ECE 321 - Bonus Homework Solutions

1) Connecting a test capacitor of 100 fF to the output:

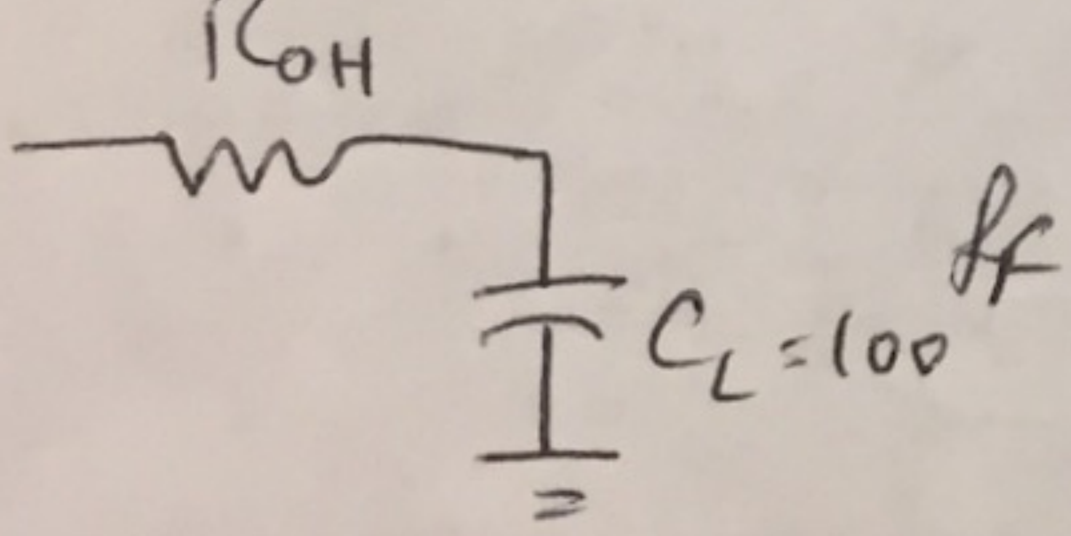
$$I_{av} = \frac{1}{2}\left[I_{DS}(V_{out}=0) + I_{DS}\left(V_{out}=\frac{V_{DD}}{2}\right)\right]$$

$$V_{out}=0 \rightsquigarrow I_{DS} = \frac{K'_p}{2}\left(\frac{W}{L}\right)_p (V_{gs}+|V_{tp}|)^2(1+\lambda V_{DS}) = 802\ \mu A$$

$$V_{out}=\frac{V_{DD}}{2} \rightsquigarrow I_{DS} = K'_p\left(\frac{W}{L}\right)_p\left[(V_{gs}-|V_{tp}|)\cdot\frac{V_{DD}}{2} - \frac{(V_{DD}/2)^2}{2}\right] = 555\ \mu A$$

$$\Rightarrow I_{av} = \frac{1}{2}(802\ \mu A + 555\ \mu A) = 678.5\ \mu A$$

$$\Rightarrow t_{PLH} = \frac{C_L (V_{DD}/2)}{I_{av}} = \frac{(100\ fF)\left(\frac{1.5}{2}\right)V}{678.5\ \mu A} = 11.05\ ns$$



$$t_{PLH} = 0.69\, R_{OH} C_L$$

$$\Rightarrow R_{OH} = \frac{t_{PLH}}{0.69\, C_L} = \frac{11.05\ ns}{0.69 * 100\ fF}$$

$$\Rightarrow R_{OH} = 1579\ \Omega$$

2) $\tau = (1.579^{k\Omega} + 120^{\Omega})\, 75^{fF}$

$+ (1.579^{k\Omega} + 120^{\Omega})(15^{fF} + 45^{fF} + 50^{fF})$

$+ (1.579^{k\Omega} + 120^{\Omega} + 165^{\Omega})\, 100^{fF}$

$+ (1.579^{k\Omega} + 120^{\Omega} + 165^{\Omega})\, 95^{fF}$

$+ (1.579^{k\Omega} + 120^{\Omega} + 165^{\Omega} + 105^{\Omega})\, 65^{fF}$

$+ (1.579^{k\Omega} + 120^{\Omega} + 165^{\Omega} + 105^{\Omega} + 220^{\Omega})\, 200^{fF}$

$\Rightarrow \tau = 1.24^{ns} \Rightarrow t_{PLH} = 0.69\tau = 0.69 * 1.24^{ns}$

$\Rightarrow t_{PLH} = 871^{ps}$

3) $V_A = V_B = 2^V, \quad V_C = 0$

$Q_{initial} = 125^{fF} * 2^V = 250^{fC}$

Case 1: $Q_{final} = (125^{fF} + 20^{fF} + 8^{fF})\, V_X$

$Q_{initial} = Q_{final} \Rightarrow V_X = 1.634^V > V_{DD} - V_T = 2 - 0.7 = 1.3^V$

impossible

Case 2:

$$Q_{final} = 125^{fF} * V_x + \underbrace{(V_{DD}-V_T)}_{2-0.7} * 20^{fF} + \underbrace{(V_{DD}-V_T)}_{2-0.7} * 8^{fF}$$

$$\Rightarrow Q_{final} = 125^{fF} * V_x + 36.4^{fC}$$

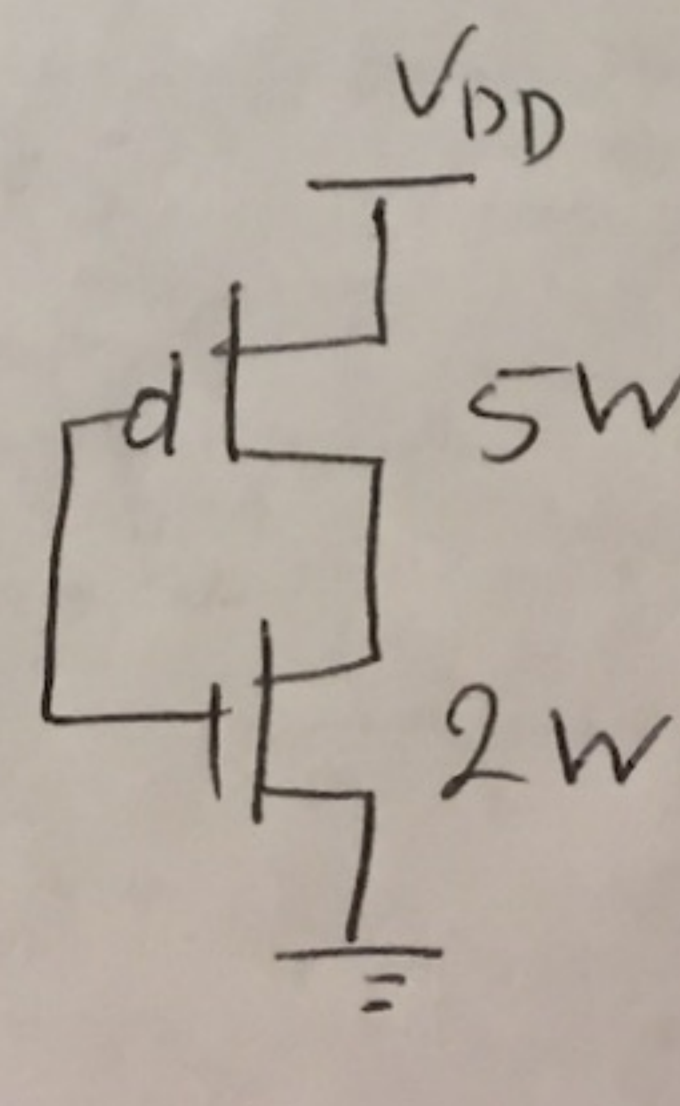
$$Q_{initial} = Q_{final} \Rightarrow 125^{fF} * V_x + 36.4^{fC} = 250^{fC}$$

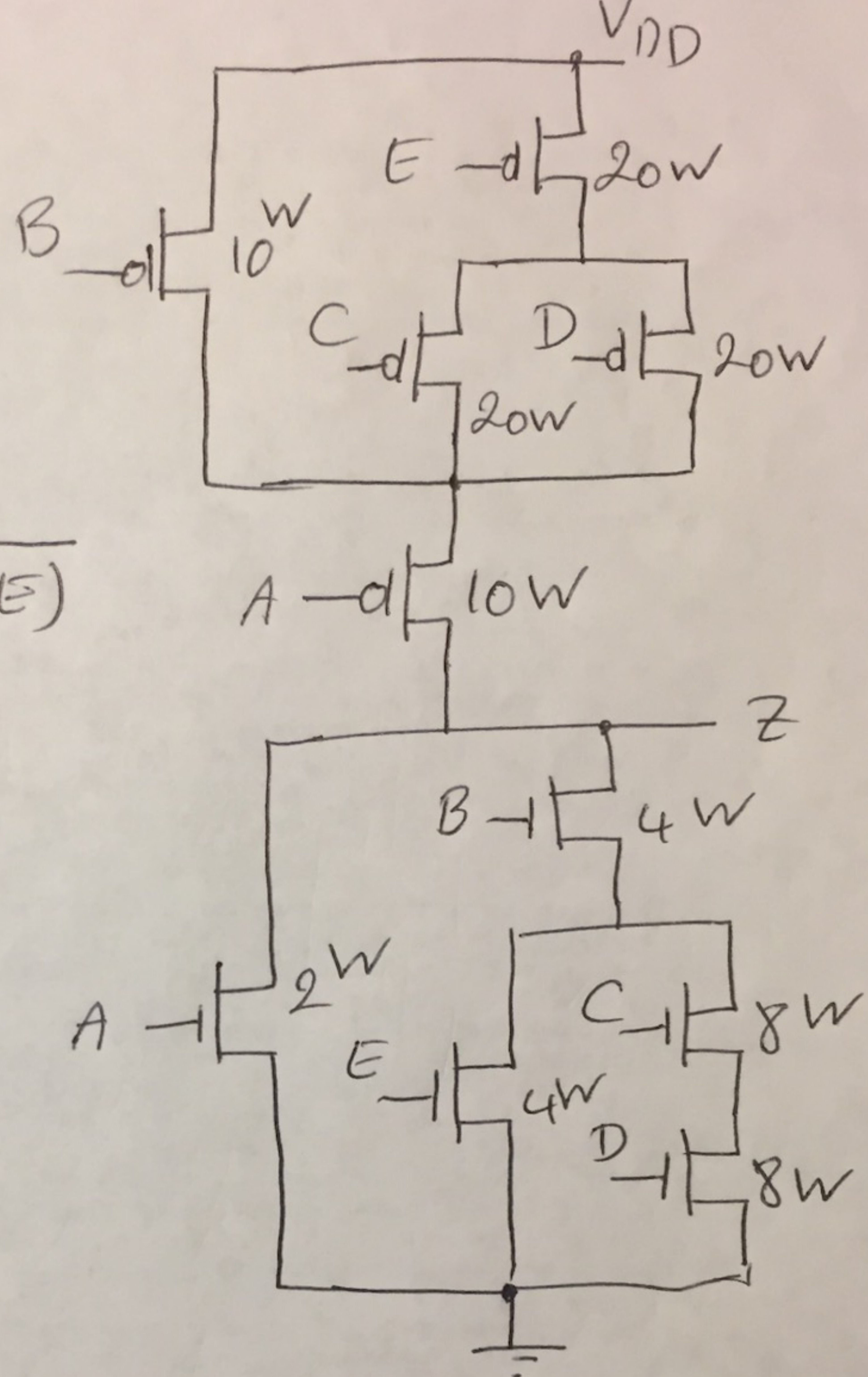
$$\Rightarrow V_x = 1.7088^{V}$$

$$\Rightarrow \begin{cases} V_{125fF} = 1.7088^{V} \\ V_{20fF} = 1.3^{V} \\ V_{8fF} = 1.3^{V} \end{cases}$$

4)

Reference:





$Z = \overline{A + B(CD+E)}$

5) $P_{leak} = \frac{1}{2} V_{DD} [I_{off(NMOS)} + I_{off(PMOS)}]$

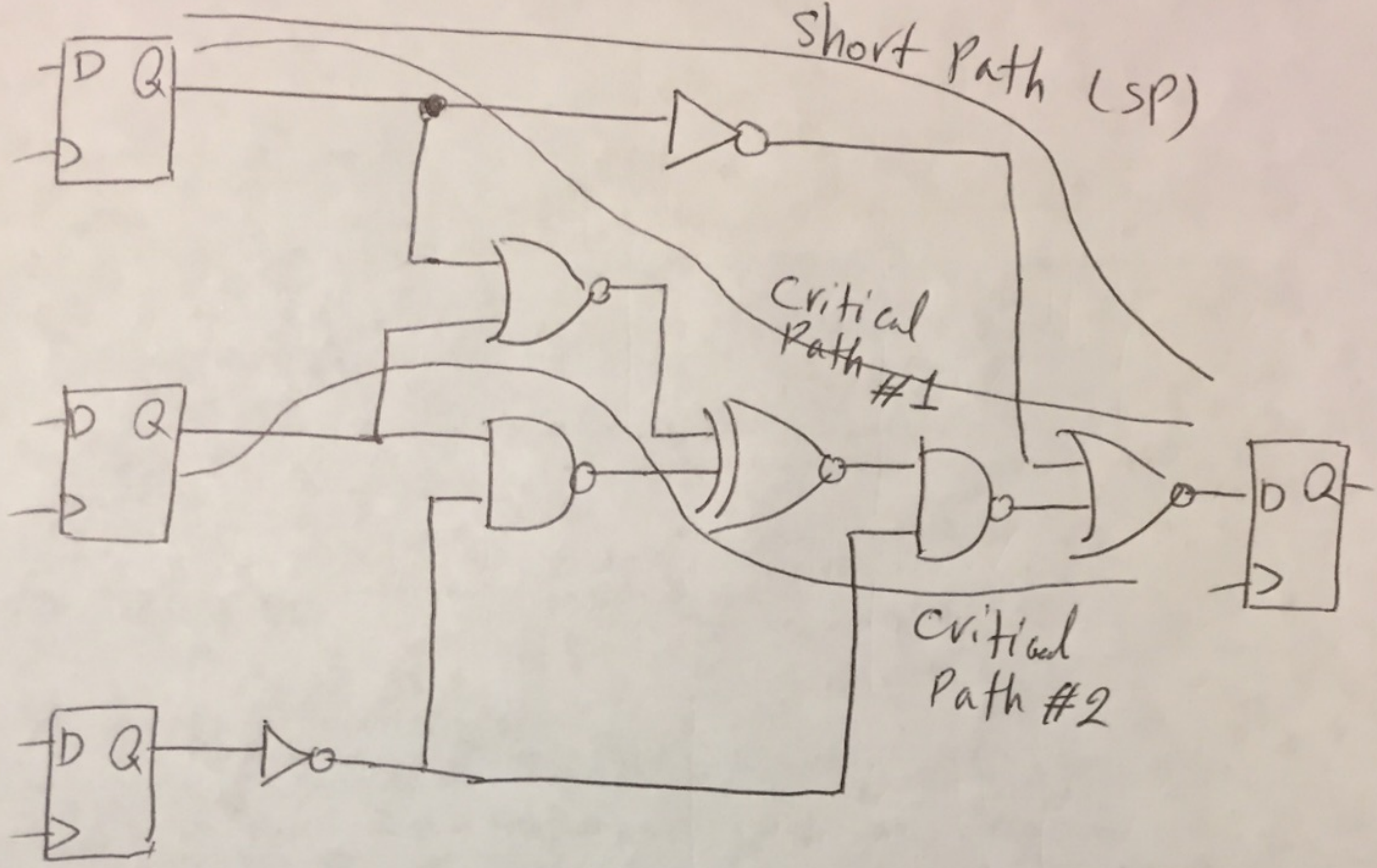
$W_n = 58.5^{\mu m}$ ⟶ $I_{off(NMOS)} = 702^{nA}$

$W_p = 85.5^{\mu m}$ ⟶ $I_{off(PMOS)} = 2223^{nA}$

$\Rightarrow P_{leakage} = \frac{1}{2} * 1.2 (702^{nA} + 2223^{nA}) = 1.755^{\mu W}$

6)

a)



b) there are two Critical Paths (#1 and #2) and one short Path. (SP)

$$T > t_{C2Q} + t_{logic} + t_{su}$$

$$\Rightarrow T > 1^{ns} + (550^{ps} + 750^{ps} + 350^{ps} + 450^{ps}) + 3^{ns}$$

$$\Rightarrow T > 6.1^{ns} \rightarrow f_{Max} = \frac{1}{T_{min}} = \frac{1}{6.1^{ns}} = 164^{MHZ}$$

c) SP is a short Path.

d) $t_{hold} \overset{?}{<} t_{c2Q} + t_{logic,CD}$

$2^{ns} \overset{?}{<} 1^{ns} + (150^{ps} + 450^{ps})$

$\Rightarrow 2^{ns} \not< 1.6^{ns}$ ⇝ this is a hold time Violation!

7) a) $Z = \overline{A(BC+DE)}$

b) ABCDE = 11010

c) if ABCDE = 11000 , $V_{DD} = 1.2^{V}$

Case 1:

$$\begin{cases} Q_{initial} = 20^{fF} * 1.2^{V} \\ Q_{final} = (20^{fF} + 3^{fF} + 2^{fF} + 2^{fF}) * V_x \end{cases}$$

$Q_{initial} = Q_{final} \Rightarrow V_x = 0.88 > V_{DD} - V_T = 1.2 - 0.4 = 0.8^{V}$

impossible

Therefore:

Case 2:

$$\begin{cases} Q_{initial} = 20^{fF} * 1.2^{V} = 24^{fC} \\ Q_{final} = 20^{fF} * V_x + 3^{fF} * (1.2 - 0.4^{V}) \\ \quad + 2^{fF}(1.2^{V} - 0.4^{V}) \\ \quad + 2^{fF}(1.2^{V} - 0.4^{V}) \\ \quad = 20^{fF} * V_x + 5.6^{fF} \end{cases}$$

$$\Rightarrow 20^{fF} V_x + 5.6^{fF} = 24^{fC} \Rightarrow V_x = 0.92^{V}$$

$$\Rightarrow \begin{cases} V_{20fF} = 0.92^{V} \\ V_{2fF} = V_{3fF} = V_{2fF} = 1.2^{V} - 0.4^{V} = 0.8^{V} \end{cases}$$

d) $t_{0 \to 90\%} = 250^{ps} \Rightarrow \Delta V = 0.9 * 1.2 = 1.08^{V}$

$$C_{total} = 20^{fF} + (3^{fF} + 2^{fF} + 2^{fF} + 3^{fF}) = 30^{fF}$$

$$I = \frac{K'_p}{2} \left(\frac{W}{L}\right)_p (|V_{GS}| - |V_{Tp}|)^2 = \frac{50^{\mu A/V^2}}{2} * \left(\frac{W}{L}\right)_p (1.2 - 0.5)^2$$

$$t_{0\to 90\%} = \frac{C_L \Delta V}{I} \Rightarrow 250^{ps} = \frac{30^{ff} * (1.08^V)}{\frac{50^{\mu A/V^2}}{2}\left(\frac{W}{L}\right)_p (1.2-0.5)^2}$$

$$\Rightarrow \left(\frac{W}{L}\right)_p = 10.58 \Rightarrow W_p = 1.058^{\mu m}$$

8)