ECE 238L
Digital Computers and Number Systems

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Computers are everywhere

- Computers are ubiquitous – used everywhere. Cell phones, street lights, watches, calculators, ...

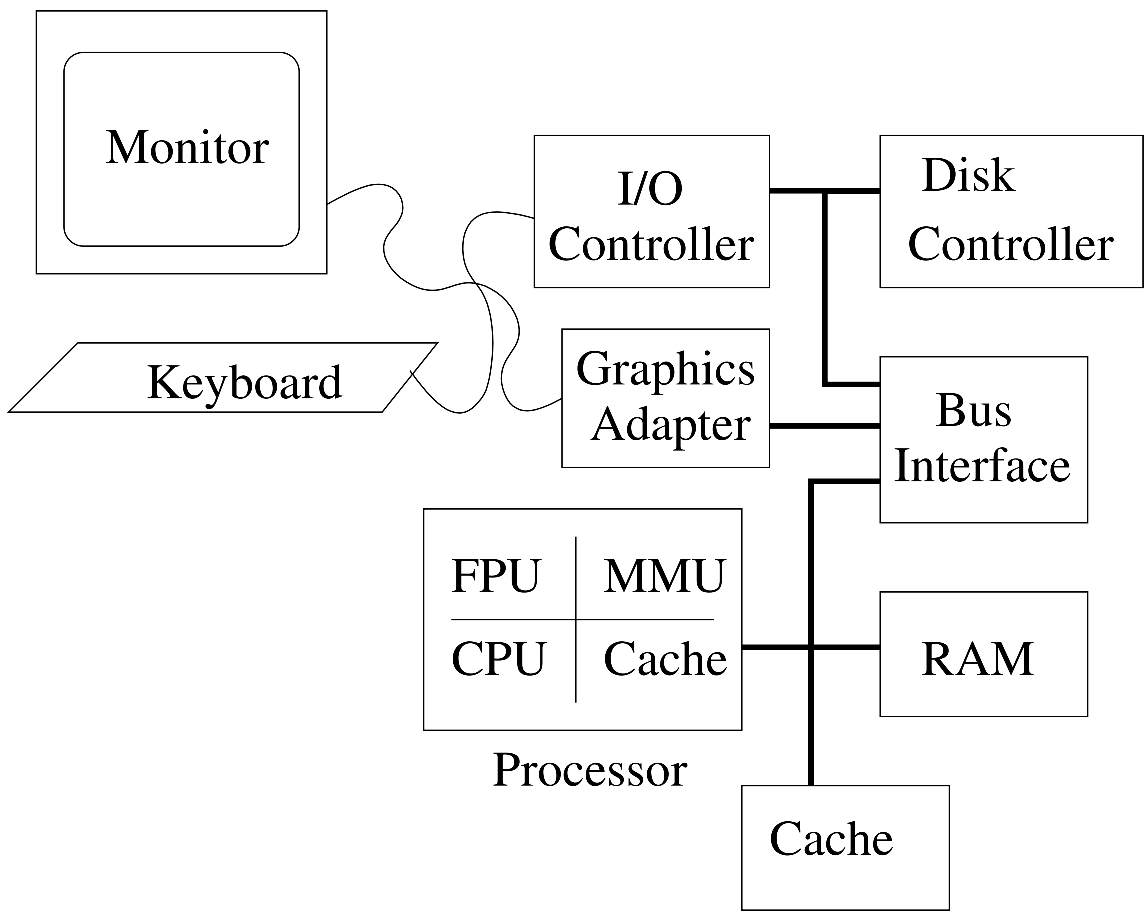
- Computers are flexible and can be reprogrammed.

- Computers operate on discrete elements (sets, information, etc.)

Discrete Elements – the basis of a computer

- Discrete elements are represented as signals.
- Most elements can have two values, e.g. binary.
- The values can be HIGH, LOW, True, False, 1, 0, etc.
- Binary representations are convenient and reliable.

Generic Computer



Positional Numbers

$$527.46_{10} = (5\times10^2)+(2\times10^1)+(7\times10^0)+(4\times10^{-1})+(6\times10^{-2})$$

$$527.46_8 = (5 \times 8^2) + (2 \times 8^1) + (7 \times 8^0) + (4 \times 8^{-1}) + (6 \times 8^{-2})$$

$527.46_5 =$ illegal – why?

$$101011.11_2 = (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2})$$

This works for binary as well...

Binary Numbers

$$101011.11_2 = (1\times 2^5) + (0\times 2^4) + (1\times 2^3) + (0\times 2^2) + (1\times 2^1) + (1\times 2^0) + (1\times 2^{-1}) + (1\times 2^{-2})$$

n	2^n	n	2^n
0	1	8	256
1	2	9	512
2	4	10	1024
3	8	11	2048
4	16	12	4096
5	32	13	8192
6	64	14	16384
7	128	15	32768

Octal and Hexadecimal

Octal is base eight. More compact than binary. Base-8 uses which digits?

$$127.4_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = 87.5_{10}$$

Hexidecimal is base 16. First 10 digits are decimal and the next 6 are from the alphabet (A $\longrightarrow$ 10, B $\longrightarrow$ 11,C $\longrightarrow$ 12,D $\longrightarrow$ 13, E $\longrightarrow$ 14, F $\longrightarrow$ 15).

$$\text{B65F}_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = 46687_{10}$$

Binary Numbers

$101011.11_2 = (1 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2})$

Convert to Base 10:

$= 32 + 0 + 8 + 0 + 2 + 1 + 1/2 + 1/4$

$= 43.75_{10}$

Convert 112 from base 10 to binary

112	
-64	1×2^6
48	
-32	1×2^5
16	
-16	1×2^4
0	
- 0	0×2^3
0	
- 0	0×2^2
0	
- 0	0×2^1
0	
- 0	0×2^0
0	

$$\downarrow$$

$$112_{10} = 1110000_2$$

An Alternate way to convert 112 from base 10 to binary.

2	112		
2	56	R	0
2	28	R	0
2	14	R	0
2	7	R	0
2	3	R	1
2	1	R	1
	0	R	1

$\uparrow$

$$112_{10} = 1110000_2$$

Converting fractions from base 10 to binary: convert $.7_{10}$ to binary

$$\downarrow \qquad \begin{array}{r} .7 \\ \times 2 \\ \hline (1).4 \\ \times 2 \\ \hline (0).8 \\ \times 2 \\ \hline (1).6 \\ \times 2 \\ \hline (1).2 \\ \times 2 \\ \hline (0).4 \\ \times 2 \\ \hline (0).8 \end{array}$$

$$.7_{10} = .1\ 0110\ 0110\ 0110\ \ldots$$

$\leftarrow$ process starts repeating here.

Hexadecimal

- Commonly used for binary data

  1 hex digit $==$ 4 binary digits (bits)

- Need more digits (than decimal)
  - Use 0-9, A-F
    - A-F are for 10-15

      FA$2_{16} = 15 \times 16^2 + 10 \times 16^1 + 2 \times 16^0$

      FA$2_{16} = \underbrace{1111}_{\text{Each group of 4 bits } \iff \text{ 1 hex digit}} 1010\ 0010$

Numbers with different Bases

Decimal Base 10	Binary Base 2	Octal Base 8	Hexadecimal base 16
00	00000	00	00
01	00001	01	01
02	00010	02	02
03	00011	03	03
04	00100	04	04
05	00101	05	05
06	00110	06	06
07	00111	07	07
08	01000	10	08
09	01001	11	09
10	01010	12	0A
11	01011	13	0B
12	01100	14	0C
13	01101	15	0D
14	01110	16	0E
15	01111	17	0F
16	10000	20	10