ECE 238L Arithmetic Operations and Codes

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Binary Addition

 $egin{aligned} 0 + 0 &= 0 \ 0 + 1 &= 1 \ 1 + 0 &= 1 \ 1 + 1 &= 0 \ and \ carry \ 1 \ to \ the \ next \ column. \end{aligned}$

Examples:

				$1\ 1\ 1$	
	0101	5_{10}		0101	5_{10}
+	0010	2_{10}	+	0011	3_{10}
	0111	7_{10}		1000	810

Binary Addition with Overflow

Add 45_{10} and 44_{10} in binary:

1 11	<	Carries
101101	. (45)	
+ 101100) (44)	

1011001

If the operands are unsigned, you can use the final carry-out as the MSB (Most Significant Bit) of the result. Adding 2 k-bit numbers may result in a k+1 bit result.

More Binary Addition with Overflow

	$1\ 1\ 1$	
	1111	15_{10}
+	0001	1_{10}
	0000	010

If you don't want a 5-bit result, just keep the lower 4 bits.

Four bits are insufficient to hold the result (16).

So, it rolls back to 0.

Binary Subtraction

Borrows:	00000	00110
Minuend:	10110	10110
Subtrahend:	- 10010	- 10011
Difference:	00100	00011

Again the same as decimal. If the subtrahend is larger than the minuend reverse the two operands and put a minus sign on the result.

Multiplication

Multiplicand:	1011	11
Multiplier:	× 101	× 5
	1011	55
	0000	
	1011	
Product:	110111	

110111 = 32 + 16 + 4 + 2 + 1 = 55

These same approaches apply to hexadecimal as well as octal, you just need to be more careful with your digits.

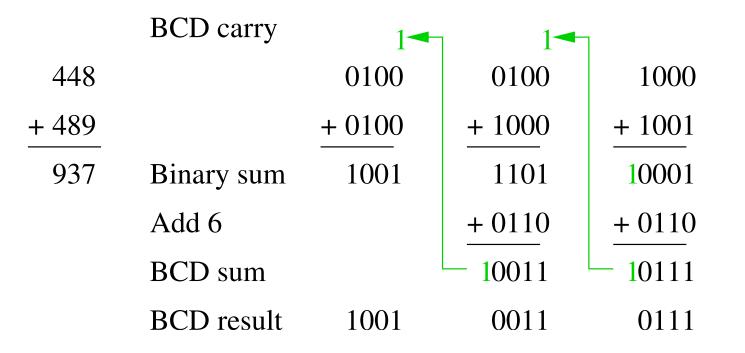
Binary Arithmetic

- Arithmetic with binary numbers is just like any other numbers
 - digit-by-digit addition
 - carries propagate to next column to left
- Overflow can occur
 - keep the extra bits
 or
 - just keep k bits (roll-over will occur)

Binary Coded Decimal (BCD)

Decimal	BCD			
0	0000	Convert 2496_{10} to BCD code:		
1	0001	2 4 9 6		
2	0010	$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$		
3	0011	0010 0100 1001 0110		
4	0100	Not this is very different from converting		
5	0101	to binary which yields:		
6	0110	100111000000_2		
7	0111	In BCD		
8	1000	0010010010010110		
9	1001			

BCD Addition



Add each digit. If the result is greater than 9, add 6 and carry any overflow to the next digit. Repeat.

Why BCD?

- BCD is common in electronic systems which display numeric values.
- Using BCD simplifies manipulation of numerical data for display.
- Each digit is a separate subcircuit, which will be true for a 7 segment display.
- Binary requires a conversion to decimal to determine the digits.

Disadvantages are 1) that addition is more difficult and requires extra circuitry. 10-15%. And 2), using 4 bits to represent 10 values can be expensive.

Binary Codes - ASCII

Character	ASCII Code	-			
С	1100011				
d	1100100				
е	1100101				
f	1100110				
g	1100111				
h	1101000	Conv	vert "help"	to ASCII	
I	1101001	h	-	I	-
j	1101010	h	е	I	р
k	1101011	110100	1100101	1101100	1111000
I	1101100				
m	1101101				
n	1101110				
0	1101111				
р	1110000				
q	1110001				

Gray Codes

		Gray
Number	Binary	Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

- Only one bit changes with each number increment
- Not a weighted code
- Useful for interfacing to some physical systems

Gray Codes are Not Unique

Gray		Gray
Code	Number	Code
000	0	000
001	1	010
011	2	110
010	3	111
110	4	011
111	5	001
101	6	101
100	7	100
	001 011 010 110 111 101	Code Number 000 0 001 1 011 2 010 3 110 4 111 5 101 6

Parity Bits

Word	Even Parity	Odd Parity
1000001	01000001	11000001
1010100	1 1010100	<mark>0</mark> 1010100

Even Parity - number of 1 bits should be even.

Odd Parity - number of 1 bits should odd.

Parity can detect any number of odd errors: 1,3,5,... Parity is also one of the simplest ways to detect errors. Communication protocols commonly include error detection and even correction.

Codes - Summary

- Bits are bits
 - Modern digital devices represent everything as collections of bits
 - A computer is one such digital device
- You can encode anything with sufficient 1's and 0's
 - Text (ASCII)
 - Computer programs (C code, assembly code, machine code)
 - Sound (.wav, .mp3, ...)
 - Pictures (.jpg, .gif, .tiff)