# Chapter 6

## CONFIDENCE INTERVALS

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A confidence interval is a range of scores believed with some level of confidence to contain the true population value.

From a random sample, we obtain a sample statistic (e.g., the sample mean, the sample correlation) that is assumed to be an estimate of the population parameter (e.g., population mean, population correlation).

The amount the sample statistic is different from the population parameter is referred to as sampling error.

- Sampling error is caused by random error and sample size.
The larger the size of the sample, the more accurate the sample will estimate the population value.

- The larger the sample size, the smaller the sampling error.

Imagine randomly sampling 25 people from a population, measuring their heights (in inches), and obtaining the sample mean and sample SD.

- Then put the 25 people back into the population.

Now imagine repeating this process for an infinite number of samples, each containing 25 people.
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All possible sample means, each of which is based on a sample size of 25 people.

\[
\begin{align*}
\overline{X}_1 &= 68.5 \\
\overline{X}_2 &= 69.25 \\
\overline{X}_3 &= 67.75 \\
\overline{X}_4 &= 67.5 \\
\overline{X}_5 &= 68.75
\end{align*}
\]

Sampling distribution of samples means, where the mean is equal to the population mean and the standard deviation is equal to the standard error of the mean (SEM).
As the sample size increases:

• the sample means will become more accurate in estimating the population mean
• the sample means will become more similar to each other
• the estimated SEM will become smaller, and
• the shape of the sampling distribution (i.e., histogram) will become narrower and taller
A sample mean is an estimate of the population mean plus or minus sampling error.

Sample Mean ± Sampling Error

- The SEM is an estimate of the sampling error.
- The size of the sampling error can be reduced by increasing the sample size.
- As sample size increases, the sample mean becomes a more accurate estimate of the population mean.
A confidence interval builds a range of values around a sample mean.

The size of a confidence interval is a function of:

- Sample size (larger N → smaller sampling error) and
- The confidence level chosen

Typically, researchers choose a confidence level of 90%, 95%, or 99%

Thus, the confidence interval is:

Sample Mean ± Sampling Error (Level of Confidence)
The formula for the 95% confidence interval is:

\[
\text{Sample Mean } \pm \text{ SEM}(t_{.05})
\]

Where:

- the SEM is an estimate of the sampling error and
- \( t_{.05} \) is the value from the \( t \)-distribution, where 5% of all \( t \)-scores are expected to be equal to or less than this score
The *t* distribution is a *z* distribution that has been adjusted for:

- Sample size and
- Not knowing the population SD (we have to estimate it using the sample SD)

**t** - scores:

- Are interpreted similarly to *z* scores (SD units), but are estimated SD units
- Have a mean equal to zero (just like *z* scores)
- Equal *z* – scores when sample size equals ∞
The $t$ distribution is located on the class website under the statistical tables link.

The value of $t_{.05}$ is found by:

1. Determining the degrees of freedom ($df$)
2. Formula: $df = N - 1$
3. Locate the $t_{.05}$ value under the 95% Confidence Interval heading

What is the value for $t_{.05}$ when $N = 25$?
When $N = 25$, the value of $t_{0.05} = \pm 2.06$
**EXAMPLE 1:**

1. Suppose a sample of 16 light trucks is randomly selected off the assembly line. The trucks are driven 1000 miles and the fuel mileage (MPG) of each truck is recorded. It is found that the mean MPG is 22 with a SD equal to 3.

   - What is the 95% confidence interval?

\[
\text{Sample Mean} \pm \text{SEM}(t_{0.05})
\]

\[
22 \pm \frac{3}{\sqrt{16}} \approx 22 \pm 0.75
\]

\[df = 16 - 1 = 15\]
EXAMPLE 1:

\[ 22 \pm 0.75 \, \hat{\theta} \, 1.31 \]

\[ 22 \pm 1.598 \]

95% Confidence Interval:

Lower Limit $\rightarrow$ 20.402  
Upper Limit $\rightarrow$ 23.598

Interpretation:

- We are 95% confident that the true fuel efficiency for all light trucks manufactured at the plant to be in the range of 20.402 MPG to 23.598 MPG. (N = 16)
The confidence level chosen (e.g., 99% versus 95%) affects the size (or range) of the confidence interval.

1. The higher the confidence level chosen, the wider the confidence interval.
   - Because we want to be more confident (99% vs. 95%), the interval must be larger.
The level of confidence chosen tells us what to expect in the *long run*.

A 95% confidence level means that we expect 5 out of 100 sample means to give us intervals that DO NOT contain the true population mean.
Suppose you want to rent an unfurnished one-bedroom apartment for next semester. The mean monthly rent for a random sample of 10 apartments advertised in the local paper is $540 with a sample SD equal to $80.

What is the 95% confidence interval?

\[
\text{Sample Mean} \pm \text{SEM}(t_{0.05})
\]

\[
540 \pm \frac{80}{\sqrt{10}} \approx 262
\]

\[
df = 16 - 1 = 15
\]
EXAMPLE 2:

540 ± 25.298

540 ± 57.225

95% Confidence Interval:

Lower Limit → $482.78  
Upper Limit → $597.23

Interpretation:

• We are 95% confident that the true fuel efficiency for all light trucks manufactured at the plant to be in the range of $482.78 to $597.23. (N = 10)
Using the same example, suppose we sample 20 adds instead of 10. What affect will this have on the confidence interval?

- **ANSWER** → The confidence interval will be smaller because the sample size is larger. A larger sample size will reduce the size of the sampling error (or SEM)

\[
540 \pm \frac{80}{\sqrt{20}} \pm 0.093
\]

\[df = 20 - 1 = 19\]
Interpretation:

- We are 95% confident that the true monthly mean rent for unfurnished one-bedroom apartments in the community is in the range of $502.56 to $577.44. (N = 20)
Increasing the sample size from $N = 10$ to $N = 20$ reduced the size of the confidence interval by nearly $\$20$ on each side of the interval.
Using the same example where N = 10, suppose we choose a confidence level of 99% instead of 95%. What affect will this have on the size of the confidence interval?

- **ANSWER**: The confidence interval will be larger because we want to be even more certain that the interval contains the true population mean.

\[
540 \pm \frac{80}{\sqrt{10}} \times 2.821
\]

\[t_{0.05}\text{ under 99\% column in } t\text{ table}\]

\[df = 10 - 1 = 9\]
EXAMPLE 2:

\[ 540 \pm 25.298 \oslash 0.821 \]

\[ 540 \pm 71.366 \]

95% Confidence Interval:

Lower Limit $\rightarrow$ $468.63$  
Upper Limit $\rightarrow$ $611.37$

Interpretation:

- We are 99% confident that the true monthly mean rent for unfurnished one-bedroom apartments in the community is in the range of $468.63 to $611.37. ($N = 10$)
THAT’S IT FOR CHAPTER 7