# HW #11

1.

$V_{DD} = 1.5\,V$, $k_n' = 100\ \mu A/V^2$, $V_{tn} = 0.4\,V$ $\lambda_n = 0.1\ V^{-1}$ $(\frac{W}{L})_n = 10$

$k_p' = 60\ \mu A/V^2$, $V_{tp} = -0.4\,V$, $\lambda_p = 0.2\,V^{-1}$ $(\frac{W}{L})_p = 17$

a)

$V_{OH} = V_{DD} = 1.5\ V$ $V_{OL} = 0\ V$

To find $V_m$: $I_{Dn} = |I_{Dp}|$

$$\frac{k_n'}{2}\left(\frac{W}{L}\right)_n (V_M - V_{tn})^2 (1 + \lambda_n V_{DS}) = \frac{k_p'}{2}\left(\frac{W}{L}\right)_p (V_{GS} - |V_{tp}|)^2 (1 + |\lambda_p||V_{DS}|)$$

$$\frac{100\ \mu A/V^2}{2}(10)(V_m - 0.4)^2(1 + 0.1 \times V_m) = \frac{60\ \mu A/V^2}{2}(17)(1.5 - V_M - 0.4)^2(1 + (0.2)V_m)$$

$$\boxed{V_M = 0.758\ V}$$

b)

$$g = \frac{-2}{\lambda_n + |\lambda_p|}\left[\frac{1}{V_M - V_{tn}} + \frac{1}{V_{DD} - V_M - |V_{tp}|}\right]$$

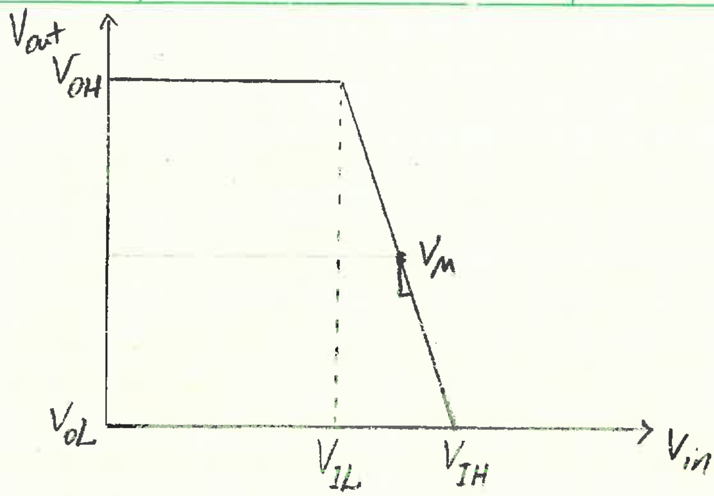
$$g = \frac{-2}{0.1 + 0.2}\left[\frac{1}{0.758 - 0.4} + \frac{1}{1.5 - 0.758 - 0.4}\right] = -38.045$$

c)

$$V_{IL} = V_M + \frac{V_{DD} - V_M}{V_g} = 0.758 + \frac{1.5 - 0.752}{-38.045} = 0.738\ V$$

$$V_{IH} = V_M - \frac{V_M}{V_g} = 0.758 - \left[\frac{0.758}{-38.045}\right] = 0.778\ V$$

d)




e) $NM_H = V_{OH} - V_{IH} = 1.5 - 0.778 = 0.722\ V$

$NM_L = V_{IL} - V_{OL} = 0.738 - 0 = 0.738\ V$

f)

$$I_{av} = \frac{1}{2}\left[ I_{DS}(V_{out} = 0) + I_{DS}\left(V_{out} = \frac{V_{DD}}{2}\right)\right]$$

$V_{out} = 0 \Rightarrow I_{DS} = \frac{k_p'}{2}\left(\frac{W}{L}\right)_p \left[V_{GS} - |V_{tp}|\right]^2 (1 + \lambda_p V_{DS})$

$$I_{DS} = \frac{60\ \mu A/V^2}{2}(17)\left[1.5 - 0.4\right]^2 (1 + 0.2 \times 1.5) = 802.23\ \mu A$$

$V_{out} = \frac{V_{DD}}{2} \Rightarrow I_{DS} = \frac{k_p'}{2}\left(\frac{W}{L}\right)_p \left[2(V_{GS} - |V_{tp}|)\frac{V_{DD}}{2} - \frac{V_{DD}^2}{4}\right] = 555\ \mu A$

$$\boxed{I_{av_p} = \frac{1}{2}(802.23\ \mu A + 555\ \mu A) = 678.6\ \mu A}$$

$$t_{PLH} = \frac{C_L \cdot (V_{DD}/2)}{I_{av_p}} = \frac{100 \times 10^{-15} \cdot (1.5/2)}{678.6 \times 10^{-6}} = 11.05 \times 10^{-11}\ s$$

$V_{out} = 0 \Rightarrow I_{DS} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_n \left[V_{GS} - V_{tn}\right]^2 (1 + \lambda_n V_{DS}) = 696\ \mu A$

$V_{out} = \frac{V_{DD}}{2} \Rightarrow I_{DS} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_p \left[2(V_{GS} - V_{tn})\frac{V_{DD}}{2} - \frac{V_{DD}^2}{4}\right] = 544\ \mu A$

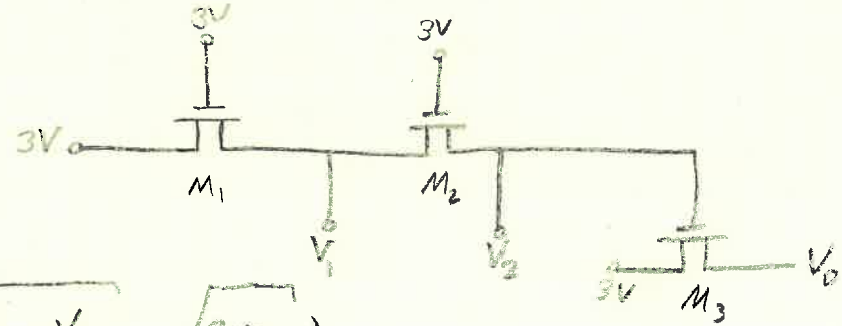
$$I_{av_n} = \frac{1}{2}(696\,\mu A + 544\,\mu A) = 620\,\mu A$$

$$t_{PHL} = \frac{100\times10^{-15}\times(1.5/2)}{620\times10^{-6}} = 120.96\ ps$$

g) $$t_r = \frac{C_L(0.9V_{DD} - 0.1V_{DD})}{I_{av}} = \frac{C_L\times0.8\times V_{DD}}{I_{av}} = \frac{100\times10^{-15}\times0.8\times1.5}{620\times10^{-6}} = 1.93\times10^{-10}\ s$$

$$t_f = \frac{C_L\times0.8\times V_{DD}}{I_{av}} = \frac{100\times10^{-15}\times1.5}{678.6\times10^{-6}} = 2.21\times10^{-10}\ s$$

2.




$M_1$

$$\begin{cases} V_1 = 3 - V_{T_1} \\ V_{T_1} = V_{T_0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right) \\ V_{SB} = V_1 \end{cases}$$

$$V_1 = 3 - \left(0.5 + 0.3\left(\sqrt{0.7 + V_1} - \sqrt{0.7}\right)\right)$$

$$\boxed{V_1 = 2.2369\ V}$$

$V_2$ has the same voltage $\Rightarrow$ $\boxed{V_2 = V_1 = 2.2369\ V}$

$M_3$

$$\begin{cases} V_0 = V_2 - V_3 \\ V_{T_3} = V_{T_0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right) \quad \text{and} \quad V_{SB} = V_0 \end{cases}$$

$$V_0 = 2.2369 - \left(0.5 + 0.3\left(\sqrt{0.7 + V_0} - \sqrt{0.7}\right)\right)$$

$$\boxed{V_0 = 1.539\ \text{V}}$$