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# **Logic and Computer Design Fundamentals**

## **Adders and Multiplication**

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# Overview

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- **Iterative combinational circuits**
- **Binary adders**
  - Half and full adders
  - Ripple carry and carry lookahead adders
- **Binary subtraction**
- **Binary multiplication**
- **Other arithmetic functions**

# Enabling Function

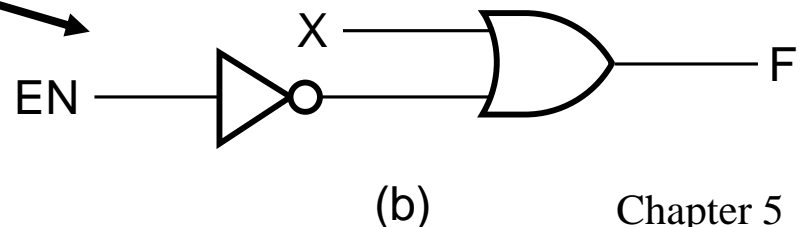
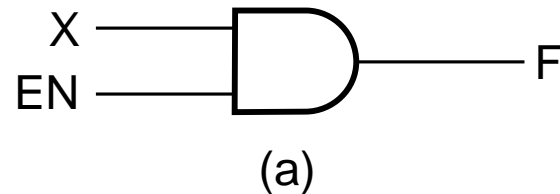
- ***Enabling*** permits an input signal to pass through to an output
- ***Disabling*** blocks an input signal from passing through to an output, replacing it with a fixed value

- The value on the output when it is disabled can be Hi-Z (as for three-state buffers and transmission gates), 0, or 1

- When disabled, 0 output

- When disabled, 1 output

- See Enabling App in text



# Decoding

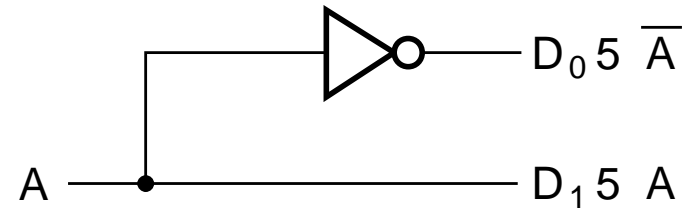
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- **Decoding - the conversion of an  $n$ -bit input code to an  $m$ -bit output code with  $n \leq m \leq 2^n$  such that each valid code word produces a unique output code**
- **Circuits that perform decoding are called *decoders***
- **Here, functional blocks for decoding are**
  - **called  $n$ -to- $m$  line decoders, where  $m \leq 2^n$ , and**
  - **generate  $2^n$  (or fewer) minterms for the  $n$  input variables**

# Decoder Examples

- 1-to-2-Line Decoder

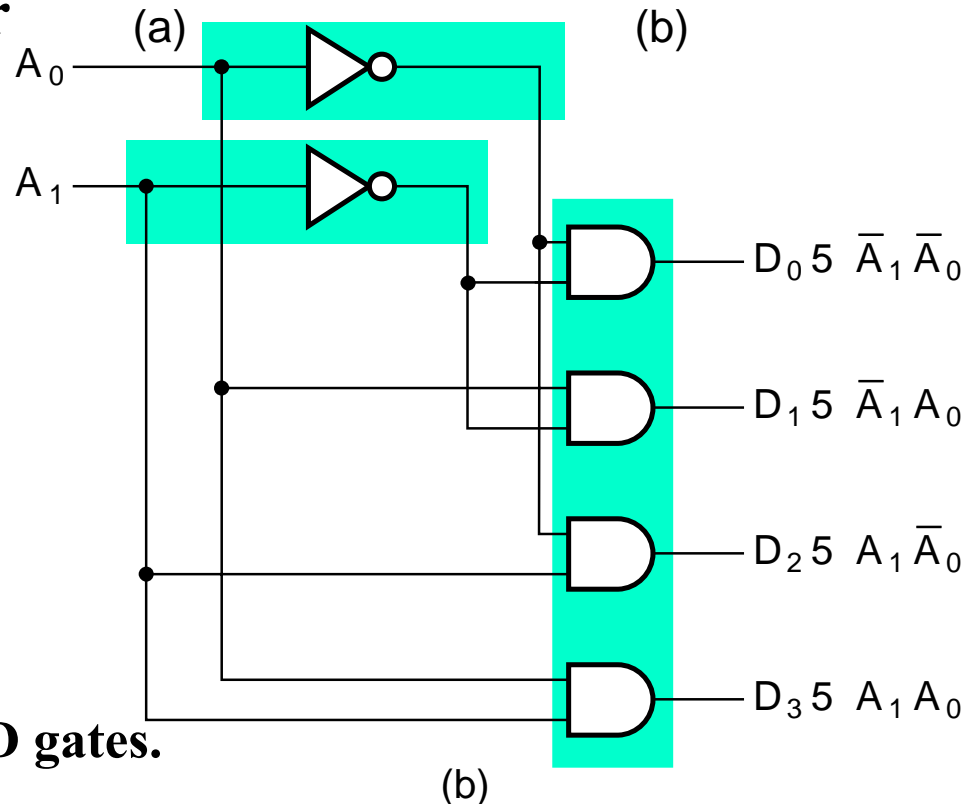
A	D <sub>0</sub>	D <sub>1</sub>
0	1	0
1	0	1



- 2-to-4-Line Decoder

A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

(a)



- Note that the 2-4-line made up of 2 1-to-2-line decoders and 4 AND gates.

# Decoder Expansion

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- General procedure given in book for any decoder with  $n$  inputs and  $2^n$  outputs.
- This procedure builds a decoder backward from the outputs.
- The output AND gates are driven by two decoders with their numbers of inputs either equal or differing by 1.
- These decoders are then designed using the same procedure until 2-to-1-line decoders are reached.
- The procedure can be modified to apply to decoders with the number of outputs  $\neq 2^n$

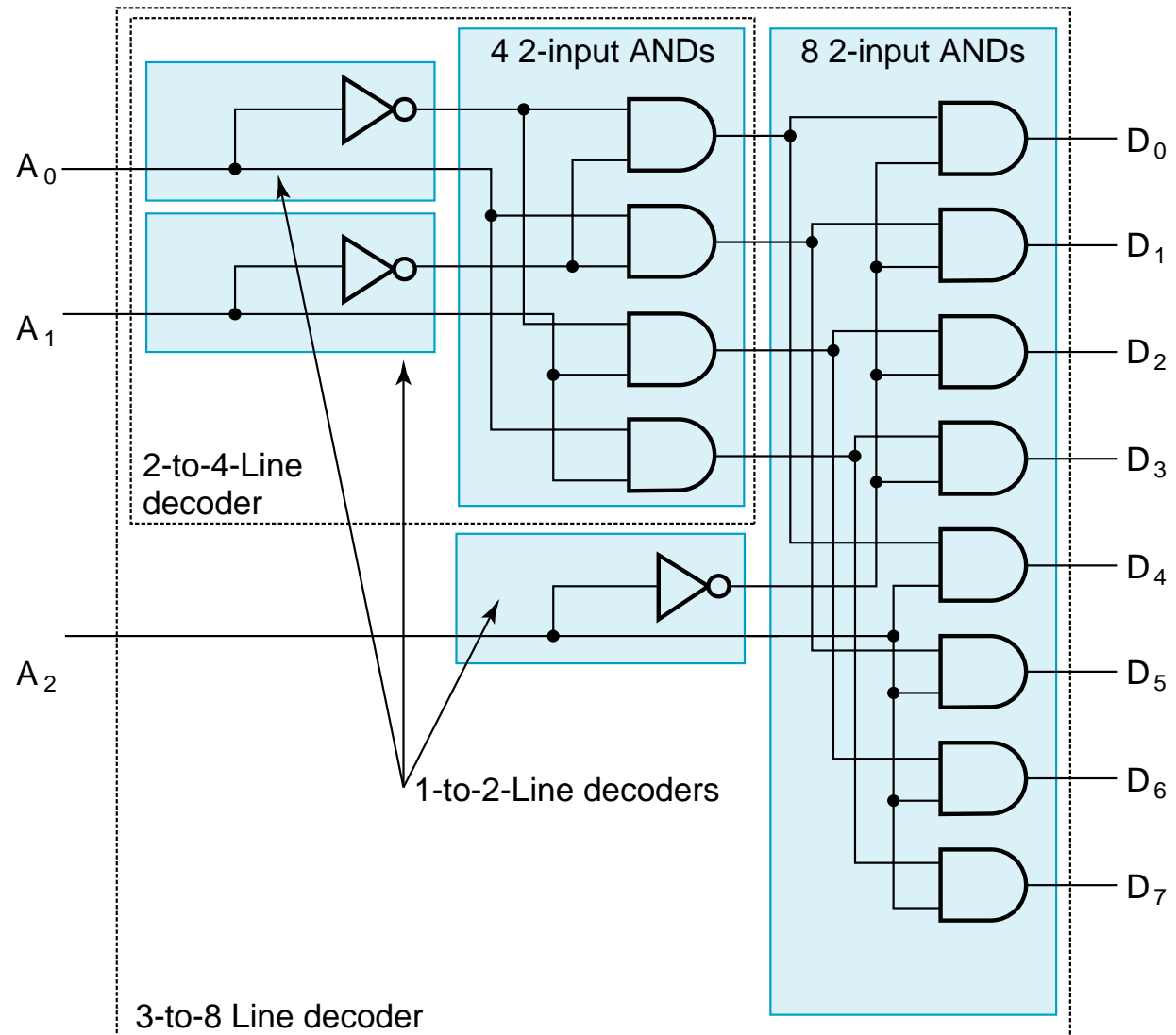
# Decoder Expansion - Example 1

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- **3-to-8-line decoder**
  - **Number of output ANDs = 8**
  - **Number of inputs to decoders driving output ANDs = 3**
  - **Closest possible split to equal**
    - **2-to-4-line decoder**
    - **1-to-2-line decoder**
  - **2-to-4-line decoder**
    - **Number of output ANDs = 4**
    - **Number of inputs to decoders driving output ANDs = 2**
    - **Closest possible split to equal**
      - **Two 1-to-2-line decoders**
- **See next slide for result**

# Decoder Expansion - Example 1

## ■ Result





# Decoder Expansion - Example 2

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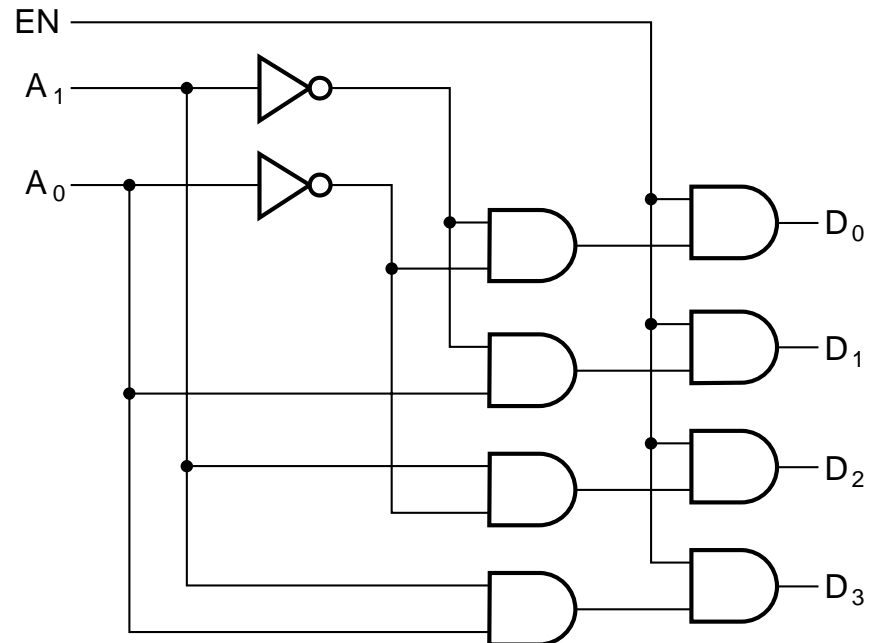
- **7-to-128-line decoder**
  - **Number of output ANDs = 128**
  - **Number of inputs to decoders driving output ANDs = 7**
  - **Closest possible split to equal**
    - **4-to-16-line decoder**
    - **3-to-8-line decoder**
  - **4-to-16-line decoder**
    - **Number of output ANDs = 16**
    - **Number of inputs to decoders driving output ANDs = 2**
    - **Closest possible split to equal**
      - **2 2-to-4-line decoders**
  - **Complete using known 3-8 and 2-to-4 line decoders**

# Decoder with Enable

- In general, attach  $m$ -enabling circuits to the outputs
- See truth table below for function
  - Note use of X's to denote both 0 and 1
  - Combination containing two X's represent four binary combinations
- Alternatively, can be viewed as distributing value of signal EN to 1 of 4 outputs
- In this case, called a *demultiplexer*

EN	A <sub>1</sub>	A <sub>0</sub>	D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
0	X	X	0	0	0	0
1	0	0	1	0	0	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1

(a)



(b)

# Encoding

---

- **Encoding - the opposite of decoding - the conversion of an  $m$ -bit input code to a  $n$ -bit output code with  $n \leq m \leq 2^n$  such that each valid code word produces a unique output code**
- **Circuits that perform encoding are called *encoders***
- **An encoder has  $2^n$  (or fewer) input lines and  $n$  output lines which generate the binary code corresponding to the input values**
- **Typically, an encoder converts a code containing exactly one bit that is 1 to a binary code corresponding to the position in which the 1 appears.**

# Encoder Example

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- **A decimal-to-BCD encoder**
  - **Inputs:** 10 bits corresponding to decimal digits 0 through 9, ( $D_0, \dots, D_9$ )
  - **Outputs:** 4 bits with BCD codes
  - **Function:** If input bit  $D_i$  is a 1, then the output ( $A_3, A_2, A_1, A_0$ ) is the BCD code for  $i$ ,
- **The truth table could be formed, but alternatively, the equations for each of the four outputs can be obtained directly.**

# Encoder Example (continued)

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- Input  $D_i$  is a term in equation  $A_j$  if bit  $A_j$  is 1 in the binary value for  $i$ .
- Equations:
$$A_3 = D_8 + D_9$$
$$A_2 = D_4 + D_5 + D_6 + D_7$$
$$A_1 = D_2 + D_3 + D_6 + D_7$$
$$A_0 = D_1 + D_3 + D_5 + D_7 + D_9$$
- $F_1 = D_6 + D_7$  can be extracted from  $A_2$  and  $A_1$   
Is there any cost saving?

# Priority Encoder

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- If more than one input value is 1, then the encoder just designed does not work.
- One encoder that can accept all possible combinations of input values and produce a meaningful result is a *priority encoder*.
- Among the 1s that appear, it selects the most significant input position (or the least significant input position) containing a 1 and responds with the corresponding binary code for that position.

# Priority Encoder Example

- Priority encoder with 5 inputs ( $D_4, D_3, D_2, D_1, D_0$ ) - highest priority to most significant 1 present - Code outputs  $A_2, A_1, A_0$  and  $V$  where  $V$  indicates at least one 1 present.

No. of Min-terms/Row	Inputs					Outputs			
	D4	D3	D2	D1	D0	A2	A1	A0	V
1	0	0	0	0	0	X	X	X	0
1	0	0	0	0	1	0	0	0	1
2	0	0	0	1	X	0	0	1	1
4	0	0	1	X	X	0	1	0	1
8	0	1	X	X	X	0	1	1	1
16	1	X	X	X	X	1	0	0	1

- Xs in input part of table represent 0 or 1; thus table entries correspond to product terms instead of minterms. The column on the left shows that all 32 minterms are present in the product terms in the table

# Priority Encoder Example (continued)

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- Could use a K-map to get equations, but can be read directly from table and manually optimized if careful:

$$A_2 = D_4$$

$$A_1 = \overline{D}_4 D_3 + \overline{D}_4 \overline{D}_3 D_2 = \overline{D}_4 F_1, \quad F_1 = (D_3 + D_2)$$

$$A_0 = \overline{D}_4 D_3 + \overline{D}_4 \overline{D}_3 \overline{D}_2 D_1 = \overline{D}_4 (D_3 + \overline{D}_2 D_1)$$

$$V = D_4 + F_1 + D_1 + D_0$$

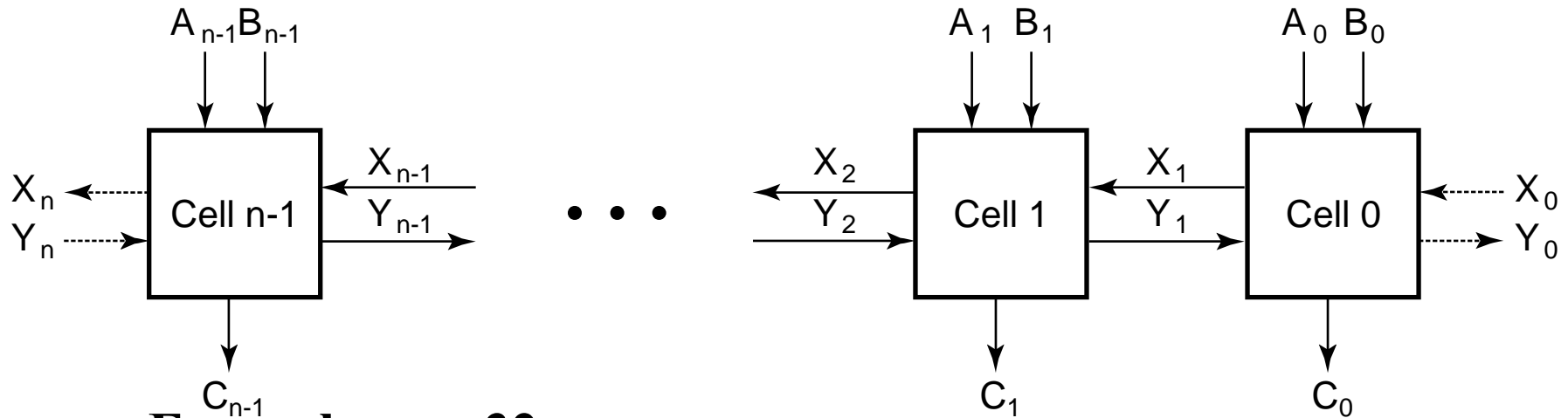


# Iterative Combinational Circuits

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- **Arithmetic functions**
  - Operate on binary vectors
  - Use the same subfunction in each bit position
- **Can design functional block for subfunction and repeat to obtain functional block for overall function**
- ***Cell* - subfunction block**
- ***Iterative array* - a array of interconnected cells**
- **An iterative array can be in a single dimension (1D) or multiple dimensions**

# Block Diagram of a 1D Iterative Array



- **Example:  $n = 32$**

- Number of inputs = ?
- Truth table rows = ?
- Equations with up to ? input variables
- Equations with huge number of terms
- Design impractical!

- **Iterative array takes advantage of the regularity to make design feasible**

# Functional Blocks: Addition

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- **Binary addition used frequently**
- **Addition Development:**
  - *Half-Adder (HA)*, a 2-input bit-wise addition functional block,
  - *Full-Adder (FA)*, a 3-input bit-wise addition functional block,
  - *Ripple Carry Adder*, an iterative array to perform binary addition, and
  - *Carry-Look-Ahead Adder (CLA)*, a hierarchical structure to improve performance.

# Functional Block: Half-Adder

- A 2-input, 1-bit width binary adder that performs the following computations:

<b>X</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>1</b>
<b>+ Y</b>	<b>+ 0</b>	<b>+ 1</b>	<b>+ 0</b>	<b>+ 1</b>
<b>C S</b>	<b>0 0</b>	<b>0 1</b>	<b>0 1</b>	<b>1 0</b>

- A half adder adds two bits to produce a two-bit sum
- The sum is expressed as a sum bit, S and a carry bit, C
- The half adder can be specified as a truth table for S and C  $\Rightarrow$

<b>X</b>	<b>Y</b>	<b>C</b>	<b>S</b>
<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>
<b>1</b>	<b>0</b>	<b>0</b>	<b>1</b>
<b>1</b>	<b>1</b>	<b>1</b>	<b>0</b>

# Logic Simplification: Half-Adder

- The K-Map for S, C is:
- This is a pretty trivial map!  
By inspection:

S		C	
	Y		Y
	0	1	1
X	1	2	3

$$S = X \cdot \overline{Y} + \overline{X} \cdot Y = X \oplus Y$$

$$S = (X + Y) \cdot (\overline{X} + \overline{Y})$$

- and

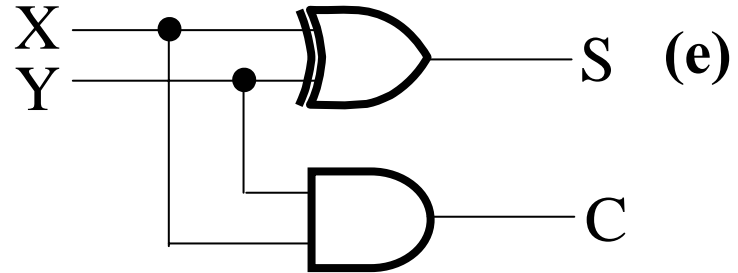
$$C = X \cdot Y$$

$$C = \overline{(\overline{X \cdot Y})}$$

# Implementations: Half-Adder

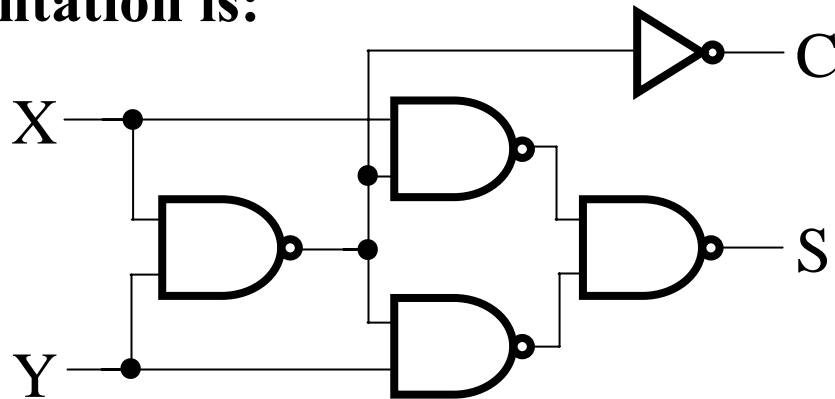
- The most common half adder implementation is:

$$S = X \oplus Y$$
$$C = X \cdot Y$$



- A NAND only implementation is:

$$S = \overline{(X + Y)} \cdot C$$
$$C = \overline{(X \cdot Y)}$$



# Functional Block: Full-Adder

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- A full adder is similar to a half adder, but includes a carry-in bit from lower stages. Like the half-adder, it computes a sum bit, S and a carry bit, C.

- For a carry-in (Z) of 0, it is the same as the half-adder:

Z	0	0	0	0
X	0	0	1	1
<u>+ Y</u>	<u>+ 0</u>	<u>+ 1</u>	<u>+ 0</u>	<u>+ 1</u>
C S	0 0	0 1	0 1	1 0

- For a carry- in (Z) of 1:

Z	1	1	1	1
X	0	0	1	1
<u>+ Y</u>	<u>+ 0</u>	<u>+ 1</u>	<u>+ 0</u>	<u>+ 1</u>
C S	0 1	1 0	1 0	1 1

# Logic Optimization: Full-Adder

- Full-Adder Truth Table:

X	Y	Z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

- Full-Adder K-Map:

S		Y			
X		0	1	3	2
		1		1	
		4	5	7	6
		Z			

C		Y			
X		0	1	3	2
			1	1	
		4	1	1	6
		Z			



# Equations: Full-Adder

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- From the K-Map, we get:

$$S = X \bar{Y} \bar{Z} + \bar{X} Y \bar{Z} + \bar{X} \bar{Y} Z + X Y Z$$

$$C = X Y + X Z + Y Z$$

- The S function is the three-bit XOR function (Odd Function):

$$S = X \oplus Y \oplus Z$$

- The Carry bit C is 1 if both X and Y are 1 (the sum is 2), or if the sum is 1 and a carry-in (Z) occurs. Thus C can be re-written as:

$$C = X Y + (X \oplus Y) Z$$

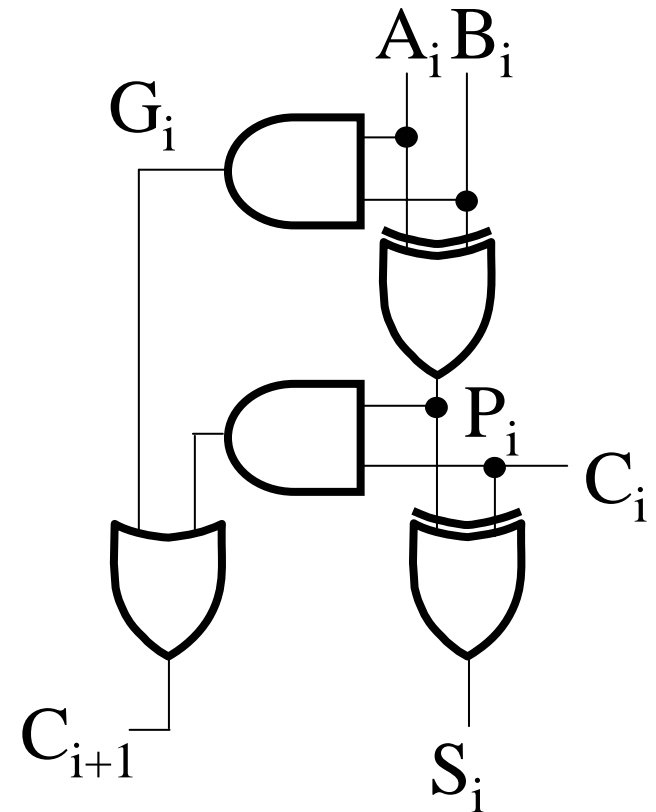
- The term  $X \cdot Y$  is *carry generate*.
- The term  $X \oplus Y$  is *carry propagate*.

# Implementation: Full Adder

- **Full Adder Schematic**
- Here X, Y, and Z, and C (from the previous pages) are A, B,  $C_i$  and  $C_o$ , respectively. Also,  
    **G = generate and**  
    **P = propagate.**
- **Note:** This is really a combination of a 3-bit odd function (for S) and Carry logic (for  $C_o$ ):

**(G = Generate) OR (P = Propagate AND  $C_i$  = Carry In)**

$$C_o = G + P \cdot C_i$$



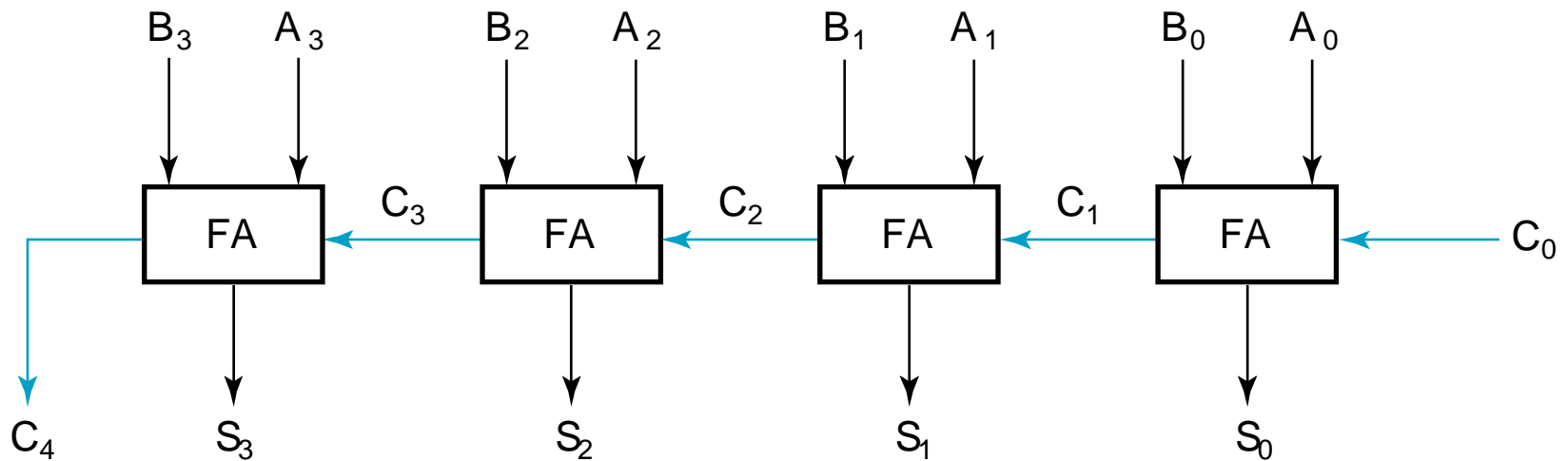
# Binary Adders

- To add multiple operands, we “bundle” logical signals together into vectors and use functional blocks that operate on the vectors
- **Example: 4-bit ripple carry adder:** Adds input vectors  $A(3:0)$  and  $B(3:0)$  to get a sum vector  $S(3:0)$
- **Note:** carry out of cell  $i$  becomes carry in of cell  $i + 1$

Description	Subscript 3 2 1 0	Name
Carry In	0 1 1 0	$C_i$
Augend	1 0 1 1	$A_i$
Addend	<u>0 0 1 1</u>	$B_i$
Sum	1 1 1 0	$S_i$
Carry out	0 0 1 1	$C_{i+1}$

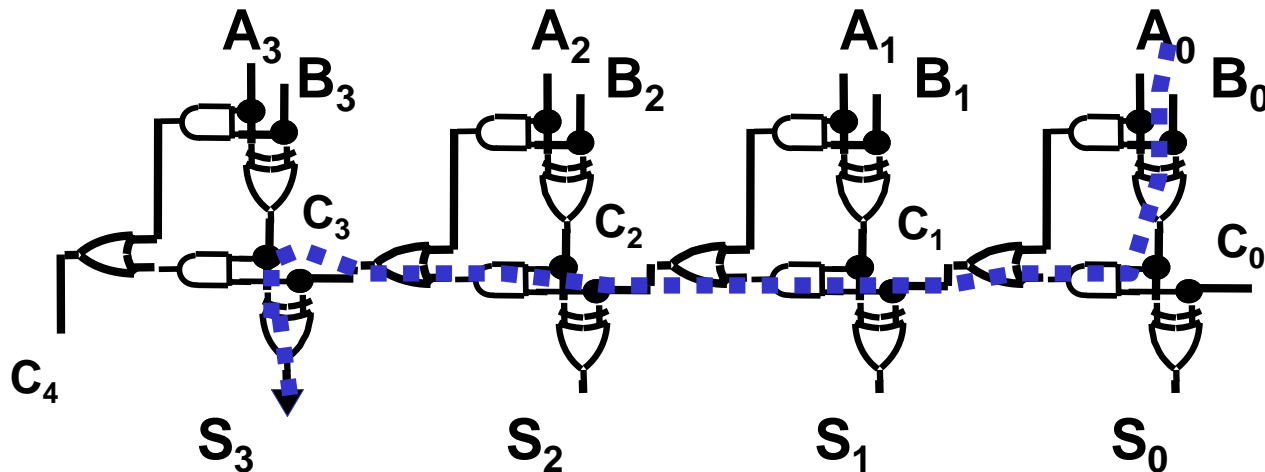
# 4-bit Ripple-Carry Binary Adder

- A four-bit Ripple Carry Adder made from four 1-bit Full Adders:



# Carry Propagation & Delay

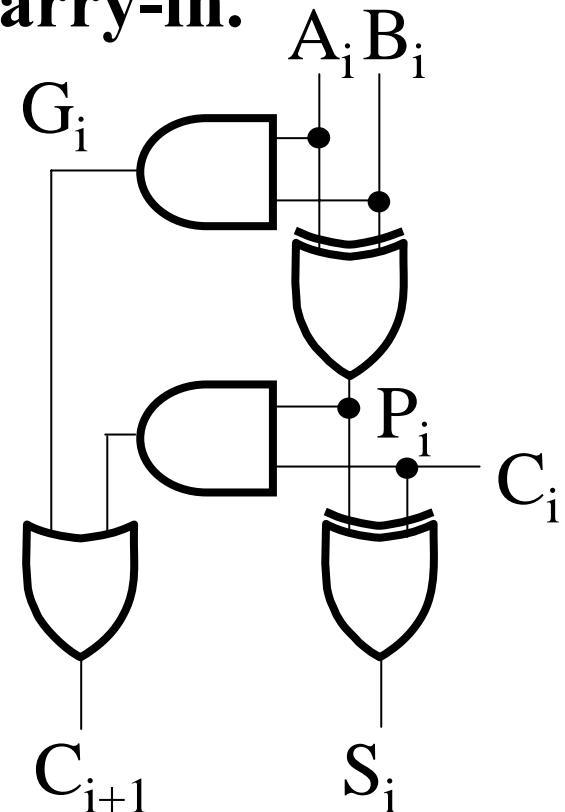
- One problem with the addition of binary numbers is the length of time to propagate the ripple carry from the least significant bit to the most significant bit.
- The gate-level propagation path for a 4-bit ripple carry adder of the last example:



- Note: The "long path" is from  $A_0$  or  $B_0$  through the circuit to  $S_3$ .

# Carry Lookahead

- Given Stage  $i$  from a Full Adder, we know that there will be a carry generated when  $A_i = B_i = "1"$ , whether or not there is a carry-in.
- Alternately, there will be a carry propagated if the “half-sum” is “1” and a carry-in,  $C_i$  occurs.
- These two signal conditions are called *generate*, denoted as  $G_i$ , and *propagate*, denoted as  $P_i$  respectively and are identified in the circuit:



# Carry Lookahead (continued)

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- In the ripple carry adder:
  - $G_i$ ,  $P_i$ , and  $S_i$  are local to each cell of the adder
  - $C_i$  is also local each cell
- In the carry lookahead adder, in order to reduce the length of the carry chain,  $C_i$  is changed to a more global function spanning multiple cells
- Defining the equations for the Full Adder in term of the  $P_i$  and  $G_i$ :

$$P_i = A_i \oplus B_i$$

$$G_i = A_i B_i$$

$$S_i = P_i \oplus C_i$$

$$C_{i+1} = G_i + P_i C_i$$

# Carry Lookahead Development

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- $C_{i+1}$  can be removed from the cells and used to derive a set of carry equations spanning multiple cells.
- Beginning at the cell 0 with carry in  $C_0$ :

$$C_1 = G_0 + P_0 C_0$$

$$\begin{aligned} C_2 &= G_1 + P_1 C_1 = G_1 + P_1(G_0 + P_0 C_0) \\ &= G_1 + P_1 G_0 + P_1 P_0 C_0 \end{aligned}$$

$$\begin{aligned} C_3 &= G_2 + P_2 C_2 = G_2 + P_2(G_1 + P_1 G_0 + P_1 P_0 C_0) \\ &= G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 C_0 \end{aligned}$$

$$\begin{aligned} C_4 &= G_3 + P_3 C_3 = G_3 + P_3 G_2 + P_3 P_2 G_1 \\ &\quad + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 C_0 \end{aligned}$$



# Group Carry Lookahead Logic

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- Figure 5-6 in the text shows the implementation of these equations for four bits. This could be extended to more than four bits; in practice, due to limited gate fan-in, such extension is not feasible.
- Instead, the concept is extended another level by considering *group generate* ( $G_{0-3}$ ) and *group propagate* ( $P_{0-3}$ ) functions:

$$G_{0-3} = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 P_0 G_0$$

$$P_{0-3} = P_3 P_2 P_1 P_0$$

- Using these two equations:

$$C_4 = G_{0-3} + P_{0-3} C_0$$

- Thus, it is possible to have four 4-bit adders use one of the same carry lookahead circuit to speed up 16-bit addition

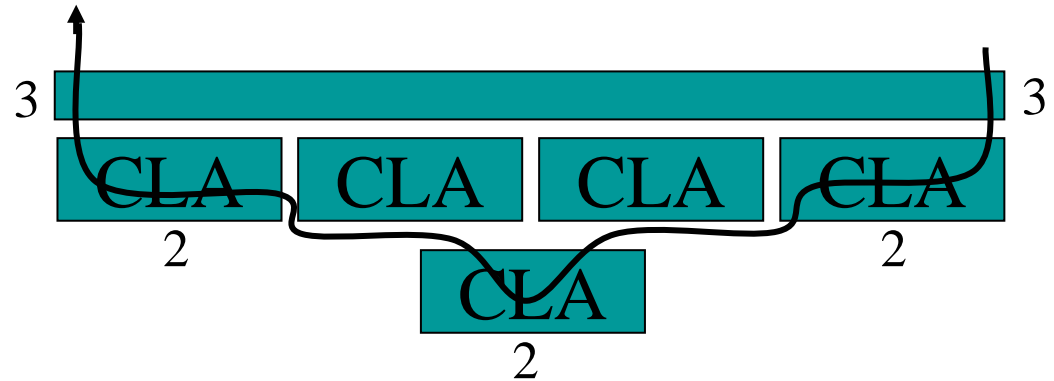
# Carry Lookahead Example

## ■ Specifications:

- 16-bit CLA

- Delays:

- NOT = 1
- XOR = Isolated AND = 3
- AND-OR = 2



## ■ Longest Delays:

- Ripple carry adder\* =  $3 + 15 \times 2 + 3 = 36$
- CLA =  $3 + 3 \times 2 + 3 = 12$

\*See slide 16

# Unsigned Subtraction

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## ■ Algorithm:

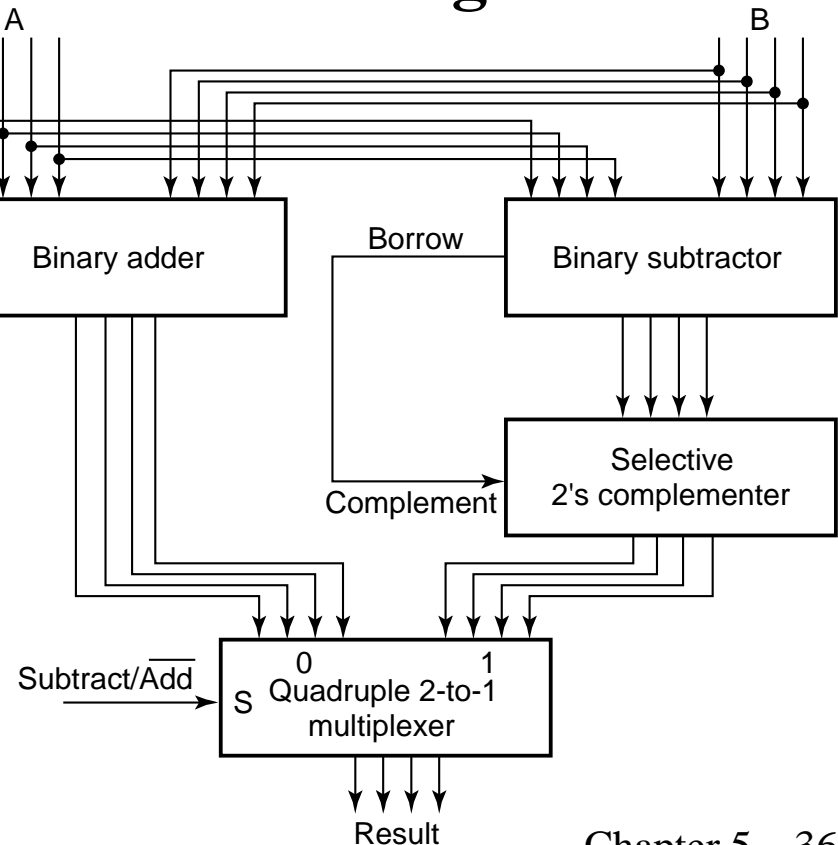
- Subtract the subtrahend  $N$  from the minuend  $M$
- If no end borrow occurs, then  $M \geq N$ , and the result is a non-negative number and correct.
- If an end borrow occurs, the  $N > M$  and the difference  $M - N + 2n$  is subtracted from  $2n$ , and a minus sign is appended to the result.

## ■ Examples:

0	1
1001	0100
- 0111	- 0111
<u>0010</u>	<u>1101</u>
	10000
	- 1101
	<u>(-) 0011</u>

# Unsigned Subtraction (continued)

- The subtraction,  $2^n - N$ , is taking the 2's complement of  $N$
- To do both unsigned addition and unsigned subtraction requires:
- Quite complex!
- Goal: Shared simpler logic for both addition and subtraction
- Introduce complements as an approach



# Binary 2's Complement

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- For  $r = 2$ ,  $N = 01110011_2$ ,  $n = 8$  (8 digits), we have:

$$(r^n) = 256_{10} \text{ or } 100000000_2$$

- The 2's complement of 01110011 is then:

$$\begin{array}{r} 100000000 \\ - 01110011 \\ \hline 10001101 \end{array}$$

- Note the result is the 1's complement plus 1, a fact that can be used in designing hardware

# Subtraction with 2's Complement

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- For n-digit, unsigned numbers M and N, find  $M - N$  in base 2:
  - Add the 2's complement of the subtrahend N to the minuend M:
$$M + (2^n - N) = M - N + 2^n$$
  - If  $M \geq N$ , the sum produces end carry  $r^n$  which is discarded; from above,  $M - N$  remains.
  - If  $M < N$ , the sum does not produce an end carry and, from above, is equal to  $2^n - (N - M)$ , the 2's complement of  $(N - M)$ .
  - To obtain the result  $-(N - M)$ , take the 2's complement of the sum and place a  $-$  to its left.

# Unsigned 2's Complement Subtraction Example 1

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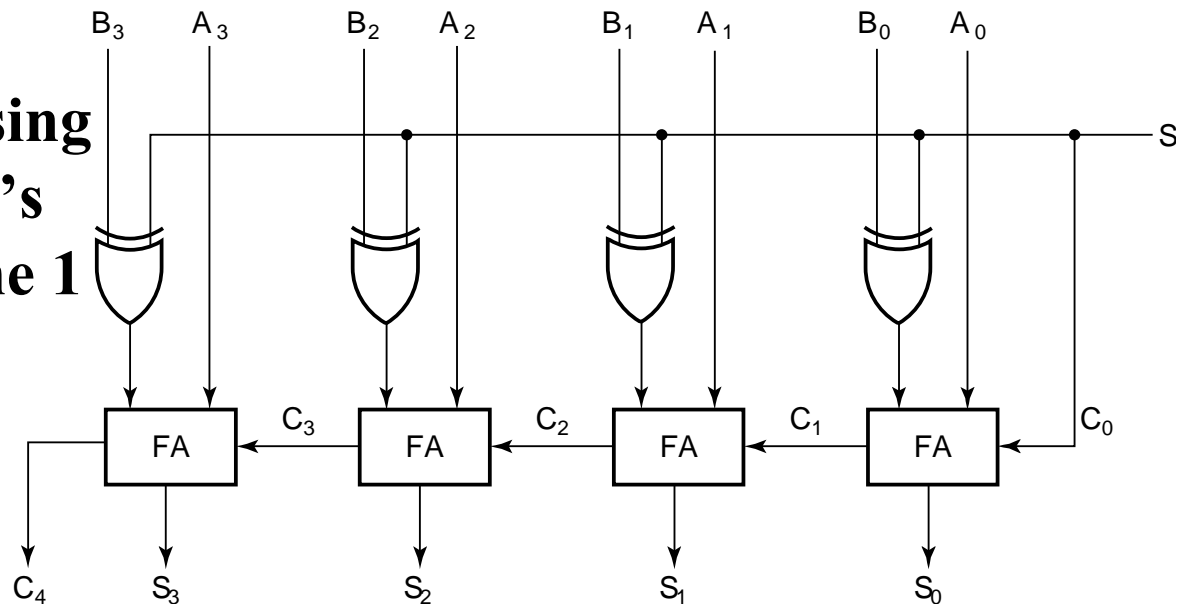
- Find  $01010100_2 - 01000011_2$

$$\begin{array}{r} 01010100 \\ - 01000011 \\ \hline \end{array} \xrightarrow{\text{2's comp}} \begin{array}{r} 101010100 \\ + 10111101 \\ \hline 00010001 \end{array}$$

- The carry of 1 indicates that no correction of the result is required.

# 2's Complement Adder/Subtractor

- Subtraction can be done by addition of the 2's Complement.
  1. Complement each bit (1's Complement.)
  2. Add 1 to the result.
- The circuit shown computes  $A + B$  and  $A - B$ :
- For  $S = 1$ , subtract, the 2's complement of  $B$  is formed by using XORs to form the 1's comp and adding the 1 applied to  $C_0$ .
- For  $S = 0$ , add,  $B$  is passed through unchanged





# Overflow Detection

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- ***Overflow* occurs if  $n + 1$  bits are required to contain the result from an  $n$ -bit addition or subtraction**
- **Overflow can occur for:**
  - Addition of two operands with the same sign
  - Subtraction of operands with different signs
- **Signed number overflow cases with correct result sign**

$$\begin{array}{cccc} 0 & 0 & 1 & 1 \\ + \underline{0} & - \underline{1} & - \underline{0} & + \underline{1} \\ 0 & 0 & 1 & 1 \end{array}$$

- **Detection can be performed by examining the result signs which should match the signs of the top operand**

# Overflow Detection

- Signed number cases with carries  $C_n$  and  $C_{n-1}$  shown for correct result signs:

$$\begin{array}{r}
 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \\
 0 \quad 0 \quad 1 \quad 1 \\
 + \underline{0} \quad - \underline{1} \quad - \underline{0} \quad + \underline{1} \\
 \hline
 0 \quad 0 \quad 1 \quad 1
 \end{array}$$

- Signed number cases with carries shown for erroneous result signs (indicating overflow):

$$\begin{array}{r}
 0 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \\
 0 \quad 0 \quad 1 \quad 1 \\
 + \underline{0} \quad - \underline{1} \quad - \underline{0} \quad + \underline{1} \\
 \hline
 1 \quad 1 \quad 0 \quad 0
 \end{array}$$

- Simplest way to implement overflow  $V = C_n \oplus C_{n-1}$
- This works correctly only if 1's complement and the addition of the carry in of 1 is used to implement the complementation! Otherwise fails for  $-10 \dots 0$

# Binary Multiplication

---

- The binary digit multiplication table is trivial:

$(a \times b)$	$b = 0$	$b = 1$
$a = 0$	0	0
$a = 1$	0	1

- This is simply the Boolean AND function.
- Form larger products the same way we form larger products in base 10.

# Review - Decimal Example: $(237 \times 149)_{10}$

- Partial products are:  $237 \times 9$ ,  $237 \times 4$ , and  $237 \times 1$

- Note that the partial product summation for  $n$  digit, base 10 numbers requires adding up to  $n$  digits (with carries).

- Note also  $n \times m$  digit multiply generates up to an  $m + n$  digit result.

		2	3	7					
	×		1	4	9				
		2	1	3	3				
		9	4	8	-				
	+	2	3	7	-	-			
		3	5	3	1	3			

# Binary Multiplication Algorithm

---

- **We execute radix 2 multiplication by:**
  - Computing partial products, and
  - Justifying and summing the partial products. (same as decimal)
- **To compute partial products:**
  - Multiply the row of multiplicand digits by each multiplier digit, one at a time.
  - With binary numbers, partial products are very simple! They are either:
    - all zero (if the multiplier digit is zero), or
    - the same as the multiplicand (if the multiplier digit is one).
- **Note: No carries are added in partial product formation!**

## Example: (101 x 011) Base 2

- Partial products are:  $101 \times 1$ ,  $101 \times 1$ , and  $101 \times 0$
- Note that the partial product summation for  $n$  digit, base 2 numbers requires adding up to  $n$  digits (with carries) in a column.
- Note also  $n \times m$  digit multiply generates up to an  $m + n$  digit result (same as decimal).

	1	0	1
×	0	1	1
<hr/>			
	1	0	1
1	0	1	
0	0		
<hr/>			
1	1	1	1

# Multiplier Boolean Equations

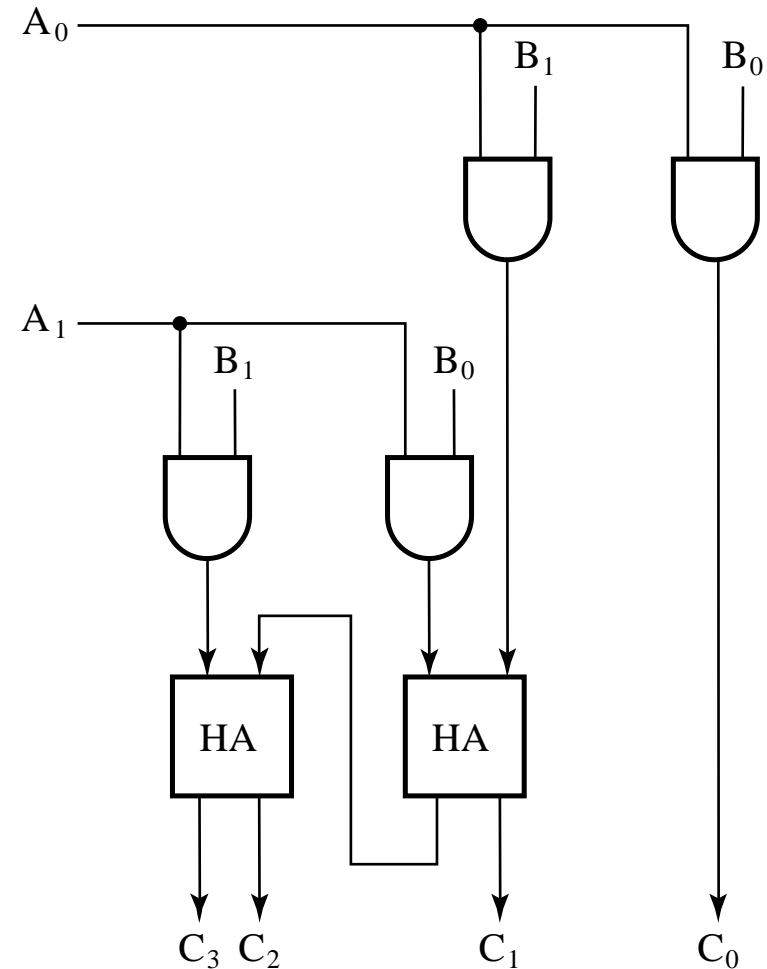
- We can also make an  $n \times m$  “block” multiplier and use that to form partial products.
- Example:  $2 \times 2$  – The logic equations for each partial-product binary digit are shown below:
- We need to "add" the columns to get the product bits P0, P1, P2, and P3.

- Note that some columns may generate carries.

	$b_1$	$b_0$	
	$a_1$	$a_0$	
	$\times$	$(a_0 \cdot b_1)$	$(a_0 \cdot b_0)$
$+$	$(a_1 \cdot b_1)$	$(a_1 \cdot b_0)$	
$\hline$	$\hline$	$\hline$	$\hline$
$P_3$	$P_2$	$P_1$	$P_0$

# Multiplier Arrays Using Adders

- An implementation of the  $2 \times 2$  multiplier array is shown:





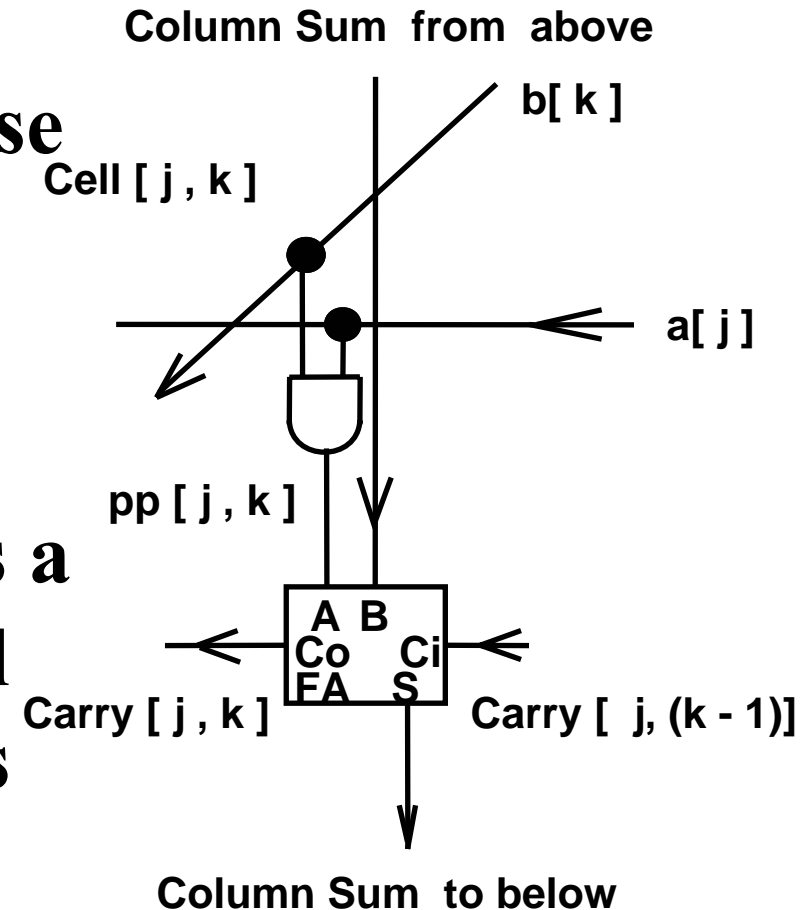
# Multiplier Using Wide Adders

---

- A more “structured” way to develop an  $n \times m$  multiplier is to sum partial products using adder trees
- The partial products are formed using an  $n \times m$  array of AND gates
- Partial products are summed using  $m - 1$  adders of width  $n$  bits
- Example: 4-bit by 3-bit adder
- Text figure 5-11 shows a  $4 \times 3 = 12$  element array of AND gates and two 4-bit adders

# Cellular Multiplier Array

- Another way to implement multipliers is to use an  $n \times m$  cellular array structure of uniform elements as shown:
- Each element computes a single bit product equal to  $a_i \cdot b_j$ , and implements a single bit full adder



# Other Arithmetic Functions

---

- Convenient to design the functional blocks by *contraction* - removal of redundancy from circuit to which input fixing has been applied
- Functions
  - Incrementing
  - Decrementing
  - Multiplication by Constant
  - Division by Constant
  - Zero Fill and Extension

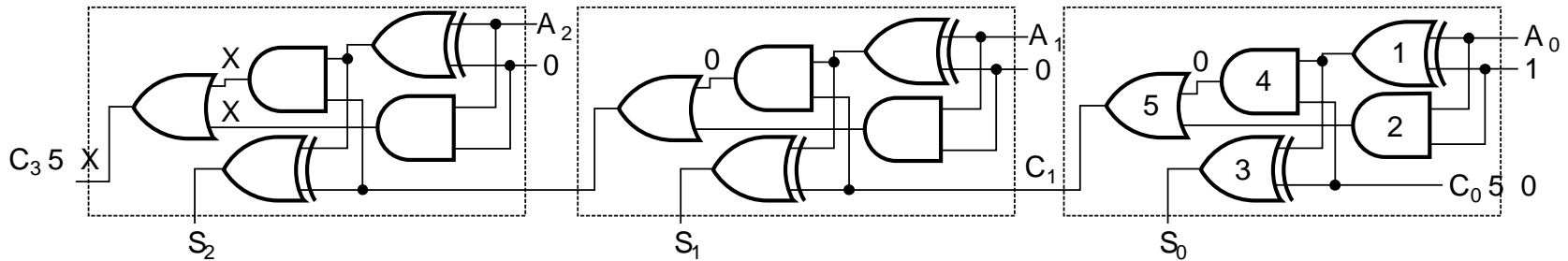
# Design by Contraction

---

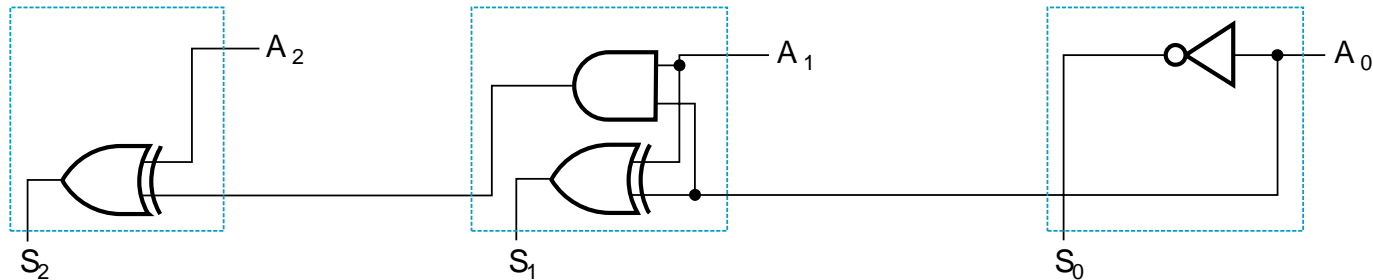
- **Contraction is a technique for simplifying the logic in a functional block to implement a different function**
  - **The new function must be realizable from the original function by applying rudimentary functions to its inputs**
  - **Contraction is treated here only for application of 0s and 1s (not for  $X$  and  $\overline{X}$ )**
  - **After application of 0s and 1s, equations or the logic diagram are simplified by using rules given on pages 224 - 225 of the text.**

# Design by Contraction Example

- Contraction of a ripple carry adder to incrementer for  $n = 3$ 
  - Set B = 001



(a)



(b)

- The middle cell can be repeated to make an incrementer with  $n > 3$ .

# Incrementing & Decrementing

---

## ■ *Incrementing*

- Adding a fixed value to an arithmetic variable
- Fixed value is often 1, called *counting (up)*
- Examples:  $A + 1$ ,  $B + 4$
- Functional block is called *incrementer*

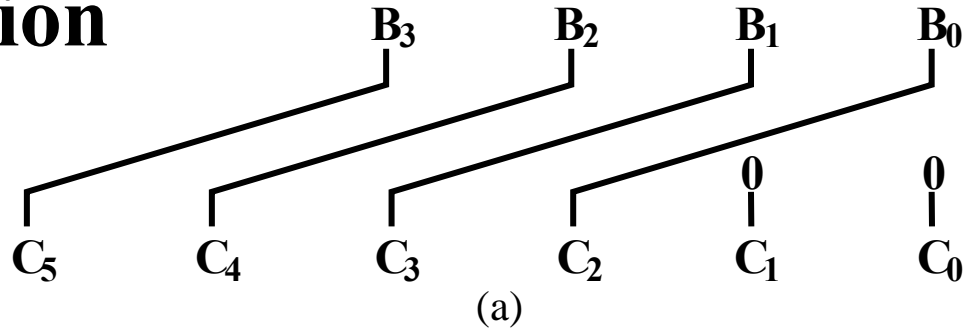
## ■ *Decrementing*

- Subtracting a fixed value from an arithmetic variable
- Fixed value is often 1, called *counting (down)*
- Examples:  $A - 1$ ,  $B - 4$
- Functional block is called *decrementer*

# Multiplication/Division by $2^n$

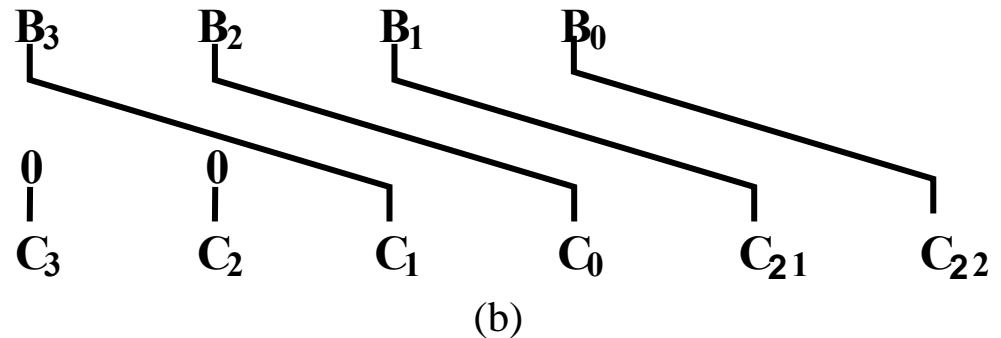
## ■ (a) Multiplication by 100

- Shift left by 2



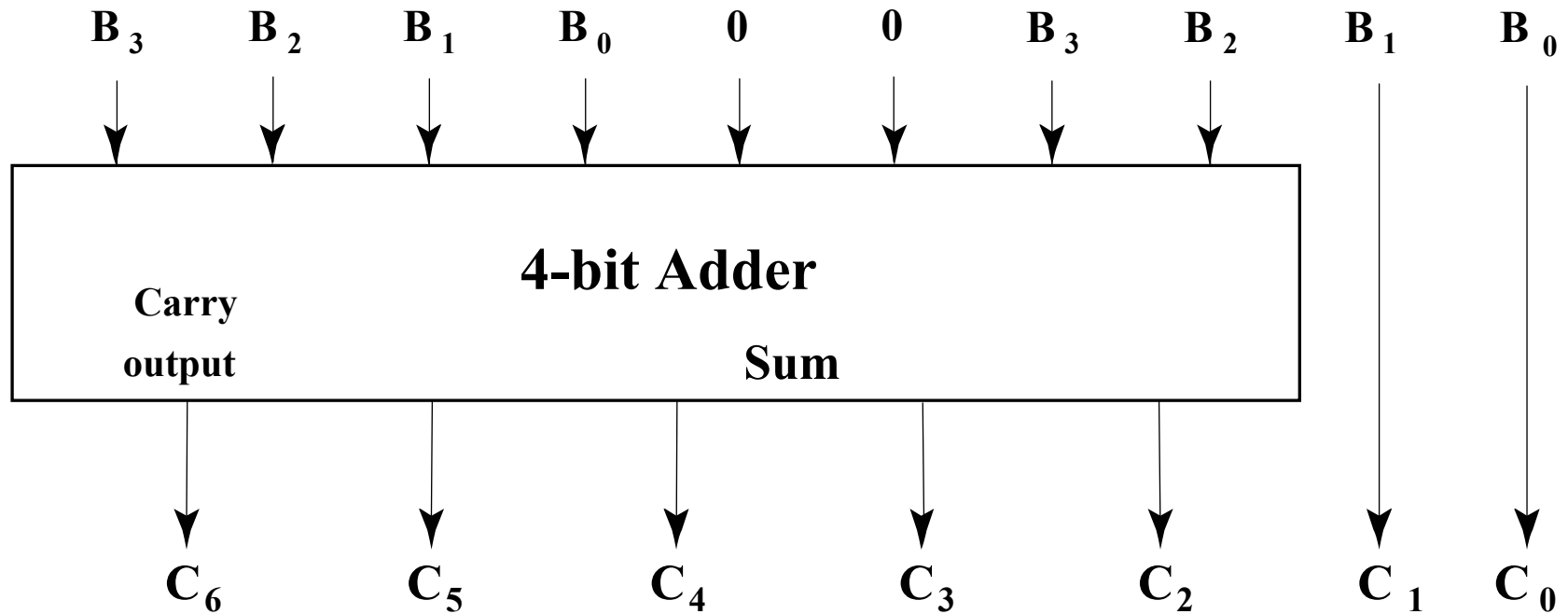
## ■ (b) Division by 100

- Shift right by 2
- Remainder preserved



# Multiplication by a Constant

- Multiplication of  $B(3:0)$  by 101
- See text Figure 513 (a) for contraction





# Zero Fill

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- ***Zero fill*** - filling an  $m$ -bit operand with 0s to become an  $n$ -bit operand with  $n > m$
- Filling usually is applied to the MSB end of the operand, but can also be done on the LSB end
- **Example: 11110101 filled to 16 bits**
  - MSB end: 0000000011110101
  - LSB end: 1111010100000000

# Extension

---

- ***Extension*** - increase in the number of bits at the MSB end of an operand by using a complement representation

- Copies the MSB of the operand into the new positions
- Positive operand example - 01110101 extended to 16 bits:

0000000001110101

- Negative operand example - 11110101 extended to 16 bits:

1111111111110101

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