Gates - Part 2

September 14, 2006

Converting English to Boolean Expressions

The air conditioner should be turned on if and only if:

- the temperature is greater than 75°,
- the time is between 8a.m. and 5 p.m.,
- and it is not a holiday.

Identify Phrases

The air conditioner should be turned on if and only if:

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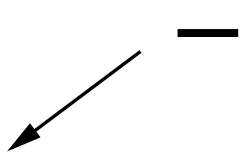
- the time is between 8a.m. and 5 p.m.,
- and it is not a holiday.

F = air conditioner should be turned on
A = temperature is greater than 75°
B = time is between 8a.m. and 5p.m.
C = it is a holiday

Identify Connective Words



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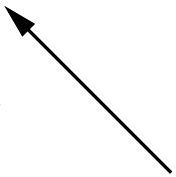

implied
and

Construct a Boolean Expression

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C = it is a holiday

$$F = A \bullet B \bullet C'$$

Draw the Network

The air conditioner should be turned on if and only if:

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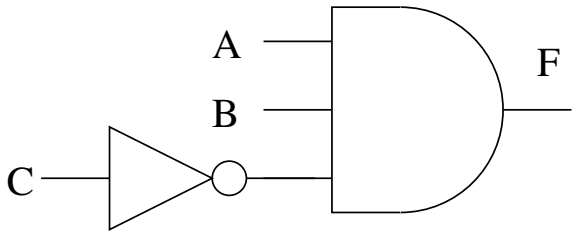
$$F = A \bullet B \bullet C'$$



Review

Converting English to Boolean

1. Identify phrases
2. Identify connective words
3. Construct a Boolean Expression
4. Draw the network

Converting English to Boolean

Be careful: Boolean algebra is precise, English is not.

The roads will be very slippery if A it snows or B rains and there is C oil on the road.

$$\mathbf{F = A + BC}$$

or

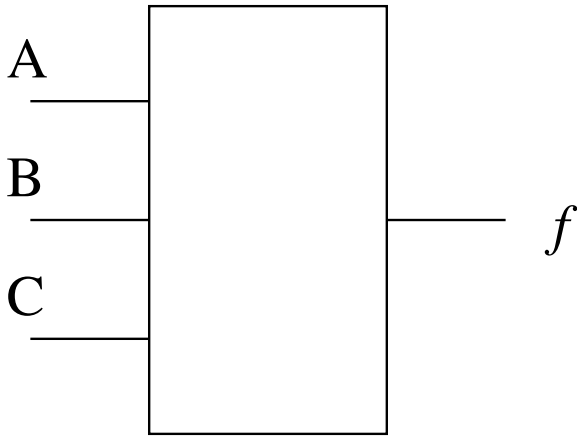
$$\mathbf{F = (A + B)C}$$

Which is it?

AND/OR vs. OR/AND Logic forms

AND/OR Logic from Truth Table

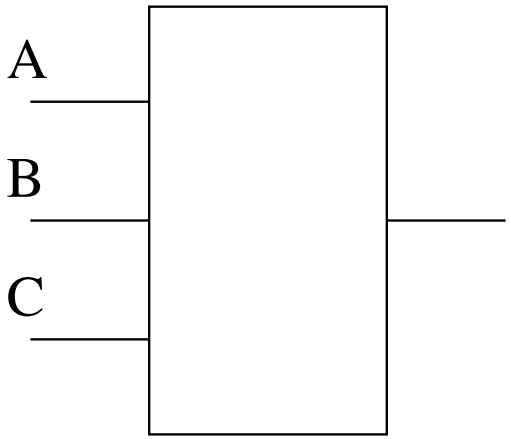
Write the SOP by inspection from f:



A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

AND/OR Logic from Truth Table

Write the SOP by inspection from f:



A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$f = \text{A'B'C} + \text{AB'C} + \text{ABC'} + \text{ABC}$$

AND/OR Logic from Truth Table

Simplify the equation

$$f = \text{A'B'C} + \text{AB'C} + \text{ABC'} + \text{ABC}$$

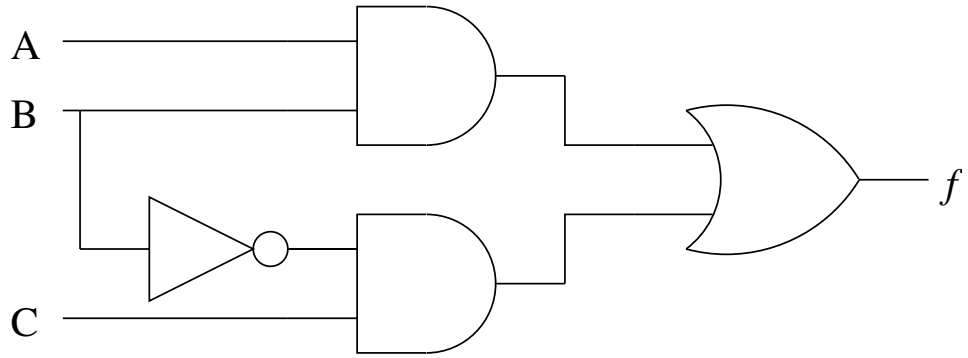
$$f = (\text{A} + \text{A'})\ \text{B'C} + \text{AB}\ (\ \text{C} + \text{C'})$$

$$f = \text{AB} + \text{B'C}$$

AND/OR Logic from Truth Table

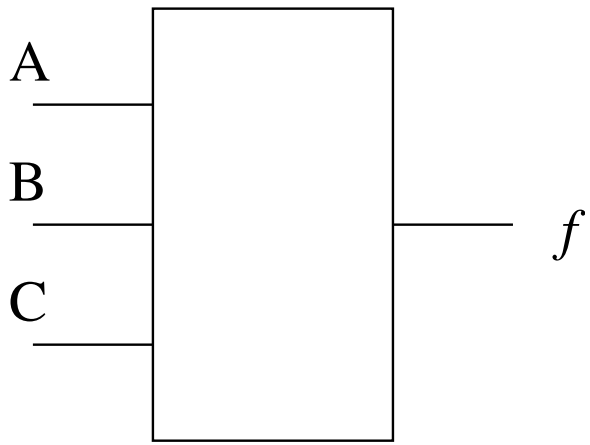
Draw the logic network

$$f = \text{AB} + \text{B'C}$$



OR/AND Logic from Truth Table

Write the POS by inspection from f:



A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

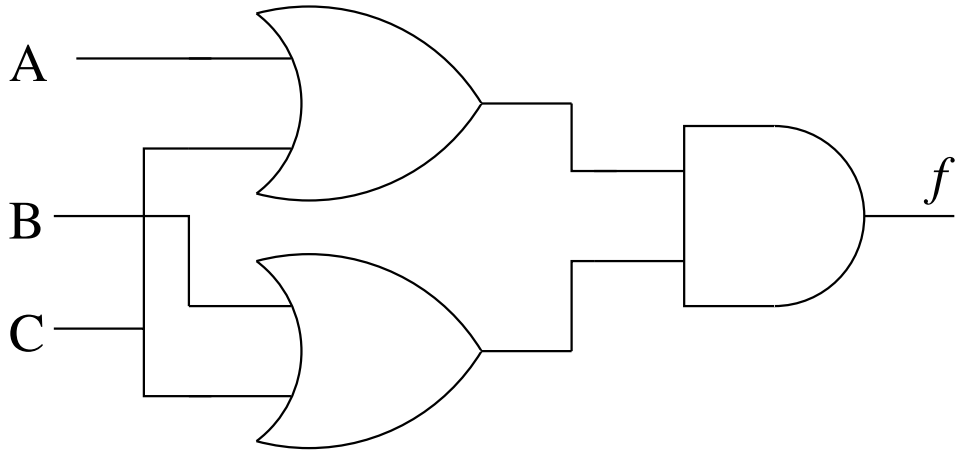
$$f' = \text{A'B'C' } + \text{A'BC' } + \text{ AB'C'}$$

$$f = (\text{A}+\text{B}+\text{C})(\text{A}+\text{B'}+\text{C})(\text{A'}+\text{B}+\text{C})$$

OR/AND Logic from Truth Table

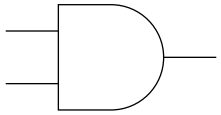
Simplify the equation: $f = (B+C)(A+C)$

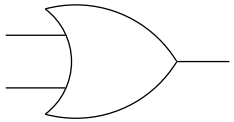
Draw the logic network:

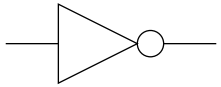


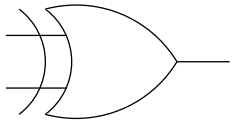
Types of gates

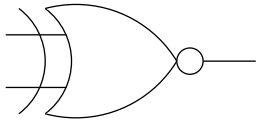
Gates already studied:

AND 

OR 

Inverters 

Exclusive–OR 

Equivalence 

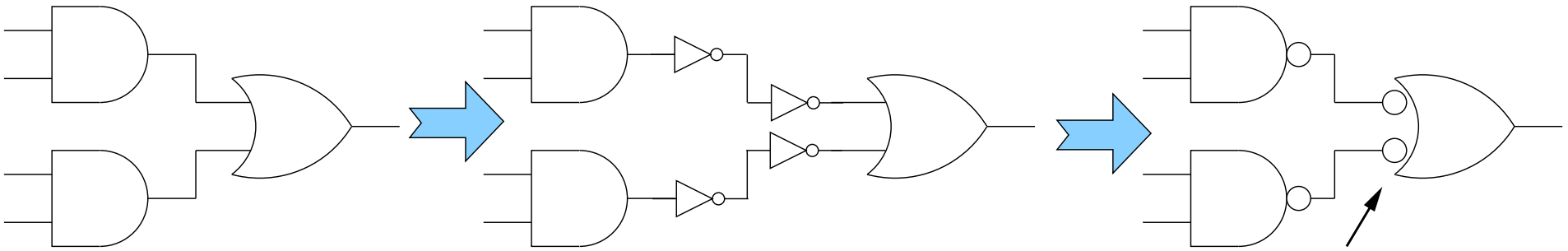
NAND/NAND and NOR/NOR Logic

AND/OR to NAND/NAND

Algebra-based:

$$\begin{aligned} AB + CD &= (AB + CD)'' \\ &= ((AB)'(CD)')' \end{aligned}$$

Schematic-based:



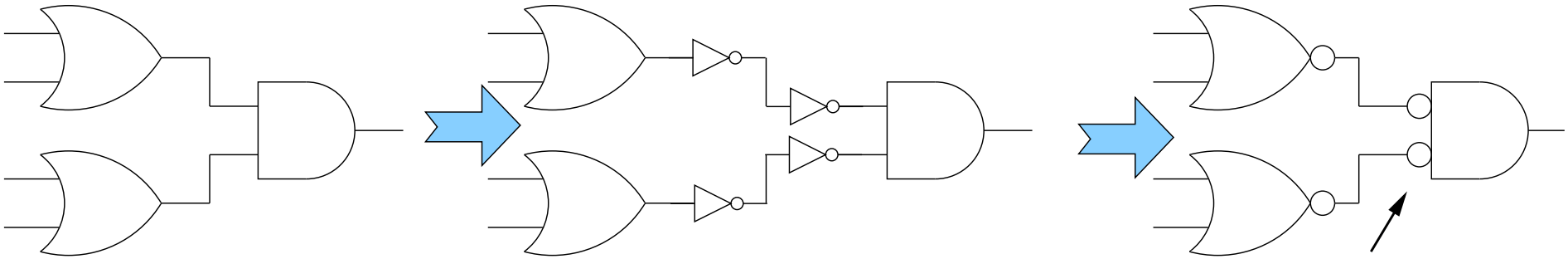
This is the preferred symbol in this context.

OR/AND to NOR/NOR

Algebra-based:

$$\begin{aligned}(A+B)(C+D) &= ((A+B)(C+D))'' \\ &= ((A+B)'+(C+D)')'\end{aligned}$$

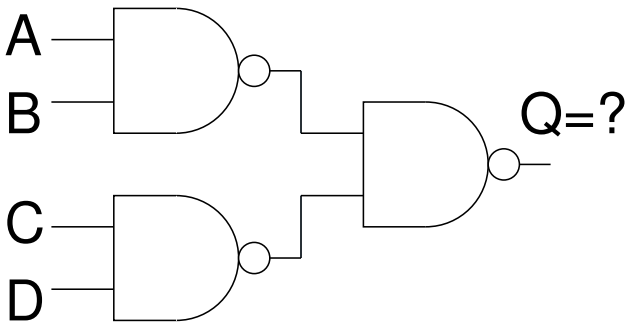
Schematic-based:

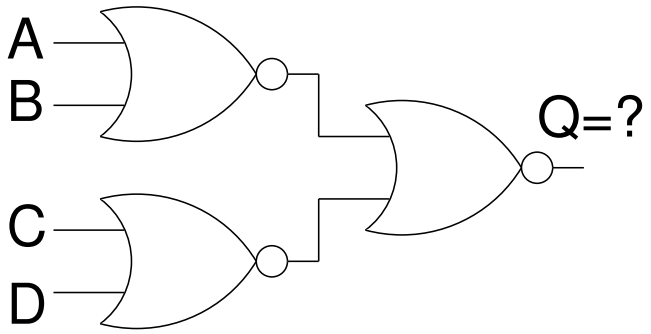


This is the preferred symbol in this context.

Alternative Gate Symbols

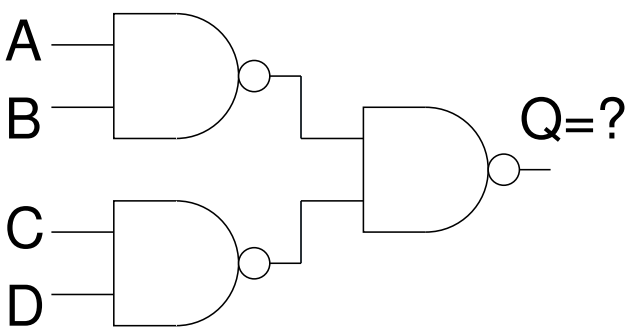
Which are easier to understand?

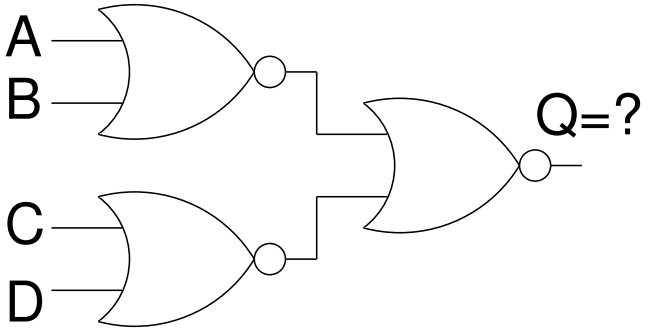


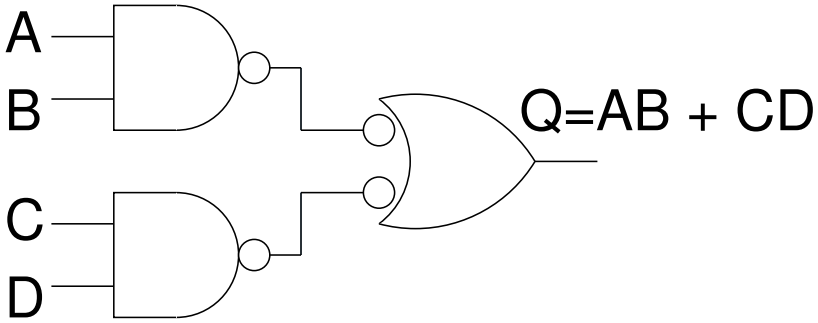


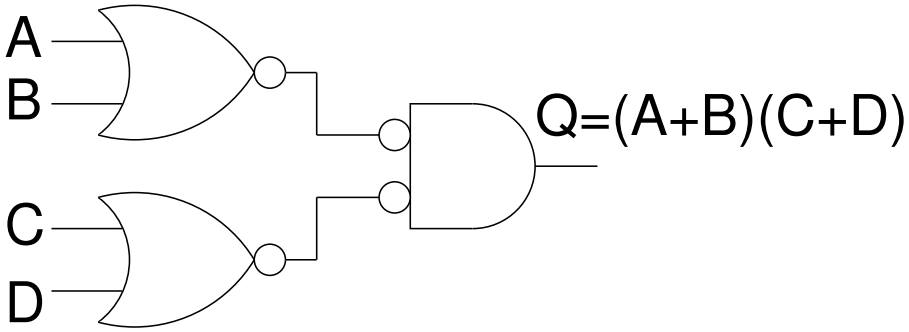
Alternative Gate Symbols

Which are easier to understand?









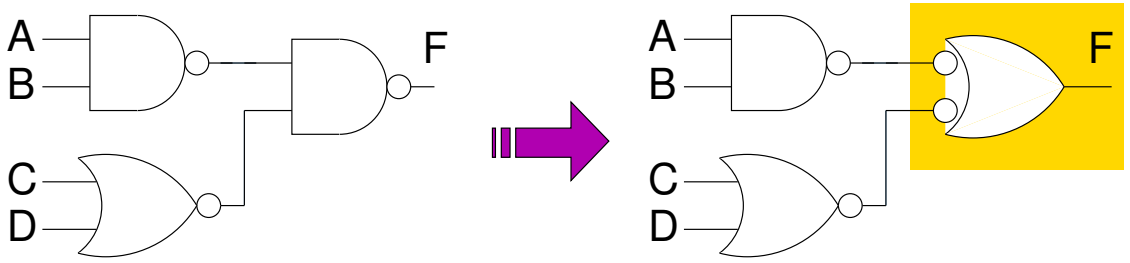
If you think of the bubbles as canceling each other out...

Bubble Matching

How to make schematics readable, understandable, maintainable, ...

Bubble Matching Rules

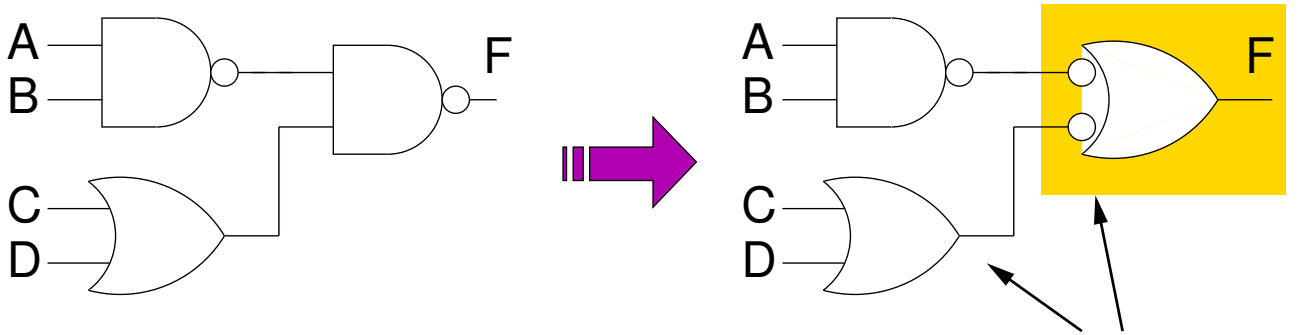
- Choose alternative symbols
- Match all interior bubbles
- More than one solution
- *Makes reading of the function trivial*



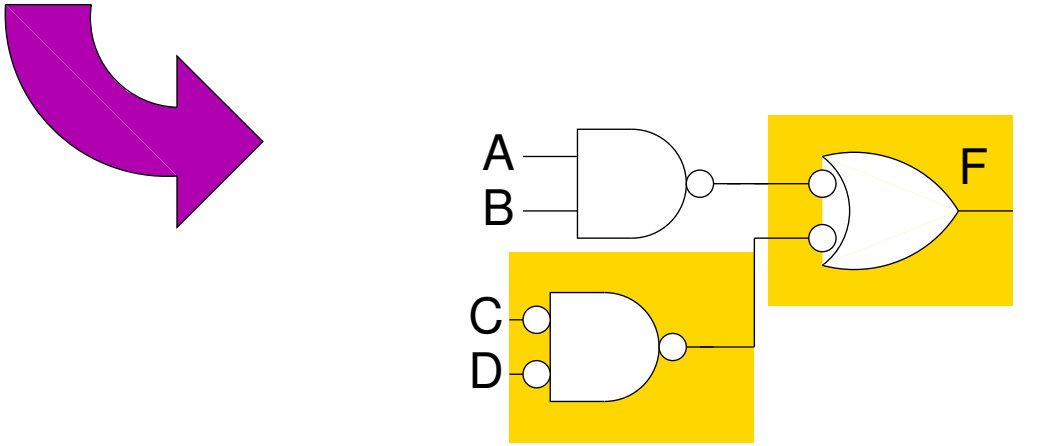
F=((AB)'(C+D)')' ???

F=AB + (C+D)=AB+C+D

More Bubble Matching

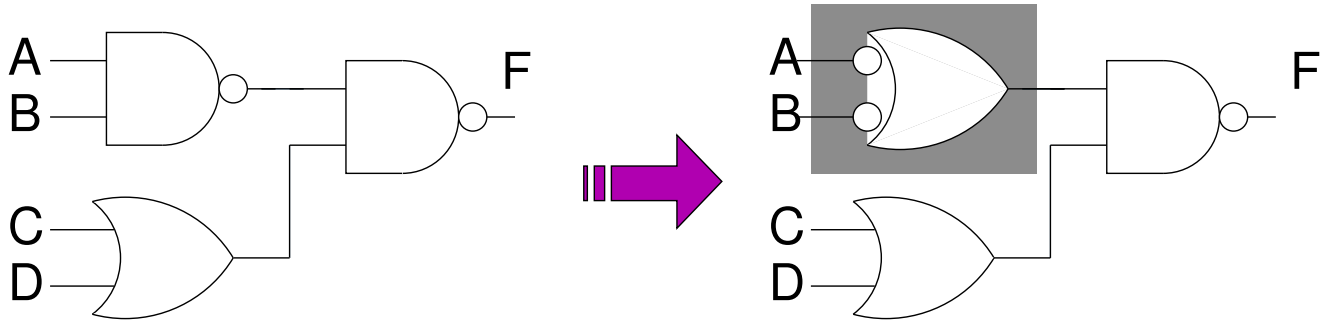


This doesn't work.
There are unmatched bubbles



This works. F = AB + C'D'

Yet More Bubble Matching



Same circuit as on previous slide ...

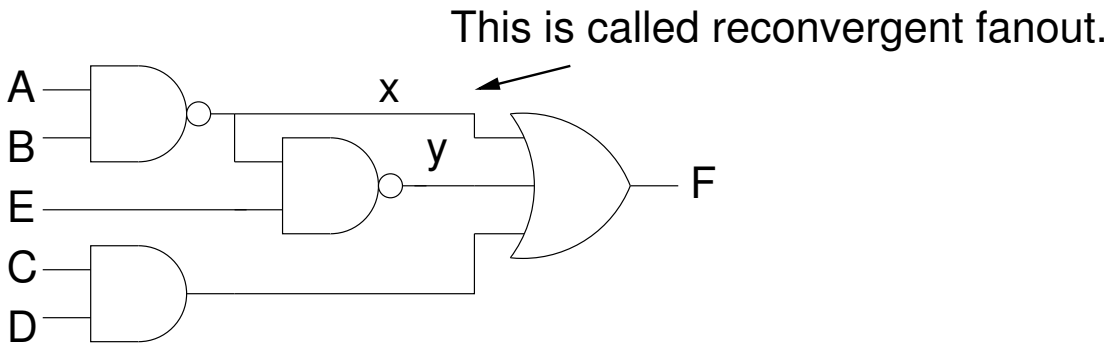
Alternative solution = convert the top-left gate.

$$F' = (A' + B')(C + D)$$
$$F = AB + C'D'$$

Same result as on previous slide.

Can Bubbles Always Be Matched?

No...



Nodes x and y both drive the final gate and so both need the same polarity (bubble or no bubble).

It is not possible to satisfy that requirement because x also drives y's input.

Can Bubbles Always Be Matched?

- Convert symbols to match bubbles
  - Two versions for each circuit
    - Inverted output
    - Non-inverted output
- Good schematic style similar to good programming style
  - Convey meaning as well as function
  - Document the design

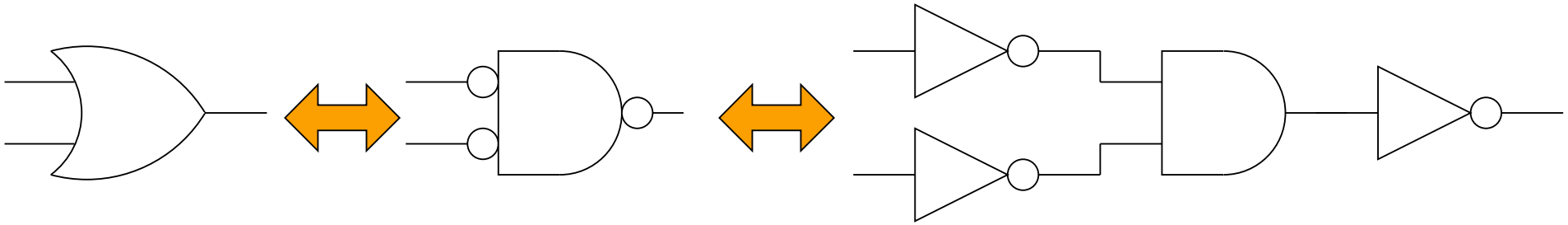
Functional Completeness

Functionally Complete

- AND, OR, and inverter are functionally complete
  - There is no truth table which cannot be implemented using AND, OR, and NOT.
  - Any set of gates which can implement AND, OR and NOT is also functionally complete.
  - Can you think of any other possible sets ???

Functionally Complete

- Is the set (AND, NOT) functionally complete?
- If I could just build an OR gate ...



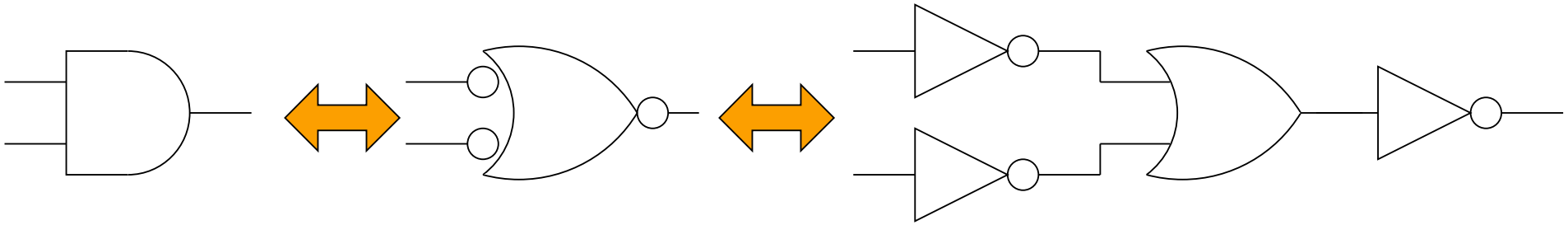
Success!

or...

$$X + Y = (X'Y')'$$

Functionally Complete

- Is the set (OR, NOT) functionally complete?
- If I could just build an AND gate ...



Success!

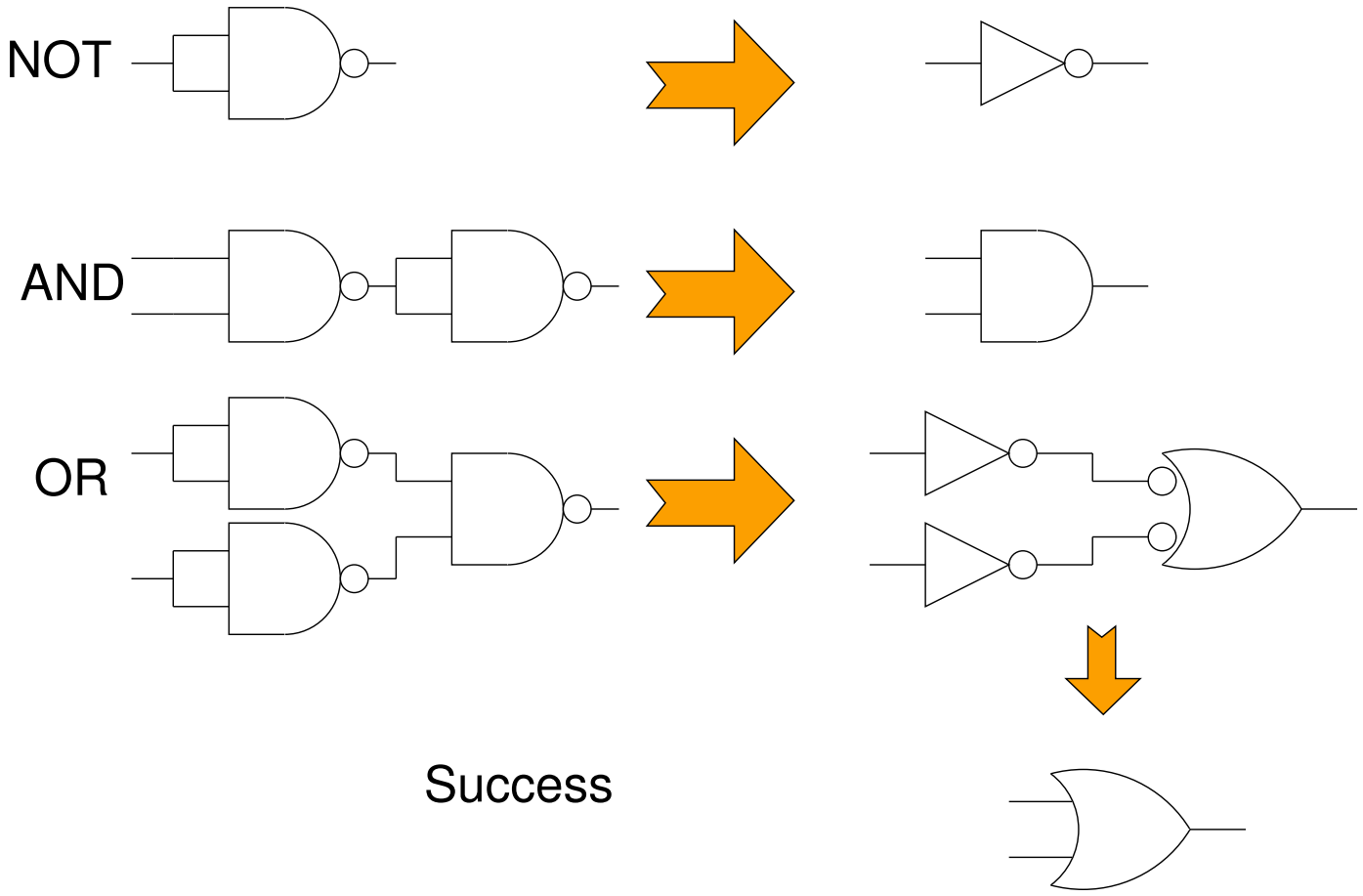
or...

XY = (X'+ Y')'

Functionally Complete

- Is the set {AND, OR} functionally complete?
- No, you cannot make a NOT from just AND and OR.

How about NAND Only?



NOR alone is also functionally complete.

Dueling Duals!

What is a Dual

Duality:

Given a logic expression, its dual is obtained by replacing all $+$ operations with $\bullet$ operations and vice versa, and by replacing all 0s with 1s and vice versa.

The dual of any true statement is also a true statement.

For example:

$$X + (X \bullet Y) = X \iff X \bullet (X + Y) = X$$

What good are duals?

1. You can more easily remember some of the boolean algebra rules.

$X + 0 = X$	$X \bullet 1 = X$
$X + 1 = 1$	$X \bullet 0 = 0$
$X + X = X$	$X \bullet X = X$
$(X')' = X$	
$X + X' = 1$	$X \bullet X' = 0$

2. Making a Dual is the same as applying DeMorgan's Theorem. So, if you have an equation that is true, its dual will also be true:

$$(X \bullet Y)' = X' + Y' \iff (X + Y)' = X' \bullet Y'$$

Dual Caveats

You cannot:

- Make a dual of part of an equation
- or just half.

It does not say that the dual of half of the equation will still equal the rest. It just says that the dual of the whole thing will still be *true*.