3. A cord of negligible mass is wrapped around the outer surface of the 50 lb cylinder and its end is subjected to a constant horizontal force of \( P = 2 \) lb. If the cylinder rolls without slipping at \( A \), determine its angular velocity in 4 s starting from rest. Neglect the thickness of the cord.

**Data:**
- \( W = 50 \) lb
- \( m = 1.5528 \) slug
- \( P = 2 \) lb

**SOLUTION**

\[
I = \frac{1}{2} m r^2 = \frac{1}{2} \times 1.5528 \times 0.6^2 = 0.2795 \text{ slug} \cdot \text{ft}^2
\]

\[
L = m(v_G)_t + \int_0^t (P - F) \, dt = m(v_G)_2
\]

\[0 + (2 - F) \times 4 = 1.5528 \times (0.6 \omega_2)\]

\[8 - 4F = 0.93168 \omega_2\]

\[\omega(4s) = ?\]

Adding (1) and (2) \[16 \times 1.3975\]

\[\omega = 4.12 \text{ rad/s}\]

Alternatively

\[\frac{5}{0.5} = 4 \times 0.25^2 \omega\]

\[\omega = 0.687 \text{ rad/s}\]

4. A 4 kg disk \( A \) is mounted on arm \( BC \), which has a negligible mass. If a torque of \( M = (5t^{0.5}) \) N \( \cdot \) m, where \( t \) is in seconds, is applied to the arm at \( C \), determine the angular velocity of \( BC \) in 2 s starting from rest. Solve the problem assuming that:

(a) The disk is set on a smooth bearing at \( B \) so that it rotates with curvilinear translation.

(b) The disk is fixed to the shaft \( BC \).

(c) The disk is given an initial freely spinning angular velocity \( \omega_B = (-80 \text{ rad/s}) \)

**SOLUTION**

\[I = \frac{1}{2} m r^2 = \frac{1}{2} \times 60 \times 0.6^2 = 0.0072 \text{ kg} \cdot \text{m}^2\]

\[\omega(2s) = ?\]

\[
0 + \int_0^2 5 e^{0.5t} \, dt = m r \omega_B = m r^2 \omega_{BC}
\]

\[17.183 = 4 \times 0.25^2 \omega + 0.0072 \omega = \omega = 0.687 \text{ rad/s}\]

\[\text{(a) Smooth bearing, disk in pure translation:}\]

\[\omega = 0.687 \text{ rad/s}\]

\[\text{(b) Fixed disk, rigid body rotation:}\]

\[\omega = 0.687 \text{ rad/s}\]

\[\text{(c) Spinning freely: (also smooth bearing)}\]

\[\omega = 0.687 \text{ rad/s}\]

4. The slender rod has a mass \( m \) and is suspended at its end \( A \) by a cord. If the rod receives a horizontal blow giving it an impulse \( I \) at its bottom \( B \), determine the location \( y \) of the point \( P \) about which the rod appears to rotate during the impact.

**SOLUTION**

\[I = \frac{1}{12} m \ell^2\]

Linear momentum:

\[m(v_{GA})_t + I = m(v_{GA})_2\]

Angular momentum:

\[I \omega + (\ell/2) I = I \omega_2\]

\[v_{GA} = 0 \Rightarrow \begin{cases} 
I = m(v_{GA})_2 \\
\ell = m \ell \omega_2\end{cases} \Rightarrow \begin{cases} 
(v_{GA})_2 = \frac{1}{m} \\
\omega_2 = \frac{61}{m \ell}\end{cases} \]

\[\omega = \frac{61}{m \ell}\]

\[d = \frac{(v_{GA})_2}{\omega_2} = \frac{1/m}{61/m \ell} = \frac{\ell}{6}\]

\[y = \frac{2\ell}{3}\]
5. The platform swing consists of a 200 lb flat plate suspended by four rods of negligible weight. When the swing is at rest, the 150 lb man jumps off the platform when his center of gravity is 10 ft from the pin at A. This is done with a horizontal velocity of 5 ft/s, measured relative to the swing at the level of G.

Determine the angular velocity he imparts to the swing just after jumping off.

**SOLUTION**

\[ I_{RA} = \frac{1}{6} m R^2 = \frac{1}{6} \times 0.15528 \times 2^2 \]

\[ I_{RA} = 0.207 \text{ slug} \cdot \text{ft}^2 \]

\[ I_{RA} = \frac{1}{6} m d^2 + m v^2 \]

\[ = (0.31677 + 0.00621) \times \left( \frac{1}{6} \times 1^2 + 2.5^2 \right) \]

\[ I_{RA} = 2.0326 \text{ slug} \cdot \text{ft}^2 \]

(in \( I_{RA} \) the mass includes both the block and the projectile)

\[ (H_A)_1 = (H_A)_2 \]

\[ 0 = -r_{G/A} m_p v_p + r_{G/A} m_p v_p + I_{RA} \]

\[ 0 = -10 \times 4.658 \times 5 + 11 \times 6.21 \times 11 \omega + 759.7 \omega \]

\[ 232.9 = 1511.1 \omega \]

\[ \omega = 0.154 \text{ rad} / \text{s} \]

6. The pendulum consists of a 5 lb slender rod \( AB \) and a 10 lb wooden block. A projectile weighing 0.2 lb is fired into the center of the block with a velocity of 1000 ft/s. If the pendulum is initially at rest, and the projectile imbeds itself into the block, determine the angular velocity of the pendulum just after impact.

**SOLUTION**

\[ I_{BA} = \frac{1}{3} m l^2 = \frac{1}{3} \times 0.15528 \times 2^2 \]

\[ I_{BA} = 0.207 \text{ slug} \cdot \text{ft}^2 \]

\[ I_{BA} = \frac{1}{3} m d^2 + m v^2 \]

\[ = (0.31677 + 0.00621) \times \left( \frac{1}{3} \times 1^2 + 2.5^2 \right) \]

\[ I_{BA} = 2.0326 \text{ slug} \cdot \text{ft}^2 \]

\( (H_A)_1 = (H_A)_2 \)

\[ (H_A)_1 = r_{B/A} m_p (v_p)_1 \]

\[ = 2.5 \times 0.00621 \times 1000 \]

\[ (H_A)_1 = 15.525 \text{ slug} \cdot \text{m}^2 / \text{s} \]

\[ (H_A)_2 = I_{BA} \omega + I_{BA} \omega \]

\[ = 0.207 \omega + 2.0326 \omega \]

\[ (H_A)_2 = 2.2396 \omega \]

\[ \omega = 6.93 \text{ rad} / \text{s} \]

Note: If we neglect the mass of the projectile in block \( B \) we get \( I_{BA} = 1.9927 \) and \( \omega = 7.06 \) (a 1.7% difference)

7. The pendulum consists of a 10 lb solid ball and 4 lb rod. If it is released from rest when \( \theta_0 = 0 \), determine the angle \( \theta_2 \) after the ball strikes the wall, rebounds, and the pendulum swings up to the point of momentary rest. Take \( e = 0.6 \).

**SOLUTION**

\[ I_{(A)} = \frac{2}{3} m s^2 = \frac{2}{3} \times 0.31056 \times 0.3^2 = 0.001118 \text{ slug} \cdot \text{ft}^2 \]

\[ I_{(B)} = \frac{1}{3} m s^2 = \frac{1}{3} \times 0.12422 \times 2^2 = 0.16563 \text{ slug} \cdot \text{ft}^2 \]

\[ I_{(C)} = I_{(A)} + m_R r^2 = 0.12118 + 0.31056 \times 2.3^2 = 1.654 \text{ slug} \cdot \text{ft}^2 \]

\[ I_A = I_{(A)} + I_{(B)} = 1.8197 \text{ slug} \cdot \text{ft}^2 \]

\[ m = m_B + m_A = 0.31056 + 0.12422 \]

\[ m = 0.43478 \text{ slug} \]

\[ r_{G/A} = \frac{0.12422 \times 1 + 0.31056 \times 2.3}{0.43478} \]

\[ r_{G/A} = 1.92858 \]

1) Velocity just before striking the wall:

\[ T_1 + V_1 = T_2 + V_2 \]

\[ 0 = \frac{1}{2} I_A \omega^2 + \frac{1}{2} m v_y^2 + W r_{G/A} \]

\[ 0 = \frac{1}{2} \times 1.8197 \omega^2 + \frac{1}{2} \times 0.43478 \times (1.92858 \omega)^2 - 14 \times 1.92858 \]

\[ \omega = -3.9639 \text{ rad} / \text{s} \]

\[ (v_{(B)})_3 = -9.1169 \text{ ft} / \text{s} \]

\[ (v_{(B)})_3 = 2.3 \omega \]
Datum \[3) \text{Maximum return angle:} \quad T_2 + V_2 = T_3 + V_3 \]

\[
\frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} m v_{G2}^2 + W r_{G,d} = 0 \quad \text{or} \quad \omega_2^2 = \frac{1}{2} \frac{1}{2} \frac{1}{2} 1.8197 \times 2.3783 \times 1.92858 \times 2.3783^2 \approx -14 \times 1.92858 = -14 \times 1.92858 \times \sin \theta_2
\]

\[\theta_2 = 39.8^\circ\]

8. The plank has a weight of 30 lb, center of gravity at \(G\), and it rests on the two sawhorses \(A\) and \(B\). If the end \(D\) is raised 2 ft above the top of the sawhorses and is released from rest, determine how high end \(C\) will rise from the top of the sawhorses after the plank falls so that it rotates clockwise about \(A\), strikes and pivots on the saw horses at \(B\), and rotates clockwise off the sawhorses at \(A\).

\[
\begin{align*}
\omega_2 &= -1.8915 \text{ rad/s} \\
\theta_2 &= 19.476 \\
\end{align*}
\]

9. The uniform plate weights 40 lb and is supported by a roller at \(A\). If a horizontal force \(F = 70 \text{ lb} \) is suddenly applied to the roller, determine the acceleration of the center of mass of the roller at the instant the force is applied. The plate has a moment of inertia about its center of mass of \(I_G = 0.414 \text{ slug \cdot ft}^2\). Neglect the weight of the roller.

\[
\begin{align*}
\alpha &= \frac{h}{3} = \frac{2 \sqrt{3}}{3} = 1.1547 \text{ ft} \\
\end{align*}
\]

\[
\begin{align*}
\sum F_i &= F = m a_G \quad \Rightarrow \quad 70 = 1.2422 a_G \\
a_G &= 56.352 \text{ ft/s}^2 \\
\end{align*}
\]

\[
\begin{align*}
\sum M_A &= 0 = -m a_G \gamma + I_G \alpha \\
-1.2422 &\times 56.352 \times 1.1547 + 0.414 \alpha = 0
\end{align*}
\]
10. The 15 kg cylinder is rotating with an angular velocity of 40 rad/s. If a force $F = 6N$ is applied to link $AB$, as shown, determine the time needed to stop the rotation. The coefficient of kinetic friction between $AB$ and the cylinder is $\mu_k = 0.4$.

**Data:**
- $m = 15$ kg
- $\omega = 40$ rad/s
- $F = 6N$
- $\mu_k = 0.4$

**SOLUTION**

To find $N$ take moments about $B$ to avoid finding those reactions.

1) $\sum M_B = F \times 0.9 - N \times 0.5 = 0$

$$N = 10.8(N)$$

$$F' = 0.4N = 0.4 \times 10.8 = 4.32N$$

2) $$(H_C)_i + \int_0^t M_C \, dt = 0$$

$$(H_C)_i = I_C \omega$$

$$I_C = \frac{1}{2}mr^2 = \frac{1}{2} \times 15 \times 0.15^2 = 0.16875 \text{ kg} \cdot \text{m}^2$$

$$I_C\omega + \int_0^t (F' r) \, dt = 0$$

$$0.1688 \times 40 - \int_0^t (4.32 \times 0.15) \, dt = 0$$

$$6.752 - 6.48t = 0 \Rightarrow t = 10.42 \text{s}$$