Karnaugh-Maps

September 14, 2006

What are Karnaugh Maps?

A simpler way to handle most (but not all) jobs of **manipulating** logic functions.

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A simpler way to handle most (but not all) jobs of **manipulating** logic functions.

Hooray!!

Karnaugh Map Advantages

- Can be completed more systematically
- Much simpler to find minimum solutions
- Easier to see what is happening (graphical)
- Almost always use instead of boolean minimization...

Truth Table Adjacencies

$F = A'$

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

These are adjacent in a gray code sense – they differ by only one bit.
We can apply $XY + XY' = X$
$A'B' + A'B = A'(B'+B) = A'(1)$
$= A'$

$F = B$

A	B	F
0	0	0
0	1	1
1	0	0
1	1	1

Same idea:
$A'B + AB = B$

Truth Table Adjacencies

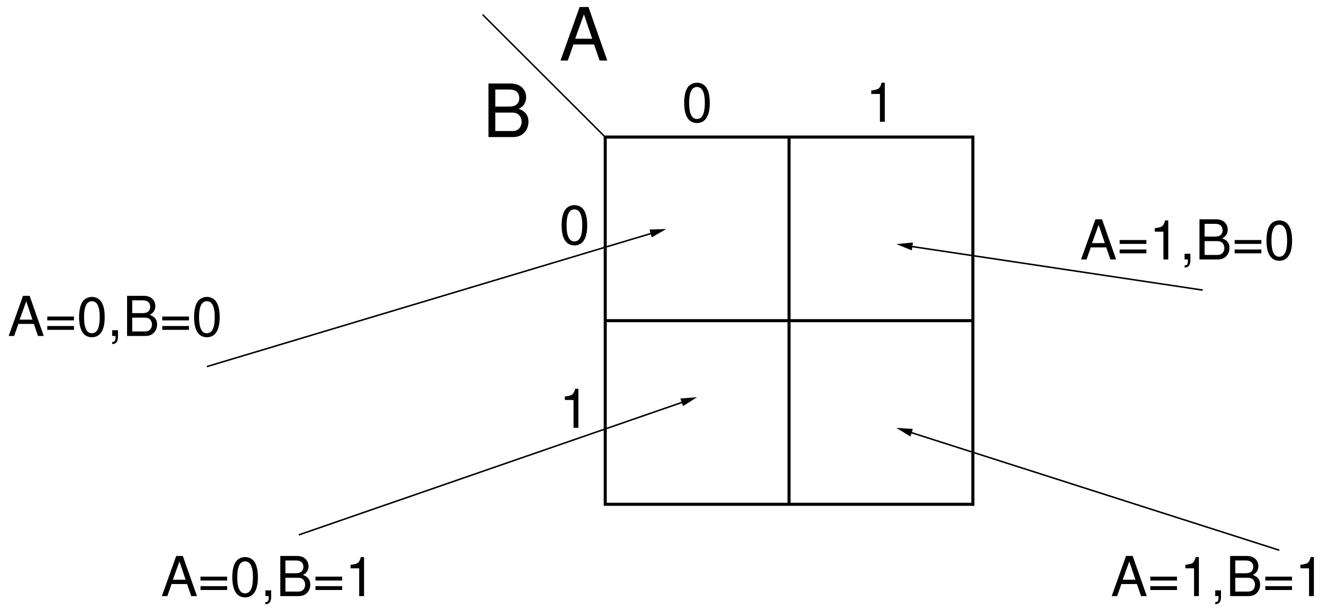
Key Idea:

Gray code adjacency allows use of simplification theorem
(e.g., XY + XY' = X).

Problem:

Physical adjacency in truth table is **not equal** to gray code adjacency.

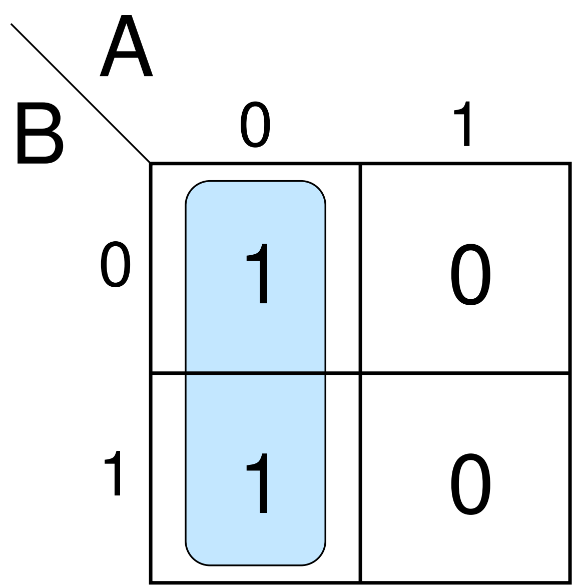
Two variable Karnaugh Map



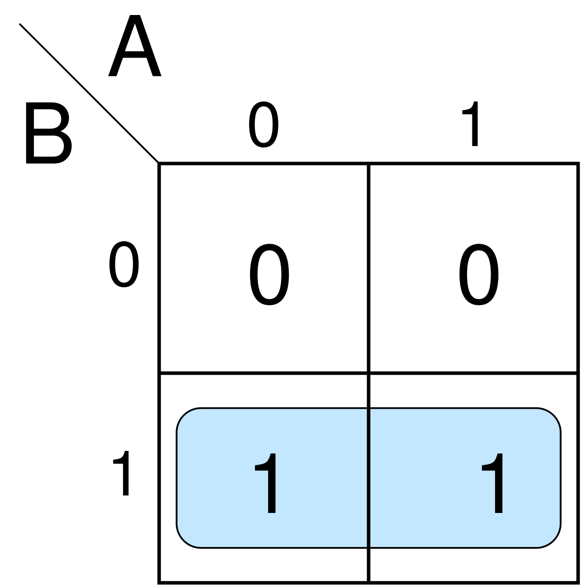
A different way to draw a truth table by folding it.

K-Map

Physical adjacency **does** imply Gray code adjacency.



$F = A'B' + A'B = A'$



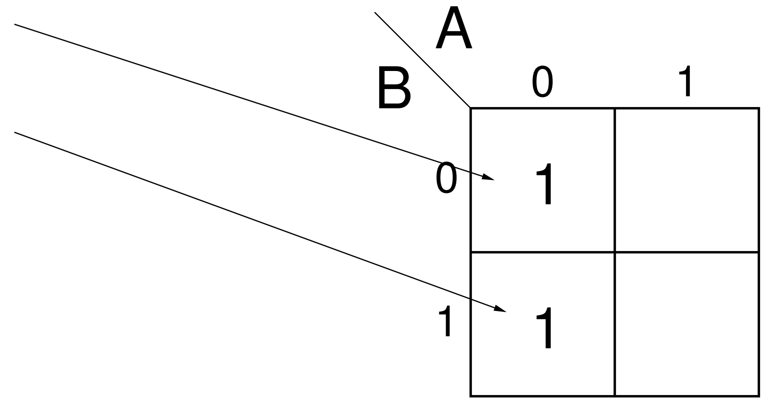
$F = A'B + AB = B$

Two Variable Karnaugh Map

A	B	F
0	0	0
0	1	0
1	0	1
1	1	1

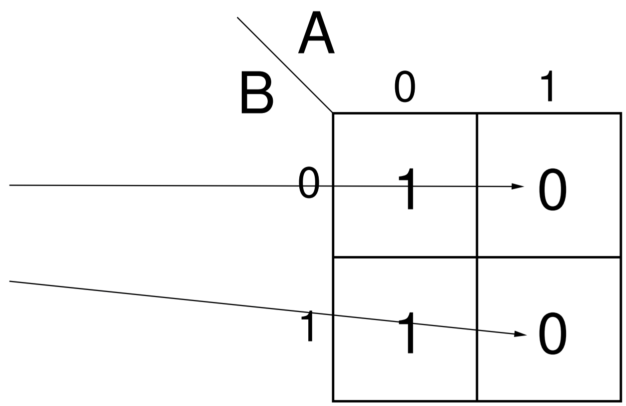
Two Variable Karnaugh Map

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0



Two Variable Karnaugh Map

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0



Two Variable Karnaugh Map

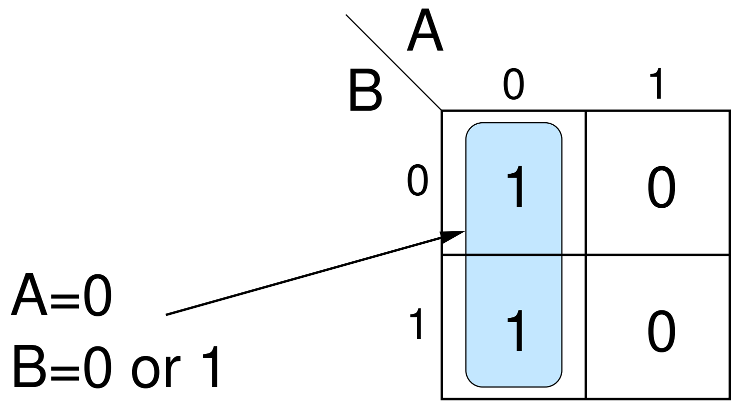
A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

B \ A	0	1
0	1	0
1	1	0

F = A'B'+A'B = A'

Two Variable Karnaugh Map

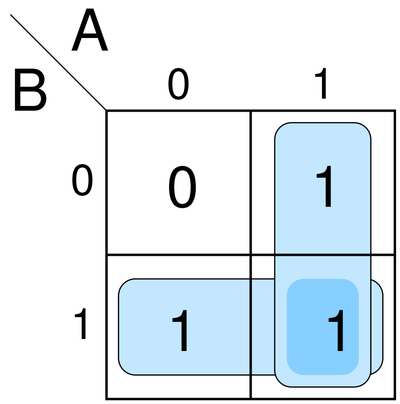
A	B	F
0	0	1
0	1	1
1	0	0
1	1	0



$$F = A'B' + A'B = A'$$

Another Two Variable Karnaugh Map

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

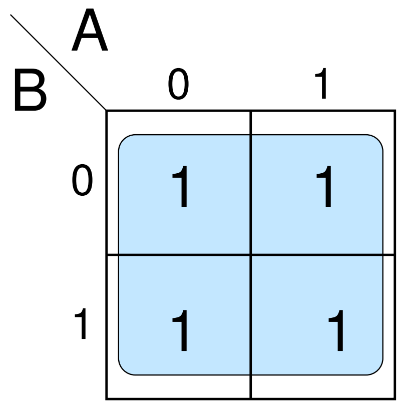


F = A'B + AB'+ AB = A+B

Each '1' must be covered at least once.

Yet Another Two Variable Karnaugh Map

A	B	F
0	0	1
0	1	1
1	0	1
1	1	1



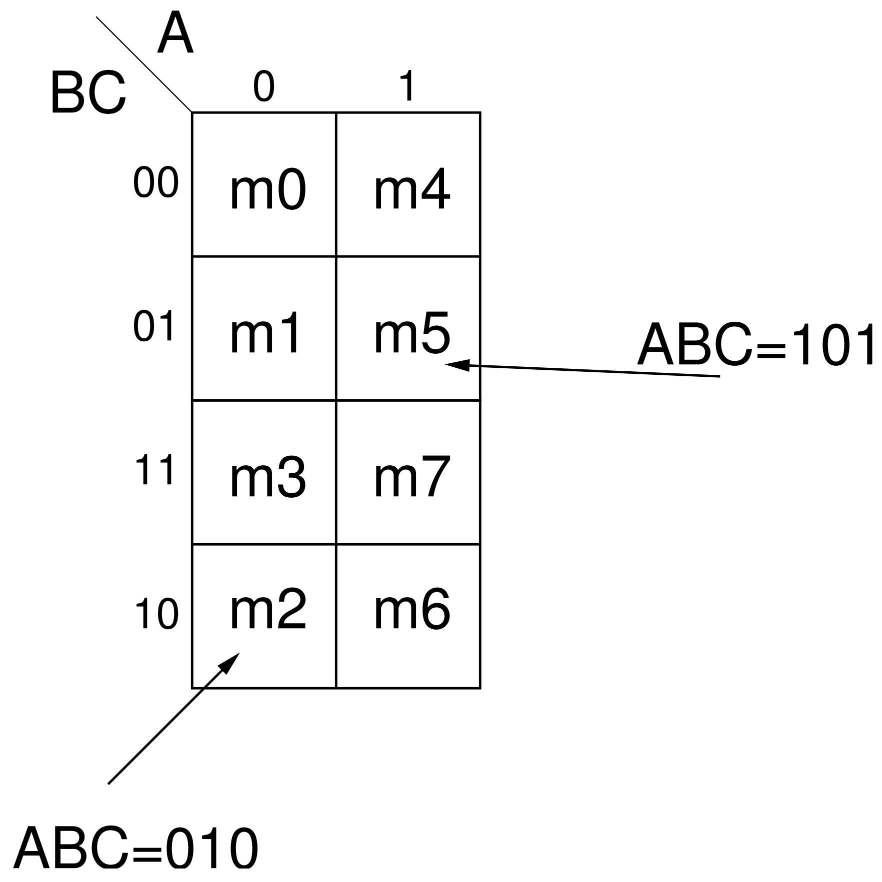
$F = A'B' + A'B + AB' + AB = 1$

Groups of more than two 1's can be combined as well.

Three Variable Karnaugh Map Showing Minterm Locations

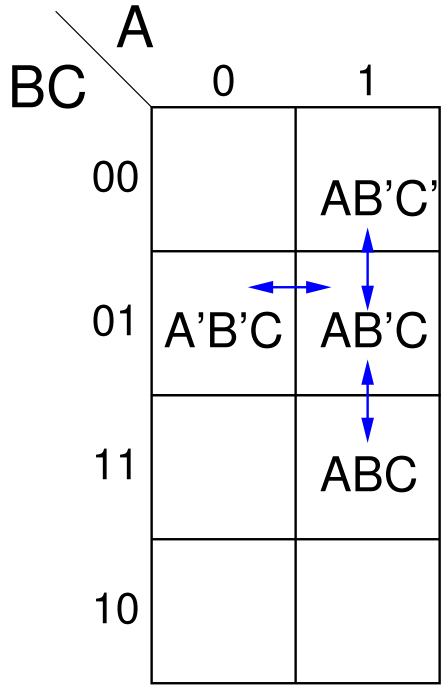
Note the order of the B C Variables:

B	C
0	0
0	1
1	1
1	0



Adjacencies

Adjacent squares differ by exactly one variable.



There is wrap-around:
top and bottom rows are adjacent.

Truth Table to Karnaugh Map

A	B	C	F	
0	0	0	0	0
0	0	1	0	1
0	1	0	1	2
0	1	1	1	3
1	0	0	0	4
1	0	1	1	5
1	1	0	0	6
1	1	1	1	7

BC \ A	0	1
00	0 (0)	0 (4)
01	0 (1)	1 (5)
11	1 (3)	1 (7)
10	1 (2)	0 (6)

Solution Example

$A'BC+A'BC' = A'B$

$\rightarrow$

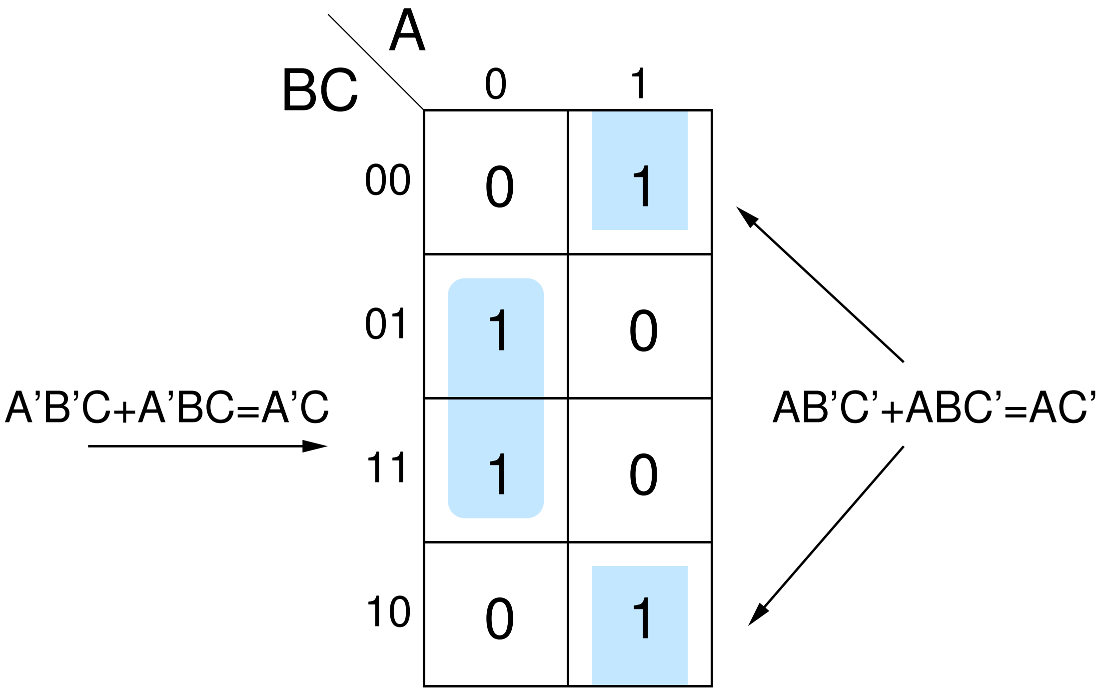
BC \ A	0	1
00	0	0
01	0	1
11	1	1
10	1	0

$AB'C+ABC = AC$

$\leftarrow$

$$F = A'B + AC$$

Another Example



$$F = A'C + AC' = A \oplus C$$

Minterm Expansion to K-Map

$$F = \sum m(1, 3, 4, 6)$$

BC \ A	0	1
00	m0	m4
01	m1	m5
11	m3	m7
10	m2	m6

BC \ A	0	1
00	0	1
01	1	0
11	1	0
10	0	1

Minterms are the 1's, everything else is 0.

Maxterm Expansion to K-Map

$$F = \prod M(0, 2, 5, 7)$$

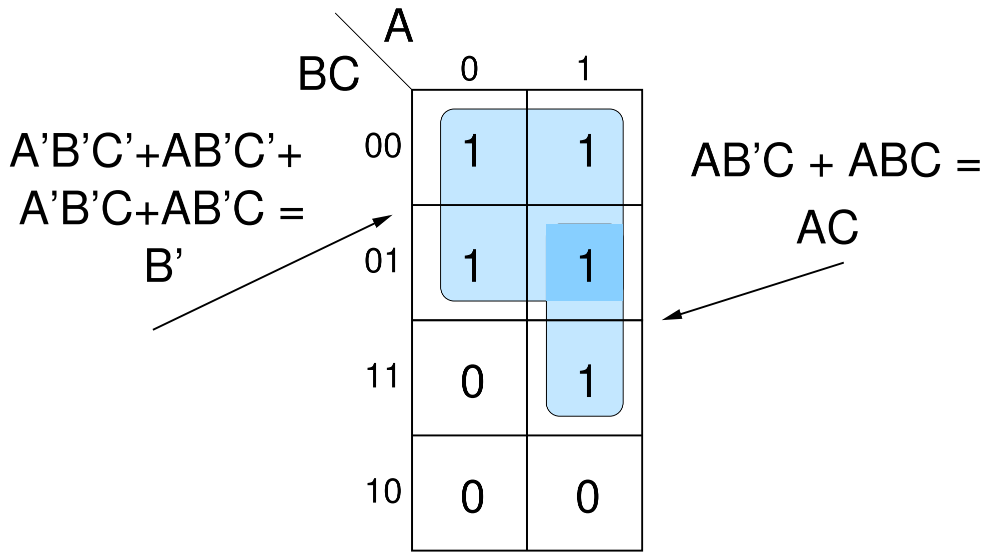
BC \ A	0	1
00	M0	M4
01	M1	M5
11	M3	M7
10	M2	M6

BC \ A	0	1
00	0	1
01	1	0
11	1	0
10	0	1

Maxterms are the 0's, everything else is 1.

Yet Another Example

2^n 1's can be circled at a time. This includes 1,2,4,8 ... .
3,... not OK.

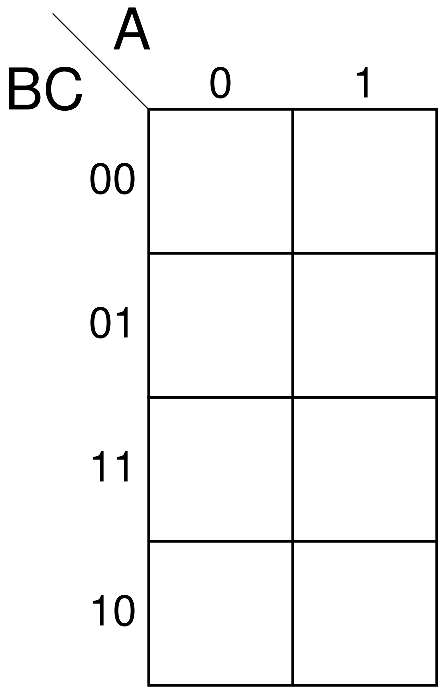


F=B'+AC

The larger the group of 1's – the simpler the resulting product term.

Boolean Algebra to Karnaugh Map

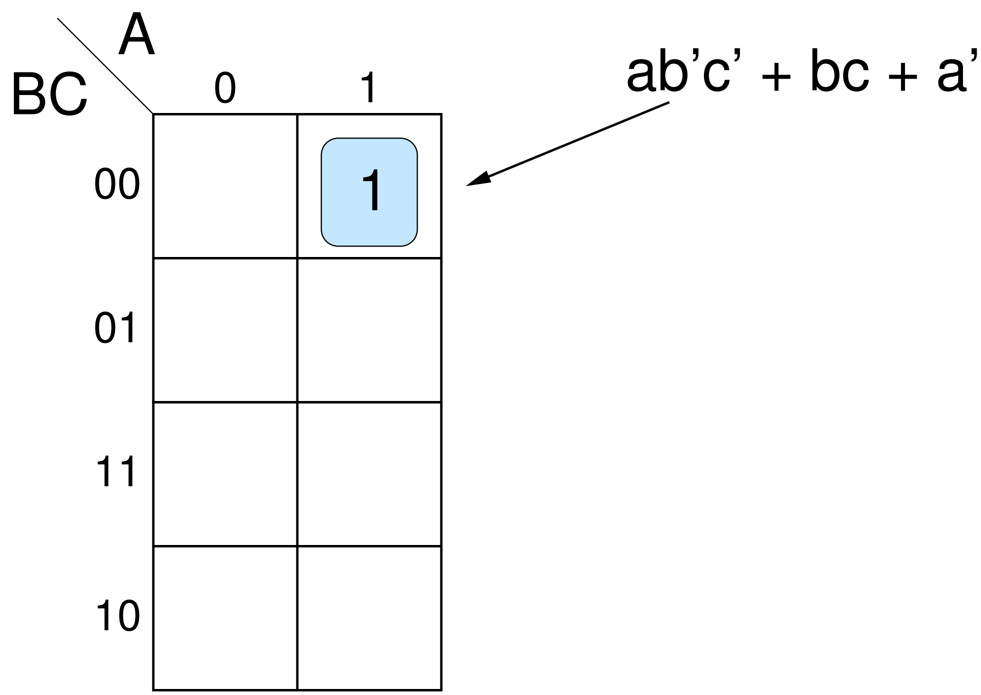
Plot:



ab'c' + bc + a'

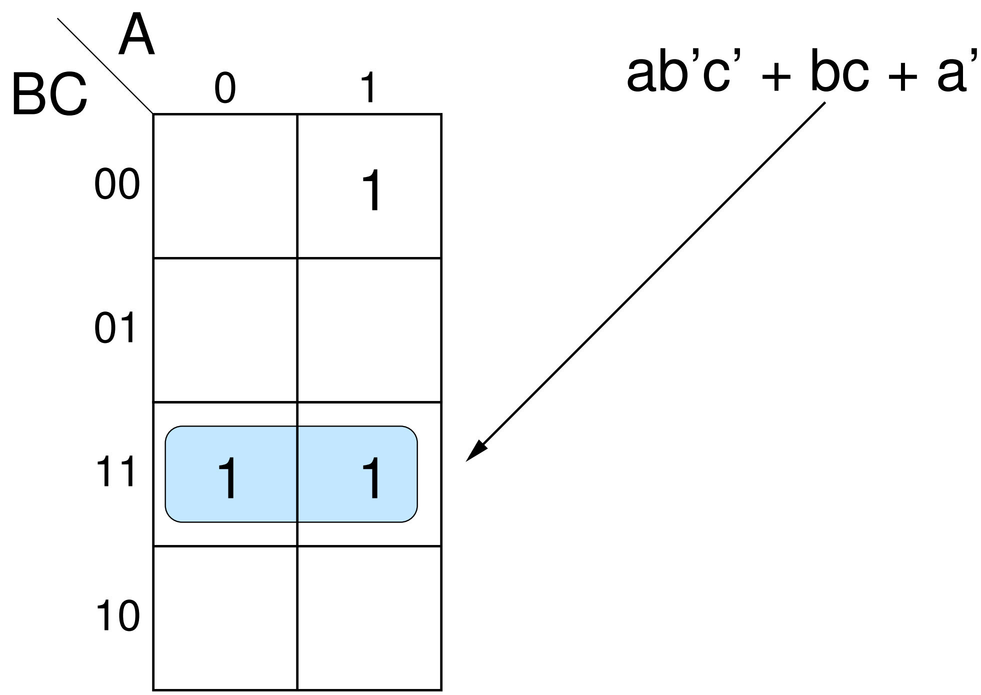
Boolean Algebra to Karnaugh Map

Plot:



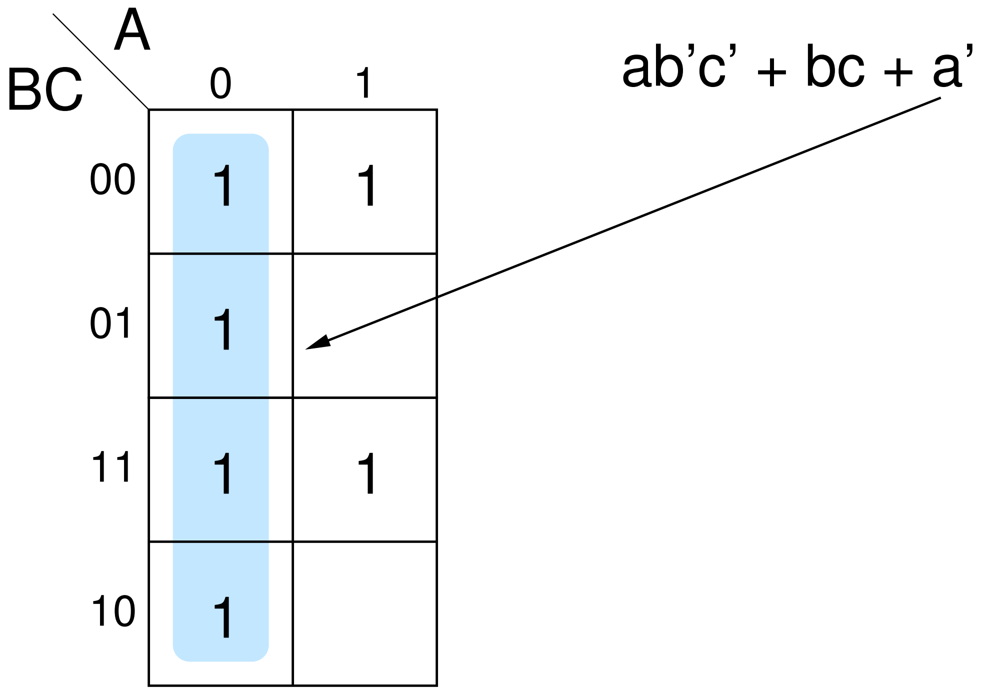
Boolean Algebra to Karnaugh Map

Plot:



Boolean Algebra to Karnaugh Map

Plot:



Boolean Algebra to Karnaugh Map

Plot:

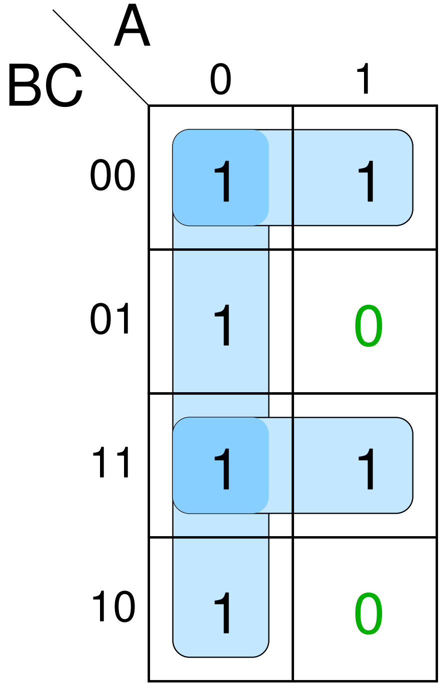
BC \ A	0	1
00	1	1
01	1	0
11	1	1
10	1	0

ab'c' + bc + a'

Remaining spaces are '0'.

Boolean Algebra to Karnaugh Map

$$F = B'C' + BC + A'$$



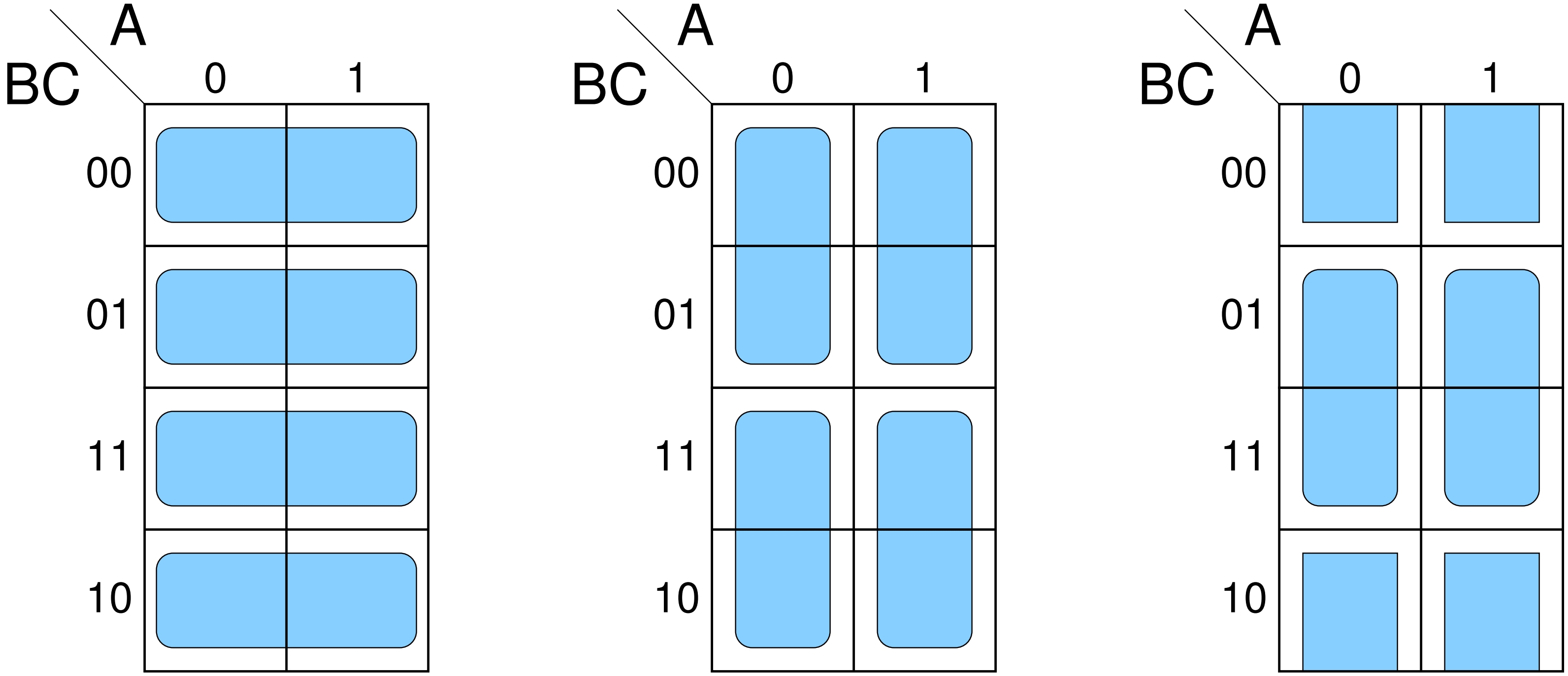
ab'c' + bc + a'

This is a simpler equation than we started with. How did we get here?

Mapping Sum of Product Terms

The three variable map has 12 possible groups of 2 spaces.

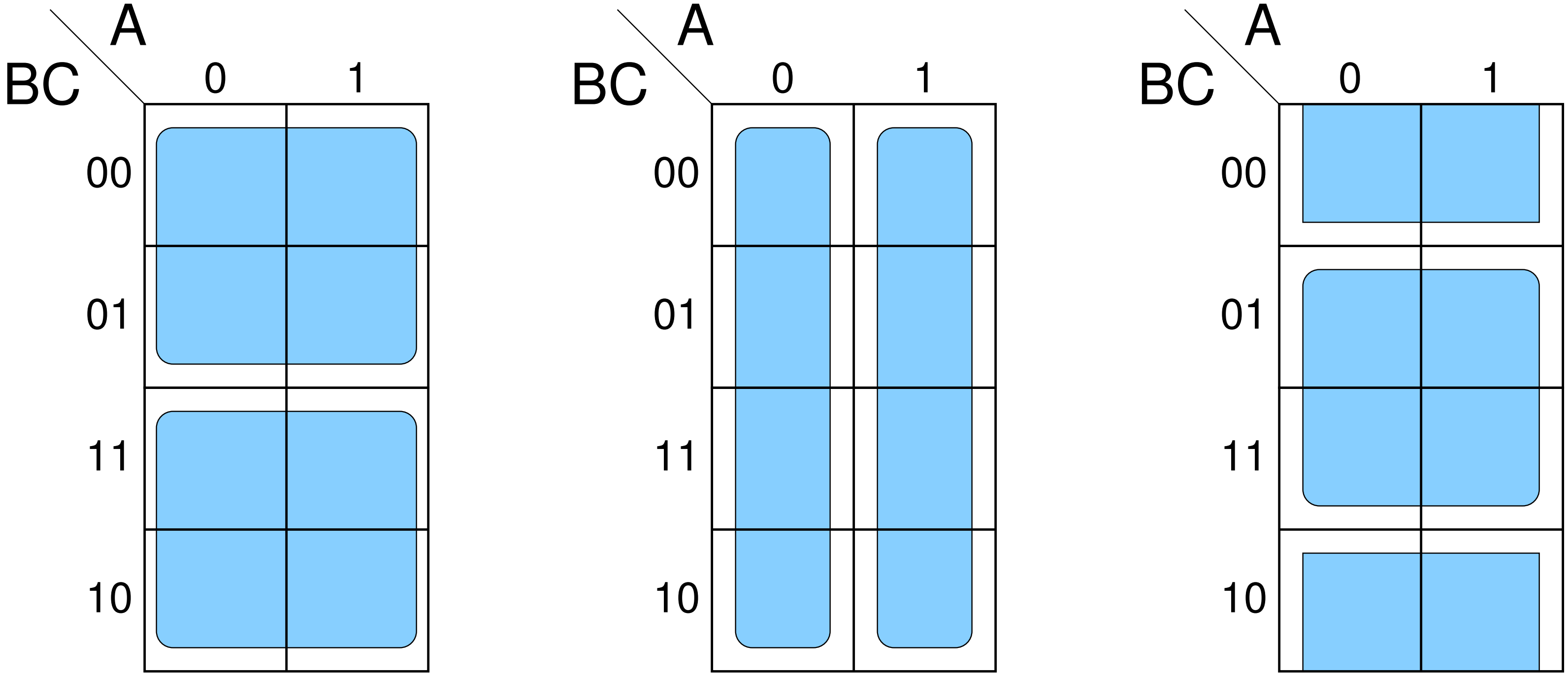
These become terms with 2 literals.



Mapping Sum of Product Terms

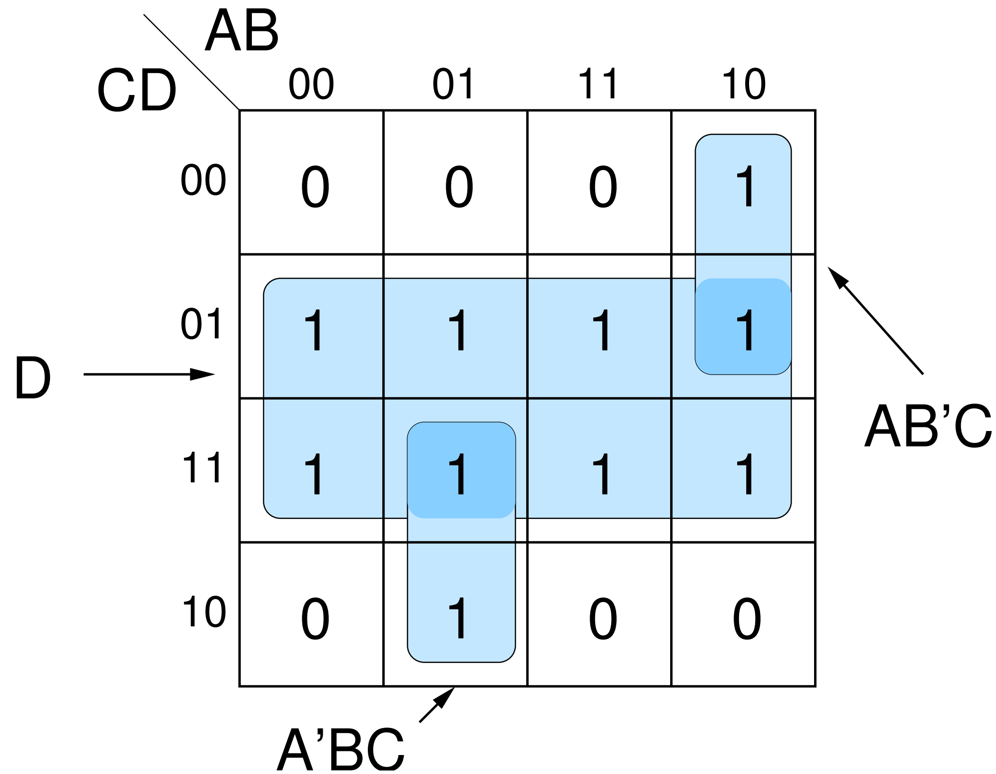
The three variable map has 6 possible groups of 4 spaces.

These become terms with 1 literal.



Four Variable Karnaugh Map

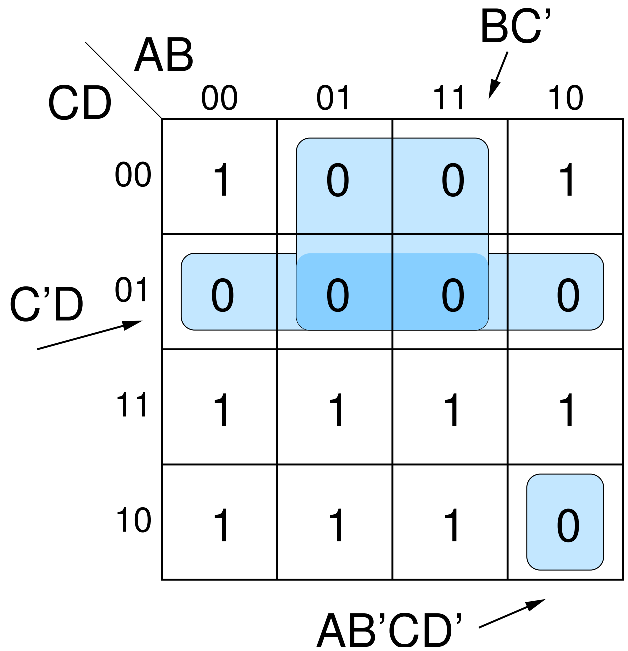
CD \ AB	00	01	11	10
00	m0	m4	m12	m8
01	m1	m5	m13	m9
11	m3	m7	m15	m11
10	m2	m6	m14	m10



F = A'BC + AB'C + D

Note the row and column numbering. This is required for adjacency.

Find a POS Solution



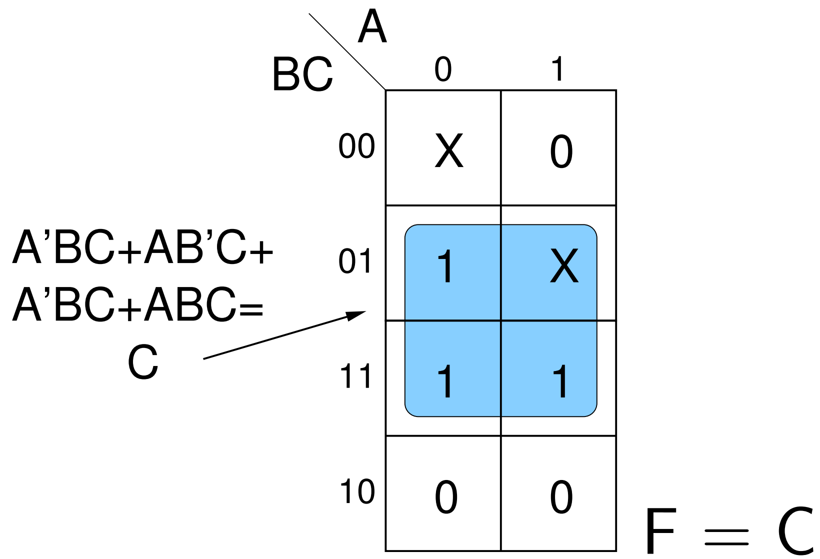
$$F' = C'D + BC' + AB'CD'$$

$$F = (C+D')(B'+C)(A'+B+C'+D)$$

Find solutions to groups of 0's to find F'. Invert to get F using DeMorgan's.

Dealing With Don't Cares

$$F = \sum m(1,3,7) + \sum d(0,5)$$



Circle the x's that help get bigger groups of 1's (or 0's if POS).
Don't circle the x's that don't help.

Minimal K-Map Solutions

Some Terminology

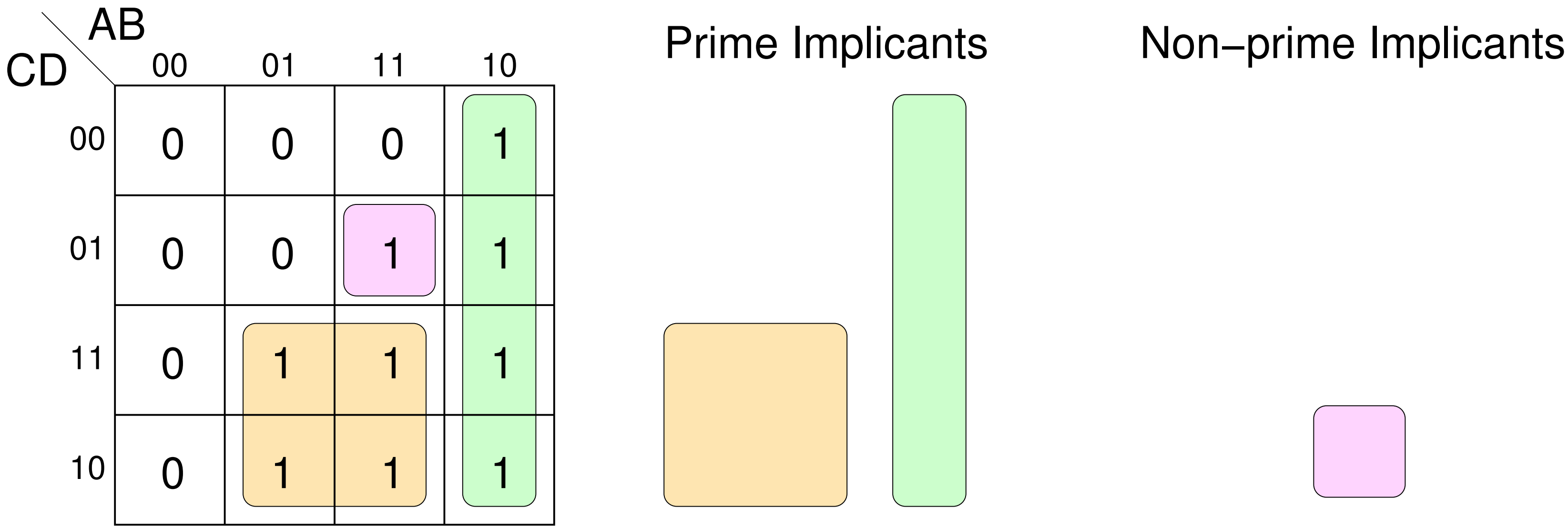
and

An Algorithm to Find Them

Prime Implicants

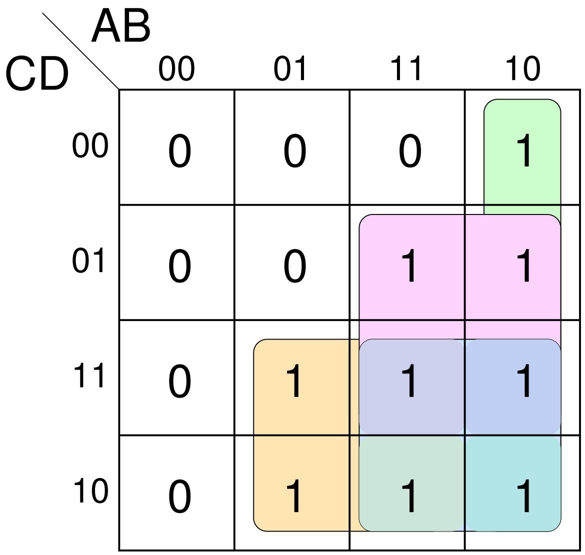
- A group of 1's which are adjacent and can be combined on a Karnaugh Map is called an implicant.
- The biggest group of 1's which can be circled to cover a 1 is called a *prime implicant*.
  - They are the only implicants we care about.

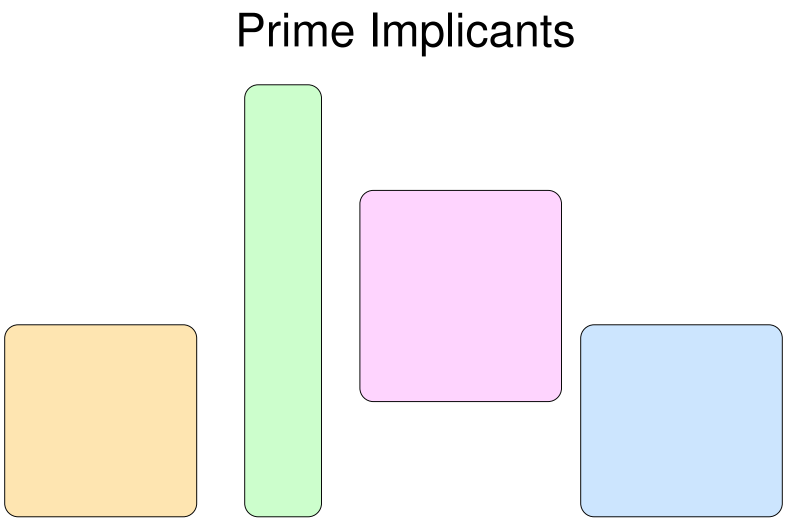
Prime Implicants



Are there any additional prime implicants in the map that are not shown above?

All the Prime Implicants





When looking for a minimal solution – **only** circle prime implicants...

A minimal solution will **never** contain non-prime implicants

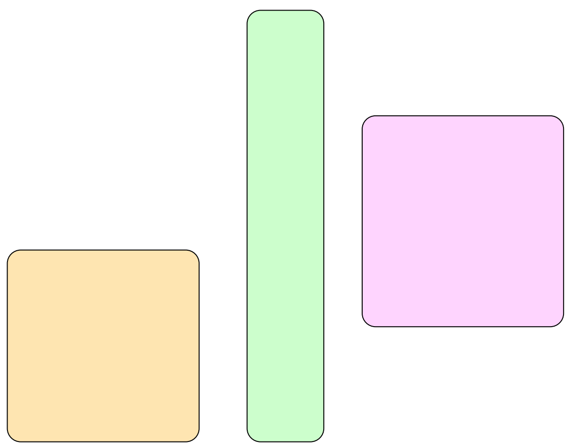
Essential Prime Implicants

CD \ AB	00	01	11	10
00	0	0	0	1
01	0	0	1	1
11	0	1	1	1
10	0	1	1	1

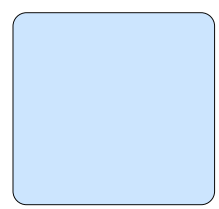
Not all prime implicants are required ...

A prime implicant which is the only cover of some 1's is **essential** – a minimal solution requires it.

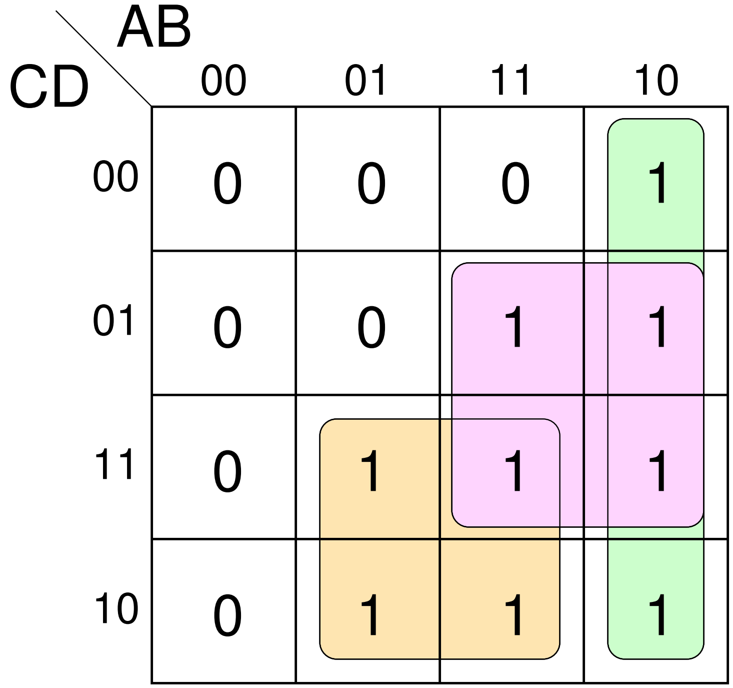
Essential Prime Implicants



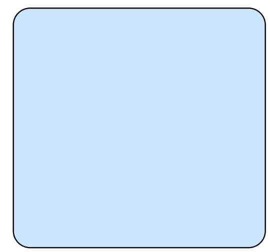
Non–essential Prime Implicants



A Minimal Solution Example



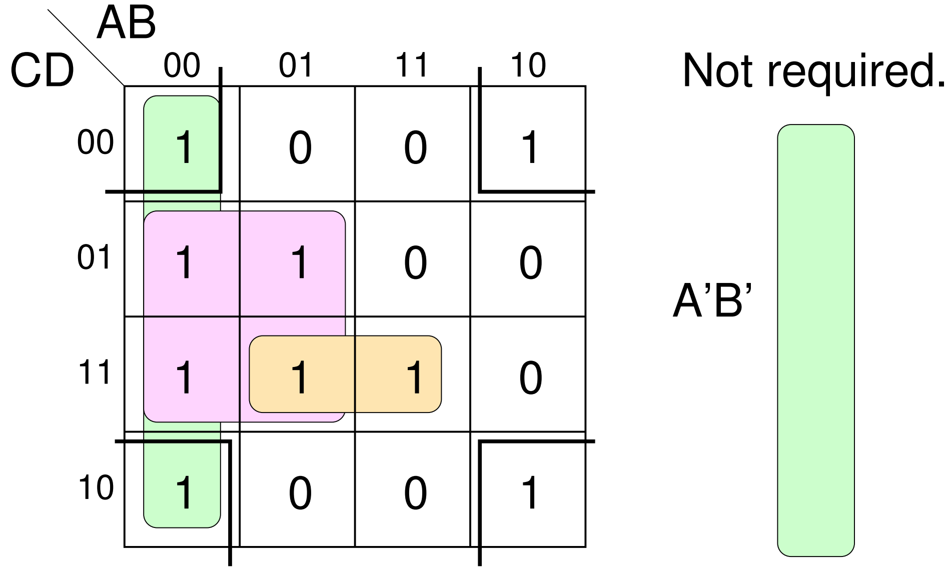
Not required.



F = AB’+BC+AD

Minimum

Another Example



F = A'D+BCD+B'D'

Minimum

Every one one of F's locations is covered by multiple implicants.

After choosing essentials, everything is covered...

Finding the Minimum Sum of Products

1. Find each *essential* prime implicant and include it in the solution.

2. Determine if any minterms are not yet covered.

3. Find the minimal number of *remaining* prime implicants which finish the cover.

Yet Another Example

Using non-essential primes.

Essentials:

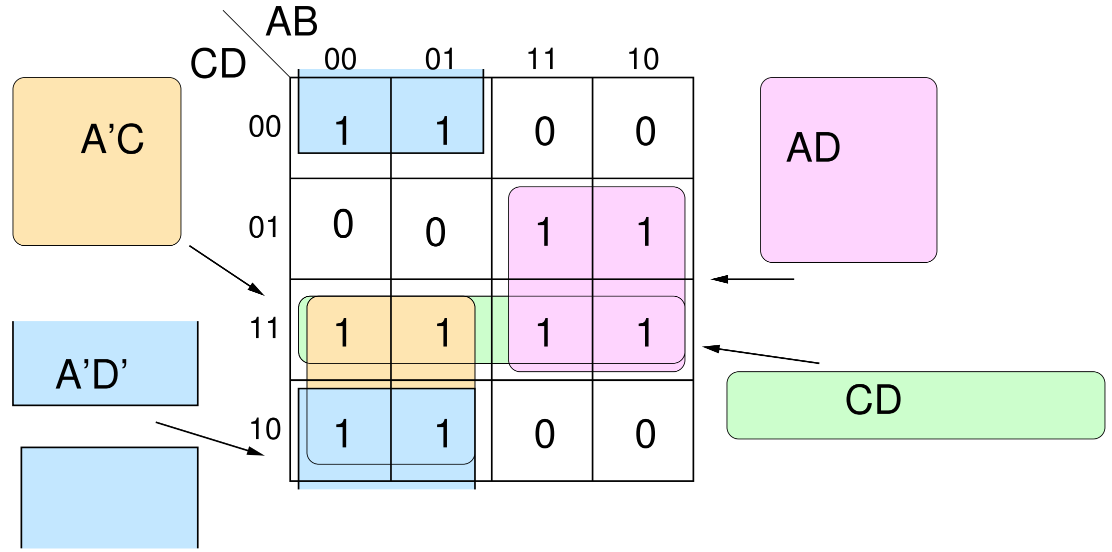
A'D' and AD

Non-essentials:

A'C and CD

Solution:

A'D'+AD+A'C
or
A'D'+AD+CD



K-Map Solution Summary

- Identify prime implicants.
- Add essentials to solution.
- Find minimum number non-essentials required to cover rest of map.

Five and Six Variable K-Maps

Five Variable Karnaugh Map

DE \ BC	00	01	11	10
00	m0	m4	m12	m8
01	m1	m5	m13	m9
11	m3	m7	m15	m11
10	m2	m6	m14	m10

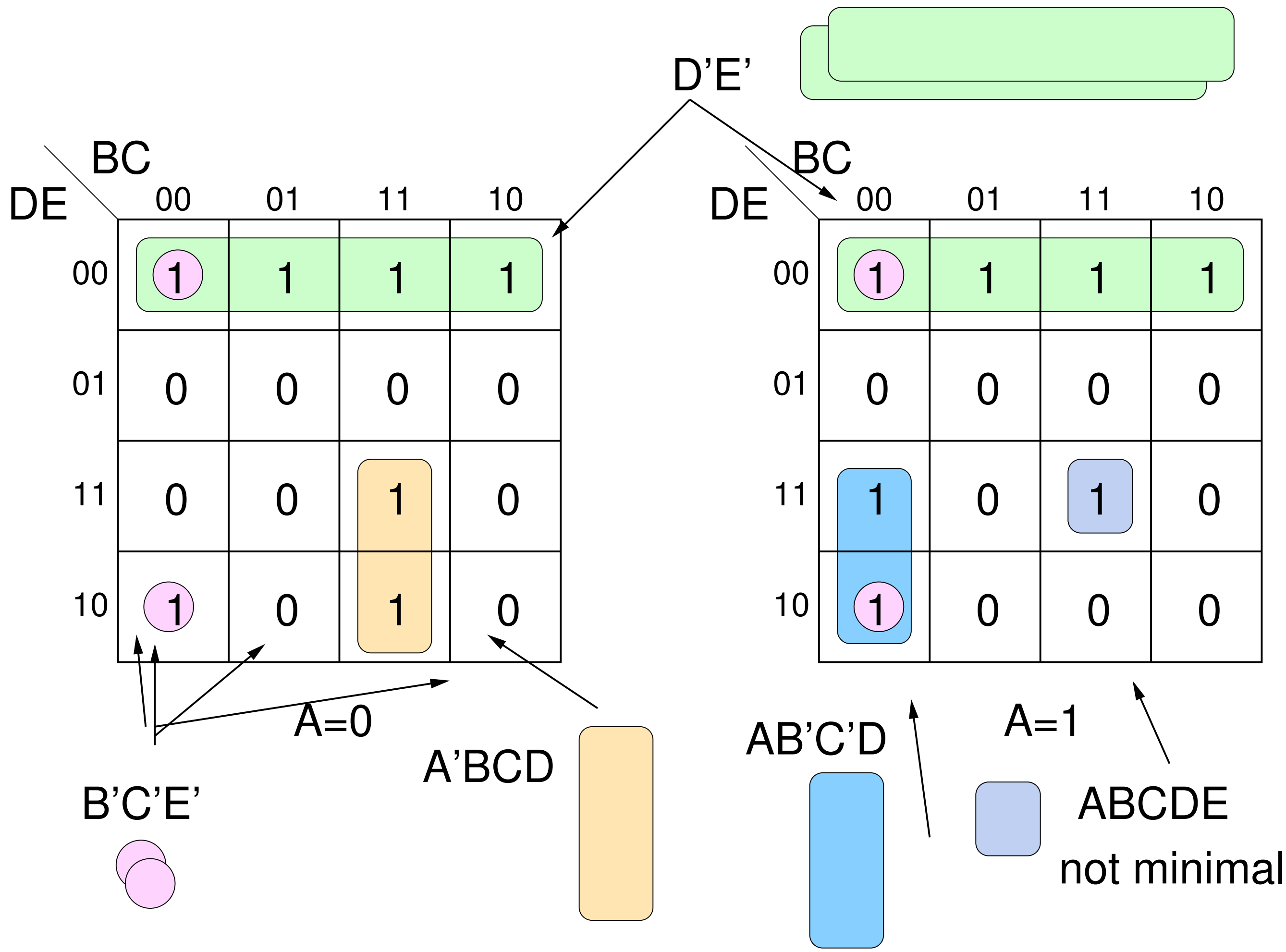
This is the A=0 plane.

DE \ BC	00	01	11	10
00	m16	m20	m28	m24
01	m17	m21	m29	m25
11	m19	m23	m31	m27
10	m18	m22	m30	m26

This is the A=1 plane.

The planes are adjacent to one another (one is above the other in 3D).

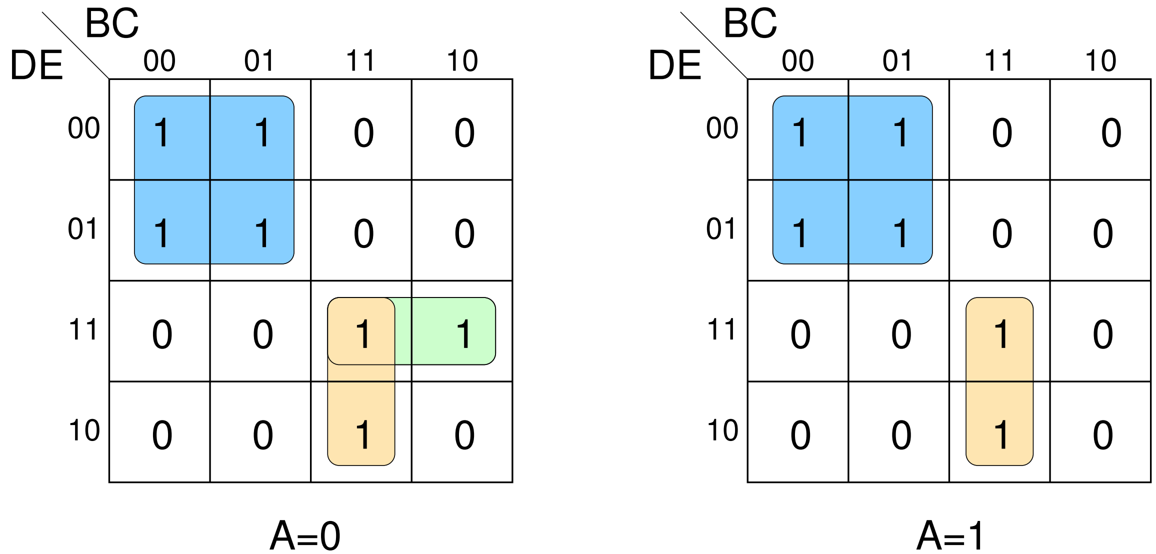
Some Implicants in a Five Variable K-Map



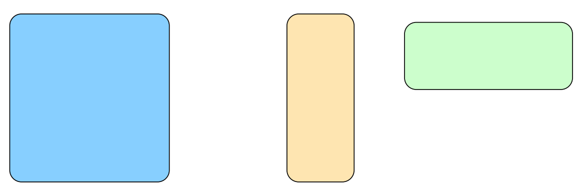
Some Implicants in a Five Variable K-Map

Find the minimum sum-of-products for:

$$F = \sum m(0, 1, 4, 5, 11, 14, 15, 16, 17, 20, 21, 30, 31)$$



F = B'D' + BCD + A'BDE



Six Variable K-Map

AB=00

EF \ CD	00	01	11	10
00	m0	m4	m12	m8
01	m1	m5	m13	m9
11	m3	m7	m15	m11
10	m2	m6	m14	m10

AB=10

EF \ CD	00	01	11	10
00	m32	m36	m44	m40
01	m33	m37	m45	m41
11	m35	m39	m47	m43
10	m34	m38	m46	m42

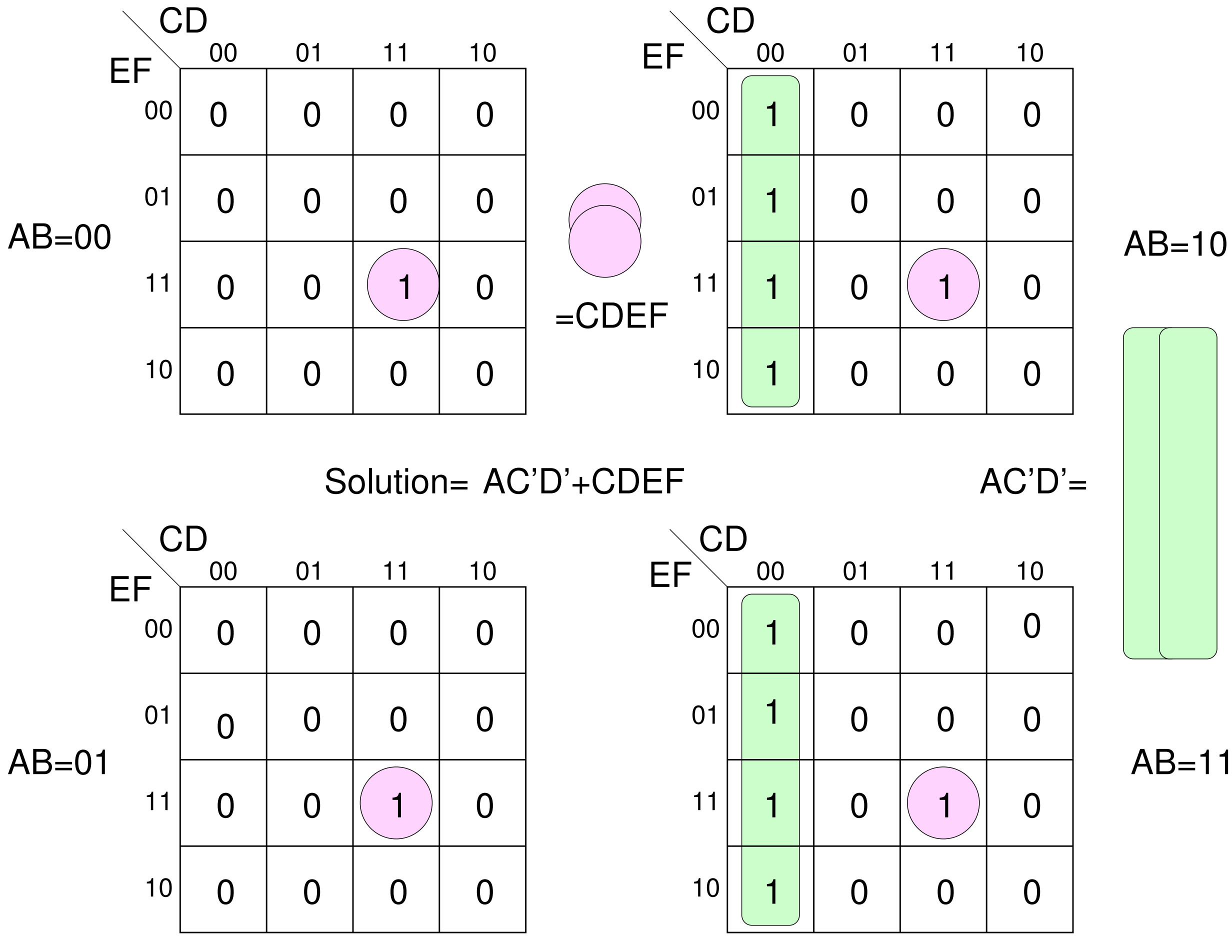
AB=01

EF \ CD	00	01	11	10
00	m16	m20	m28	m24
01	m17	m21	m29	m25
11	m19	m23	m31	m27
10	m18	m22	m30	m26

AB=11

EF \ CD	00	01	11	10
00	m48	m52	m60	m56
01	m49	m53	m61	m57
11	m51	m55	m63	m59
10	m50	m54	m62	m58

Six Variable K-Map



K-Map Summary

- A K-Map is simply a folded truth table, where physical adjacency implies logical adjacency.
- K-Maps are most commonly used hand method for logic minimization.
- K-Maps have other uses for visualizing boolean equations.