization of many important concepts in harmonic analysis and geometry, see [108], [109], [115], [58] and the references therein, in addition to the above cited papers. Another milestone was achieved in [49], which allowed for avoiding the classification rank one symmetric spaces and the heavy machinery of the semisimple Lie group theory, when studying the non-compact symmetric spaces of real rank one. Specifically, in [49] the authors considered the H-type algebras satisfying the so called J^2 condition defined in [49], see also [50].

Definition 2.2.4. We say that the H-type algebra $\mathfrak{g}$ satisfies the J^2 condition if for every $\xi_2, \xi_2' \in V_2$ which are orthogonal to each other, $< \xi_2, \xi_2' > = 0$, there exists $\xi_2'' \in V_2$ such that

$$J(\xi_2)J(\xi_2') = J(\xi_2''). \tag{2.18}$$

The key result here is the following Theorem of [49], see also [47], which can be used to show that if N is an H-type group, then the Riemannian space $S = NA$ is symmetric iff the Lie algebra $\mathfrak{n}$ of N satisfies the J^2 condition, see [[49], Theorem 6.1].

Theorem 2.2.5. *If $\mathfrak{n}$ is an H-type algebra satisfying the J^2-condition, then $\mathfrak{n}$ is an Iwasawa type algebra. In other words, the H-type groups N whose Lie algebras satisfy the J^2 condition are precisely the groups that arise as the nilpotent component in the Iwasawa decomposition of a semisimple Lie group of real rank one.*

This fundamental result has many consequences in allowing a unified proof of some classical results on symmetric spaces, in addition to some beautiful properties of extensions of the classical Cayley transform, inversion and Kelvin transform, which are of a particular importance for our goals.