NOTCH STRESS CONCENTRATION (STATIC)

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NOTCHES and changes in diameter of shaft cause stress concentrations.

The stress at the change in size is greater than the nominal stress.

\[ \sigma_{\text{nom}} = \frac{Mc}{I} \]

**COMPUTE THE OUTER FIBER STRESS USING THE SMALLER \( d \) FOR THE CALCULATIONS**

The max stress is computed using

\[ \sigma_{\text{max}} = K_t \sigma_{\text{nom}} \]

\( K_t \) is computed from the equation

\[ K_t = A \left( \frac{r}{d} \right)^b \]

Where
- \( r \) = The radius of the filled
- \( d \) = The smaller size of the shaft
- \( A \) = A value from a table in Appendix
- \( b \) = Value from Appendix C

Please note that \( K_t \) is dimensionless.

**EXAMPLE**

\[ M = 1000 \text{ N-m} \]

\[ \sigma_{\text{nom}} = \frac{Mc}{I} \]

\[ C = 15 \text{ mm} = 0.015 \text{ m} \]

\[ D = 45 \text{ mm} = 0.045 \text{ m} \]
\[ d = 30 \text{ mm} = 0.30 \text{ m} \]
\[ I = \frac{\pi d^4}{64} = \frac{\pi (0.3)^4}{64} = 3.97 \times 10^{-8} \]
\[ \sigma_{\text{nom}} = \frac{M c}{I} = \frac{1000 (0.015)}{3.97 \times 10^{-8}} = 378 \text{ MPa} \]
\[ K_t = A \left( \frac{r}{d} \right)^{0.254} \]
\[ r = 3 \text{ mm} = 0.003 \]
\[ K_t = 0.938 \left( \frac{0.003}{0.03} \right)^{0.254} = 0.93838 \]
\[ K_t = 1.70 \]

\[ \sigma_{\text{max}} = K_t \sigma_{\text{nom}} = 1.70 \times 378 = 642 \text{ MPa} \]

**NOTCH SENSITIVITY (DYNAMIC LOADS)**

Materials subject to varying loads are sensitive to stress concentrations. A notch or change in diameter of a shaft can cause stress concentrations that cause the stresses above the yield point. If this is in the case, the part may be subject to earlier fatigue failure.

Interestingly enough, as the notch radius approaches zero, the notch sensitivity of the material decreases and also approaches zero.

\[ q = \frac{K_t - 1}{K_t - 1} \]

Fatigue stress concentration factor

Static stress concentration factor
Kf can be computed using the technique just demonstrated but we need to compute Kf for fatigue.

Rewriting

\[ K_f = 1 + q \left( K_t - 1 \right) \]

where

\[ q = 1 + \frac{Ta}{r} \]

We can compute \( Ta \) with bending or axial (STEEL)

\[ Ta^2 = 0.246 - 3.08 \times 10^{-3} Su + 1.51 \times 10^{-5} Su^2 - 2.67 \times 10^{-8} Su^3 \]

\( \text{Torsion} \)

\[ Ta = 0.190 - 2.51 \times 10^{-3} Su + 1.35 \times 10^{-5} Su^2 - 2.67 \times 10^{-8} Su^3 \]

Su t must be expressed in kpsi.

Note that you do not have to take the square root of \( a \). It has already been done.

After computing \( K_f \), use the formulas

\[ \sigma = K_f \sigma_{nom} \]

\[ T = K_f \tau_{nom} \]

\( K_f \) and \( K_f \tau \) will be different.
EXAMPLE

From the previous example compute the fatigue equivalent stresses.

\[ \sigma_{\text{nom}} = 378 \text{ MPa} \]
\[ r = 3 \text{ mm} \]

Previous Results

Assume we are working steel

\[ S_{ut} = 130 \text{ kpsi} = 896 \text{ MPa} \]

Using the equation,

\[ \sigma_a' = 0.246 - 3.08 \times 10^{-3} S_{ut} + 1.51 \times 10^{-5} S_{ut}^2 - 2.67 \times 10^{-8} S_{ut}^3 \]

\[ \sigma_a' = 0.246 - 3.08 \times 10^{-3} \times 130 + 1.51 \times 10^{-5} \times 130^2 - 2.67 \times 10^{-8} \times 130^3 \]

\[ \sigma_a' = 0.042 \]

Convert the radius to inches

\[ r = 3 \text{ mm} = 0.12 \text{ inches} \]

\[ q = \frac{1}{1 + 0.042/0.12} = 0.8916 \]

From the previous work

\[ K_t = 1.69 \]

\[ K_f = 1 + 0.8916(1.69 - 1) \]

\[ K_f = 1.615 \]

Previously

\[ \sigma_{\text{nom}} = 378 \text{ MPa} \]

\[ \sigma = K_f \sigma_{\text{nom}} = (1.615)(378) = 610 \text{ MPa} \]

NOTE: If the material is ductile use \( K_f \), if it is brittle, use \( K_t \).
Local Yielding

If the materials is ductile and we have local yielding at a step or a notch, we must make further corrections.

1) No Yielding
   \[ K_f |\sigma_{\text{max}}| < \sigma_y \]
   then
   \[ K_{fm} = K_f \]

2) Local Yielding
   \[ K_f |\sigma_{\text{max}}| > \sigma_y \]
   then
   \[ K_{fm} = \frac{\sigma_y - K_f \sigma_a}{\sigma_m} \]
   \[ \sigma_{\text{max}} = \sigma_m + \sigma_a \]
   \[ \sigma_{\text{min}} = \sigma_m - \sigma_a \]

3) Large scale yielding
   \[ K_f |\sigma_{\text{max}} - \sigma_{\text{min}}| > 2.5 \sigma_y \]
   \[ K_{fm} = 0 \quad \text{All hope is lost} \]

You should always try to avoid local yielding.