Homework 6 Solutions

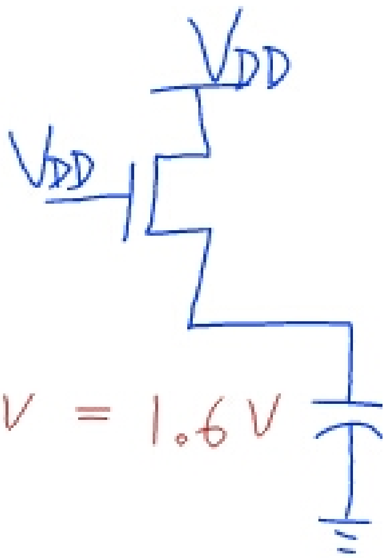
Problem 1

a) V_{OL} is limited by the PMOS. The source of the PMOS is on the output node.

$V_{SG} > V_T$ to be on

The PMOS is on when $V_G = 0V$

The output capacitor is then connected to ground and continues to discharge until $V_{SG} = V_T$ and the connection to ground is broken.



$V_{out,min} = V_{Tp}$

Similarly, $V_{OH} = V_{DD} - V_{tn}$

$V_{swing} = V_{OH} - V_{OL} = 2.1 - 0.5V = 1.6V$

b)

For input $0 \rightarrow 2.5$

Energy supplied:

$E = \int P dt = \int i(t) V_{DD} dt$

$= \int C \frac{dV_c}{dt} V_{DD} dt = C V_{DD} \int dV_c$

$= C V_{DD} V_c = C V_{DD} [V_{DD} - V_{tn} - V_{tp}]$

$= 0.6 \, pJ$

Energy dissipated:

energy in capacitor

$E = \int P dt = \int i(t) V_c dt$

$= \int C \, dV_c/dt \; V_c dt = \int C \, V_c dV_c$

$= \frac{1}{2} C V_c^2 \Big|_{|V_{tp}|}^{V_{DD}-V_{tn}}$

$= \frac{1}{2} C \left[(V_{DD} - V_{tn})^2 - V_{tp}^2 \right]$

$= 3.12 \times 10^{-13} J$

$E_{diss} = E_{supp.} - E_{cap}$

$E_{diss} = 0.288 \, pJ$

for input $2.5 \rightarrow 0$

$E_{supp} = 0 J$ Capacitor is discharging

$E_{diss} = E_{cap} = 0.312 J$

c) $t_{PLH} = C \Delta V / I_{av}$

$\Delta V = (V_{OH} + V_{OL})/2 - V_{OL} = 0.8V$

$I_{av} = (I_1 + I_2)/2$

I_1: $V_{out} = V_{OL} = 0.5V$

$V_{DS} = 2V$

$V_G = V_{DD}$

$V_{GS} = V_{DD} - V_{out} = 2V$

$V_{DS} > V_{GS} - V_{tn}$

Sat!

$I_1 = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_{tn})^2 = 768 \mu A$

I_2: $V_{out} = (V_{OH} + V_{OL})/2 = 1.3V$

$V_{DS} = 1.2V$

$V_{GS} = 1.2V$

Sat

$I_2 = \frac{1}{2} k_n \frac{W}{L} (V_{GS} - V_{tn})^2 = 192 \mu A$

Because $V_G = V_D$ for all values of V_{out} the device stays in saturation.

So then $I_{av} = (I_1 + I_2) / 2 = 480 \mu A$

$\Delta t = C \Delta V / I_{av} = (150fF)(0.8V) / 480 \mu A$

$\Delta t = 2.5 \times 10^{-10} = 0.25ns$

d) $V_{OH} = V_{DD} - V_{tn}$

$V_{tn} = V_{t0} + \gamma\left[\sqrt{|2\phi_F| + |V_{SB}|} - \sqrt{|2\phi_F|}\right]$

$V_{OH} = V_{DD} - \left(V_{t0} + \gamma\left[\sqrt{|2\phi_F| + |V_{SB}|} - \sqrt{|2\phi_F|}\right]\right)$

$V_{OH} = V_{DD} - \left(V_{t0} + \gamma\left[\sqrt{|2\phi_F| + V_{OH}} - \sqrt{|2\phi_F|}\right]\right)$

$V_{OH} = 1.791V$

Problem 2

a) $I_D = I_0 \exp(V_{GS}/(nkT/q)) (1 - \exp[-V_{DS}/(kT/q)])$

$I_{Dp} = I_{Dn}$

$|V_{GSp}| = 0.4 - V_m$

$V_{GSn} = V_m$

$|V_{DSp}| = 0.4 - V_m$

$V_{DSn} = V_m$

Focus on $(1 - \exp[-V_{DS}/(kT/q)])$ for a moment

assuming $V_m = \frac{1}{2} V_{DD} = 0.2$ V since V_m is usually near the center of the voltage rails

$V_{DSn} = |V_{DSp}| = 0.2$

$1 \gg \exp[-V_{DS}/V_t]$

so $(1 - \exp[-V_{DS}/(kT/q)]) \approx 1$

now back to $I_{Dn} = I_{Dp}$

$\exp(V_{GSn}/nV_t) = \exp(|V_{GSp}|/nV_t)$

$V_{GSn} = |V_{GSp}|$

$V_m = 0.4 - V_m$

$V_m = 0.2V$

b) $g = -\frac{1}{n}(\exp[V_{DD}/2V_t] - 1)$

$n = 1.5$

$g = -1504$

$V_{IL} = V_m + (V_{DD} - V_m)/g = 0.1999\,V$

$V_{IH} = V_m - \frac{V_m}{g} = 0.2001\,V$

c) $V_{OL} = 0V$, $V_{OH} = V_{DD}$

$NM_H = V_{OH} - V_{IH} = 0.1999\,V$

$NM_L = V_{IL} - V_{OL} = 0.1999V$