BEAMS

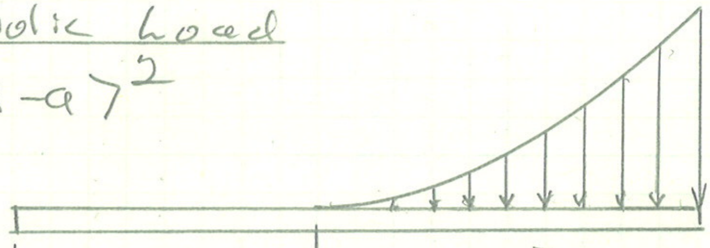
Shafts and other parts of a machine can frequently be analyzed as beams.

Beams can be easily analyzed with step functions.

EXAMPLES OF STEP FUNCTIONS

Parabolic Load

$\langle x-a \rangle^2$



Parabolic Load

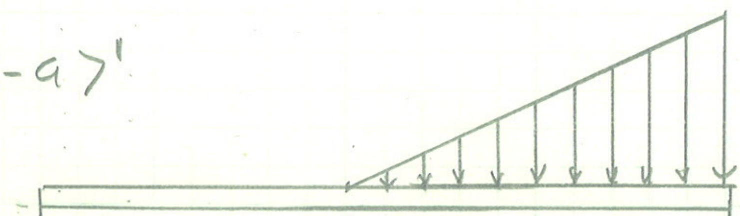
$\langle x-a \rangle^2 = 0$ ($x \leq a$)

$\langle x-a \rangle^2 = (x-a)^2$ ($x > a$)

0 a

Ramp Load

$\langle x-a \rangle^1$



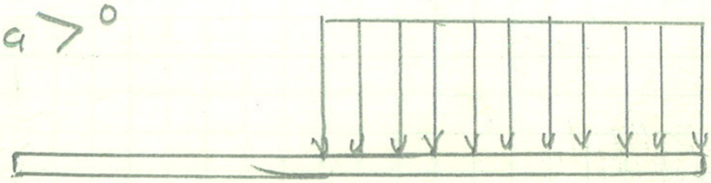
$\langle x-a \rangle^1 = 0$ ($x \leq a$)

$\langle x-a \rangle^1 = x-a$ ($x > a$)

0 a

Distributed Load

$\langle x-a \rangle^0$



$\langle x-a \rangle^0 = 0$ ($x < a$)

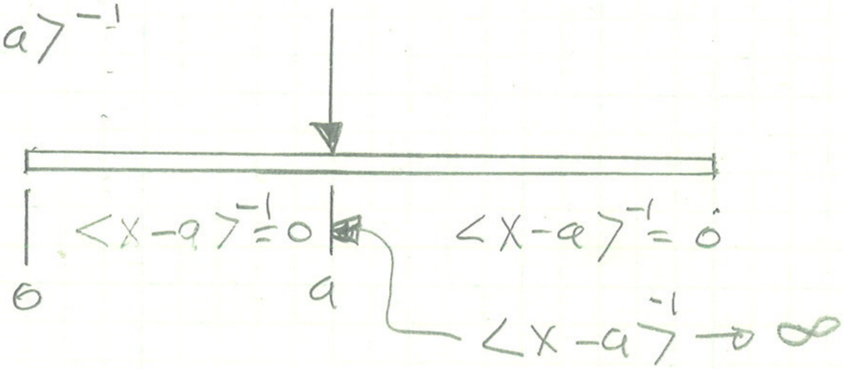
$\langle x-a \rangle^0 = 1$ ($x > a$)

0 a

$x = a$ UNDEFINED

POINT LOAD

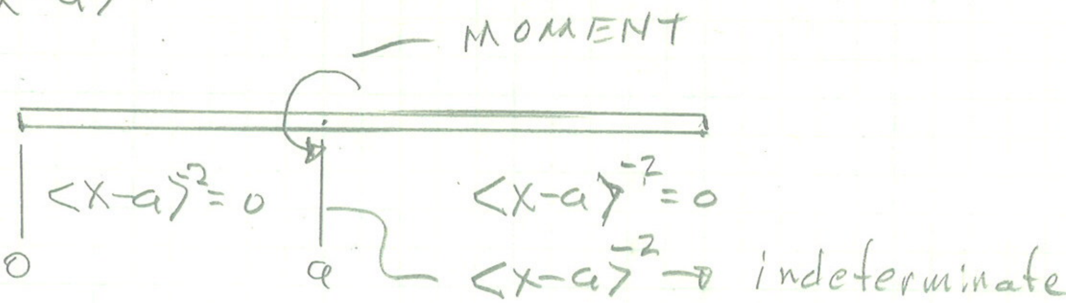
$\langle x-a \rangle^{-1}$



$\int \langle x-a \rangle^{-1} dx = 1$
@ $x = a$

CONCENTRATED MOMENT

$\langle x-a \rangle^{-2}$



These functions can be integrated

parabolic function

$$\int_{-\infty}^{x} \langle \lambda - a \rangle^{2} d\lambda = \frac{\langle x-a \rangle^{3}}{3}$$

Ramp

parabolic

$$\int_{-\infty}^{x} \langle \lambda - a \rangle^{1} d\lambda = \frac{\langle x-a \rangle^{2}}{2}$$

distributed Load

Ramp

$$\int_{-\infty}^{x} \langle \lambda - a \rangle^{0} d\lambda = \langle x-a \rangle^{1}$$

point Load   distributed Load

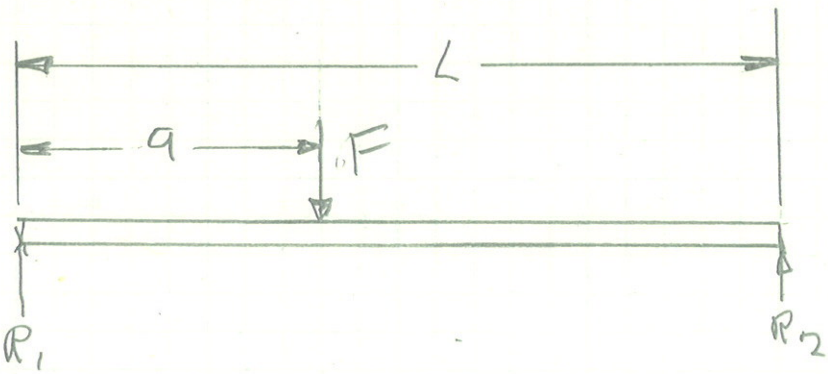
$$\int_{-\infty}^{x} \langle \lambda - a \rangle^{-1} d\lambda = \langle x - a \rangle^{0}$$

CONCENTRATED MOMENT

$$\int_{-\infty}^{x} \langle \lambda - a \rangle^{-2} d\lambda = \langle x - a \rangle^{-1}$$

POINT LOAD

EXAMPLE 1



LOAD EQUATION

$$q = R_1 \langle x - 0 \rangle^{-1} - F \langle x - a \rangle^{-1} + R_2 \langle x - L \rangle^{-1}$$

All point loads

SHEAR EQUATION

$$V = R_1 \langle x - 0 \rangle^{0} - F \langle x - a \rangle^{0} + R_2 \langle x - L \rangle^{0} + C_1$$

Distributed Load

MOMENT EQUATION

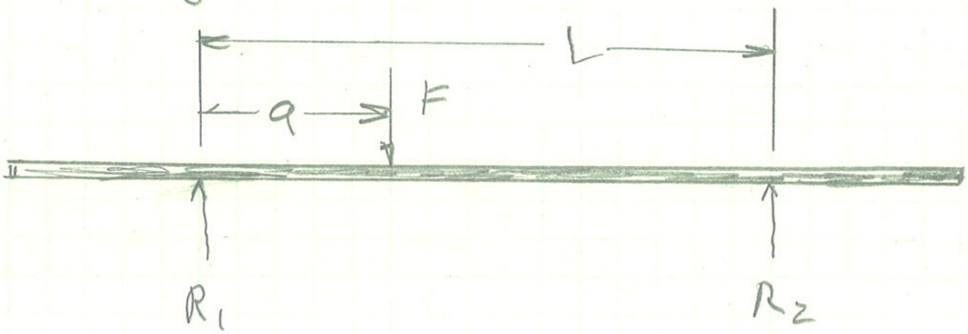
$$M = R_1 \langle x - 0 \rangle^{1} - F \langle x - a \rangle^{1} + R_2 \langle x - L \rangle^{1} + C_1 x + C_2$$

We know the Load F and we must find the two reactions C_1 and C_2 plus the Reactions.

COMPUTING C_1 and C_2

Looking at the theory we know that any shear created by the load must be canceled by the reaction forces.

If we change our beam slightly by extending it on both ends we get



We can tell by looking that to the left of R_1 and to the right of R_2 there are no shears or moments

Evaluating the shear equation at some point to the left of R_1 - say -1

$$V = 0 = R_1\langle 0-\overset{0}{0}\rangle^0 - F\langle 0-\overset{0}{a}\rangle^0 + R_2\langle 0-\overset{0}{L}\rangle^0 + C_1$$

Therefore

$$C_1 = 0$$

Using the same logic for the moment

$$M = 0 = R_1\langle -1-0\rangle^1 + F\langle -1-a\rangle^1 + R_2\langle -1-L\rangle^1 + C_2$$

Therefore

$$C_2 = 0$$

REACTIONS

We can use a similar technique for computing R_1 and R_2 but I find the methods from statics are easier.

The summation of moments around R_1 are:

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$$\sum M_{@\,x=0} = 0 = -Fa + R_2 L$$

$\therefore$

$$R_2 = \frac{Fa}{L}$$

The summation of forces in the Y direction are:

$$\sum F = 0 = R_1 + R_2 - F$$

$$R_1 = F - R_2 = F - \frac{Fa}{L}$$

Now we can compute the moments and shear at any point along the beam.

At this point, we may want to assign numbers to the variables

$L = 10 \qquad a = 4 \qquad F = 100$

The shear equation becomes

$$R_1 = F - \frac{Fa}{L} = 100 - \frac{100(4)}{10} = 60$$

$$R_2 = \frac{Fa}{L} = \frac{100(4)}{10} = 40$$

$$V = 60\langle x-0\rangle^0 - 100\langle x-4\rangle^0 + 40\langle x-10\rangle^0$$

@ $x = 0$

$V = 60(1) = 60$

@ $x = 3.99$

$V = 60(1) = 60$

@ $x = 4.01$

$V = 60(1) - 100(1) = -40$

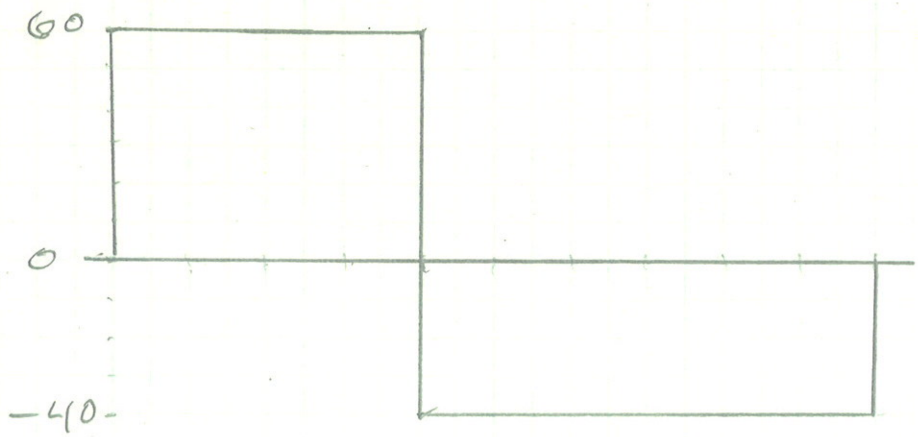
@ $X = 9.99$

$V = 60(1) - 100(1) = -40$

@ $X = 10$

$V = 60(1) - 100(1) + 40(1) = 0$

The shear diagram becomes.



The moment equation

$$M = 60\langle X-0\rangle^1 - 100\langle X-4\rangle^1 + 40\langle X-10\rangle^1$$

@ $X = 0^+$

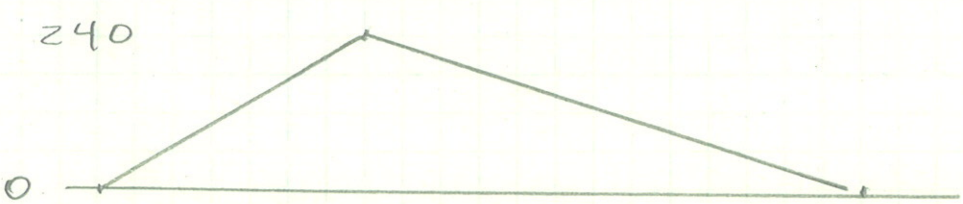
$M = 60(0) = 0$

@ $X = 4$

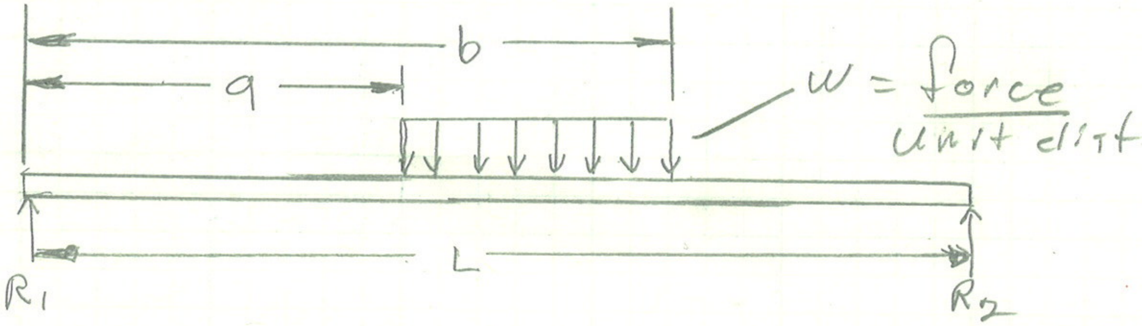
$M = 60(4) - 100(0) = 240$

@ $X = 10$

$M = 60(10) - 100(6) + 40(0) = 0$



EXAMPLE 2



Adds Load

$L = 10 \quad a = 4 \quad b = 8 \quad w = 100$

Removes Load

$$q = R_1\langle x-0\rangle^{-1} - w\langle x-a\rangle^0 + w\langle x-b\rangle^0 + R_2\langle x-L\rangle^{-1}$$

$$V = R_1\langle x-0\rangle^0 - w\langle x-a\rangle^1 + w\langle x-b\rangle^1 + R_2\langle x-L\rangle^0$$

$$M = R_1\langle x-0\rangle^1 - \frac{w}{2}\langle x-a\rangle^2 + \frac{w}{2}\langle x-b\rangle^2 + R_2\langle x-L\rangle^1$$

We have not included the constants of integration because we previously determined they are zero.

COMPUTING REACTIONS

Last time, we used moment and force equilibrium to determine R_1 and R_2. We could use that method again but this time we will demonstrate a slightly different method

Using the Moment equation and evaluating it at $X = 10$ we see

$$M(L) = R_1\langle L-0\rangle^1 - \frac{w}{2}\langle L-a\rangle^2 + \frac{w}{2}\langle L-b\rangle^2 + R_2\langle L-L\rangle^1 = 0$$

substituting in numbers

$$M(L) = 0 = R_1(10) - \frac{100}{2}(6)^2 + \frac{100}{2}(2)^2 + R_2(0)$$

$$10R_1 - 1800 + 200 + 0 = 0$$

$$R_1 = \frac{1800-200}{10} = \underline{160}$$

Using the shear equation @ $X = 10^+$

$$V(10) = R_1\langle 10-0\rangle^0 - w\langle 10-4\rangle^1 + w\langle 10-8\rangle^1 + R_2\langle 10-10\rangle^0 = 0$$

or

$$R_1(1) - 100(6) + 100(2) + R_2(1) = 0$$

substituting

$$160 - 600 + 200 + R_2 = 0$$

$$R_2 = 600 - 160 - 200 = \underline{240}$$

SHEAR AND MOMENT DIAGRAMS

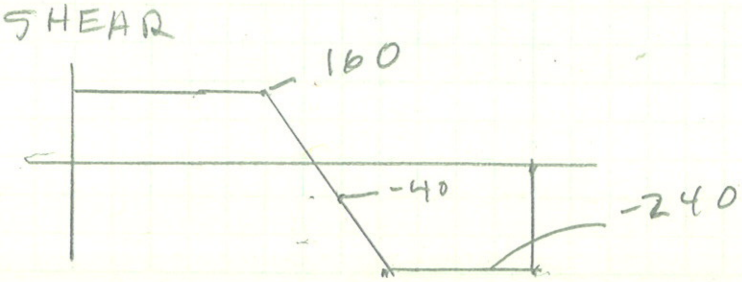
We can compute the shears and moments along the beam and plot them.

$$V(X) = 160\langle X-0\rangle^0 - 100\langle X-4\rangle^1 + 100\langle X-8\rangle^1 + 240\langle X-10\rangle^0$$

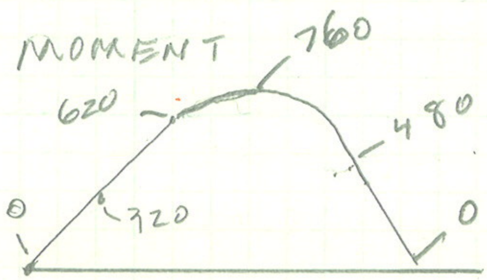
$$M(X) = 160\langle X-0\rangle^1 - 50\langle X-4\rangle^2 + 50\langle X-8\rangle^2 + 240\langle X-10\rangle^1$$

X	V(X)	M(X)
0	160	0
2	160	320
4	160	640
6	-40	760
8	-240	480
10	-240	0

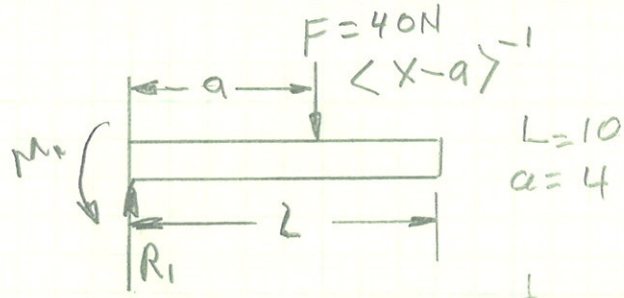
SHEAR

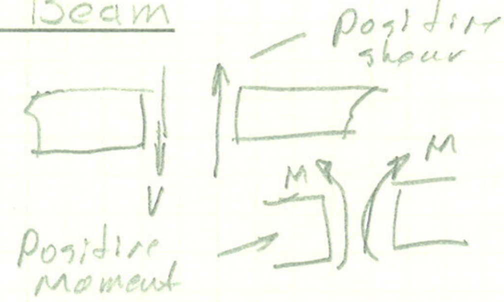


MOMENT



EXAMPLE 3 – Cantilever Beam





LOADING

Moment doublet | impulse | impulse

$$q(x) = -M_1\langle x-0\rangle^{-2} + R_1\langle x-0\rangle^{-1} - F\langle x-a\rangle^{-1}$$

SHEAR

$$V(x) = \int q\,dx = -M_1\langle x-0\rangle^{-1} + R_1\langle x-0\rangle^{0} - F\langle x-a\rangle^{0} + C_1$$

MOMENT

$$M(x) = \int V\,dx = -M_1\langle x-0\rangle^{0} + R_1\langle x-0\rangle^{1} - F\langle x-a\rangle^{1} + C_1 x + C_2$$

Looking at the shear at $x = 0^-$

$$V(0^-) = -M_1\cancelto{0}{\langle 0^- - 0\rangle^{-1}} + R_1\cancelto{0}{\langle 0^- - 0\rangle^{0}} - F\cancelto{0}{\langle 0^- - a\rangle^{0}} + C_1 = 0$$

$$\therefore \boxed{C_1 = 0}$$

Looking at the moment at $x = 0^-$

$$M(0^-) = -M_1\cancelto{0}{\langle 0^- - 0\rangle^{0}} + R_1\cancelto{0}{\langle 0^- - 0\rangle^{1}} - F\cancelto{0}{\langle 0^- - a\rangle^{0}} + C_2 = 0$$

$$\therefore C_2 = 0$$

Both of these constants are zero which is usually the case because the diagram must close to zero on both ends.

Looking at the other end.

$$V(L) = M\langle L-0\rangle^{-1} + R_1\langle L-0\rangle^{0} - F\langle L-a\rangle^{0} = 0$$

SHEAR EQN

This is a point load function that is zero everywhere except at $x = 0$

$0 = M(0) + R_1 - F$

$$\boxed{R_1 = F = 40}$$

Moment Eqn.

CONSTANT LOAD = 1

Ramp function

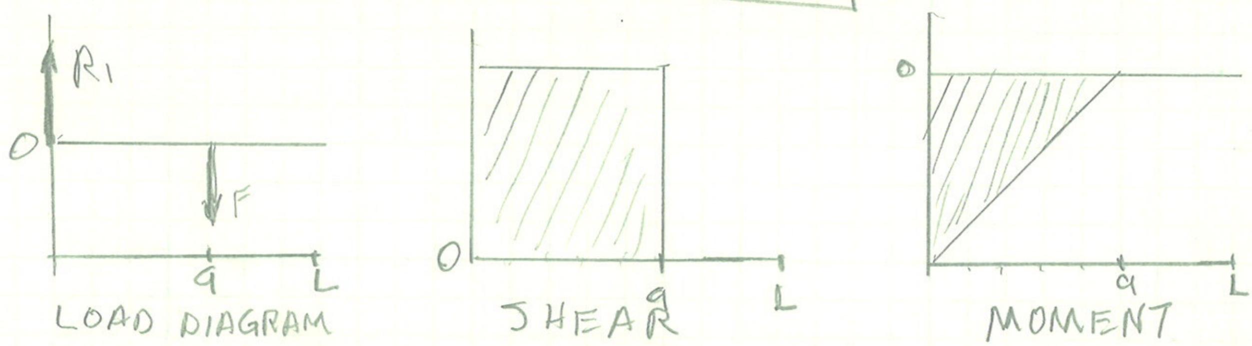
$= L$ @ $x = L$

$$M = -M_1\langle l - 0\rangle^0 + R_1\langle l - 0\rangle^1 - F\langle L - a\rangle^1 = 0$$

$$0 = -M_1 + R_1(L) - F(L-a)$$

$$M_1 = R_1(L) - F(L-a) = 40(10) - 40(10-4)$$

$$\boxed{M_1 = 160 \text{ NM CW}}$$



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