

**ECE 523/421 - Analog Electronics:**  
**University of New Mexico**  
**Solutions Homework 1**

**Problem 5.4**

**An NMOS transistor that is operated with a small  $v_{DS}$  is found to exhibit a resistance  $r_{DS}$ . By what factor will  $r_{DS}$  change in each of the following situations?**

Using the formula for the drain current for a small  $v_{DS}$ .

$$i_D = \left[ (\mu_n C_{ox}) \left( \frac{W}{L} \right) (v_{OV}) \right] v_{DS}$$

We can obtain  $r_{DS} = \frac{v_{DS}}{i_D}$

$$r_{DS} = \frac{1}{(\mu_n C_{ox}) \left( \frac{W}{L} \right) (v_{OV})}$$

**a)  $v_{OV}$  is doubled.**

$$r_{DS}' = \frac{1}{(\mu_n C_{ox}) \left( \frac{W}{L} \right) (2v_{OV})}$$
$$r_{DS}' = \frac{r_{DS}}{2}$$

The resistance is reduced by half when  $v_{OV}$  is doubled.

**b) The device is replaced with another fabricated in the same technology but with double the width.**

$$r_{DS}' = \frac{1}{(\mu_n C_{ox}) \left( \frac{2W}{L} \right) (v_{OV})}$$
$$r_{DS}' = \frac{r_{DS}}{2}$$

The resistance is reduced by half when the width  $W$  is doubled.

- c) The device is replaced with another fabricated in the same technology but with both the width and length doubled.

$$r_{DS}' = \frac{1}{(\mu_n C_{ox}) \left( \frac{2W}{2L} \right) (v_{OV})}$$

$$r_{DS}' = r_{DS}$$

The resistance remains the same when the width W the length L are doubled.

- d) The device is replaced with another fabricated in a more advanced technology for which the oxide thickness is halved and similarly for W and L (assume  $\mu_n$  remains unchanged).

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$r_{DS}' = \frac{1}{\left( \mu_n \frac{\epsilon_{ox}}{t_{ox}} \right) \left( \frac{W}{L} \right) (v_{OV})}$$

$$r_{DS}' = \frac{r_{DS}}{2}$$

The resistance is reduced by half when the oxide thickness  $t_{ox}$ , width W and length L are halved.

### Problem 5.7

An n-channel MOS device in a technology for which oxide thickness is 4 nm, minimum channel length is  $0.18\text{ }\mu\text{m}$ ,  $K'_n = 400\text{ }\mu\text{A}/\text{V}^2$ , and  $V_t = 0.5\text{ V}$  operates in the triode region, with small  $v_{DS}$  and with the gate–source voltage in the range 0V to +1.8V. What device width is needed to ensure that the minimum available resistance is  $1\text{ k}\Omega$ ?

Using the formula for the drain current for a small  $v_{DS}$ .

$$i_D = \left[ (\mu_n C_{ox}) \left( \frac{W}{L} \right) (v_{OV}) \right] v_{DS}$$

We can obtain  $r_{DS} = \frac{v_{DS}}{i_D}$

$$r_{DS} = \frac{1}{(k'_n) \left( \frac{W}{L} \right) (v_{GS} - V_t)}$$

Substituting the values into the equation (check the units).

$$1000\Omega = \frac{1}{(400 \times 10^{-6}) \left( \frac{W}{0.18 \times 10^{-6}} \right) (1.8 - 0.5)}$$

Solving for W

$$W = \frac{1}{(400 \times 10^{-6}) \left( \frac{1000}{0.18 \times 10^{-6}} \right) (1.8 - 0.5)}$$

$$W = 0.346\text{ }\mu\text{m}$$

### Problem 5.25

For the circuit in Fig. P5.25, sketch  $i_D$  versus  $v_S$  for  $v_S$  varying from 0 to  $V_{DD}$ . Clearly label your sketch.

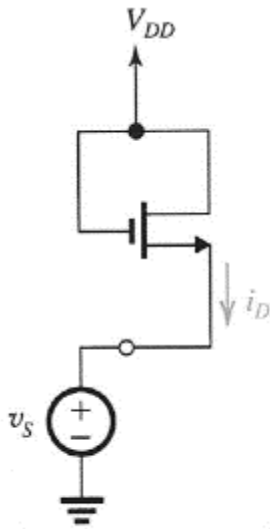


Figure P5.25

$$V_{GD} = 0 \text{ V}$$

$$V_{GS} = V_{DD} - v_S$$

For  $V_{GD} < V_{th}$  the transistor is in saturation

The drain current  $i_D$  is:

$$i_D = \frac{1}{2} k_n (v_{GS} - V_t)^2$$

Substituting  $V_{GS} = V_{DD} - v_S$

$$i_D = \frac{1}{2} k_n (V_{DD} - v_S - V_t)^2$$

$$i_D = \frac{1}{2} k_n [(V_{DD} - V_t)^2 - 2v_S(V_{DD} - V_t) + v_S^2]$$

For  $V_{GS} < V_{th}$  the transistor is in cut-off region

Substituting  $V_{GS} = V_{DD} - v_S$

$$V_{DD} - v_S < V_t$$

$$v_S > V_{DD} - V_t$$

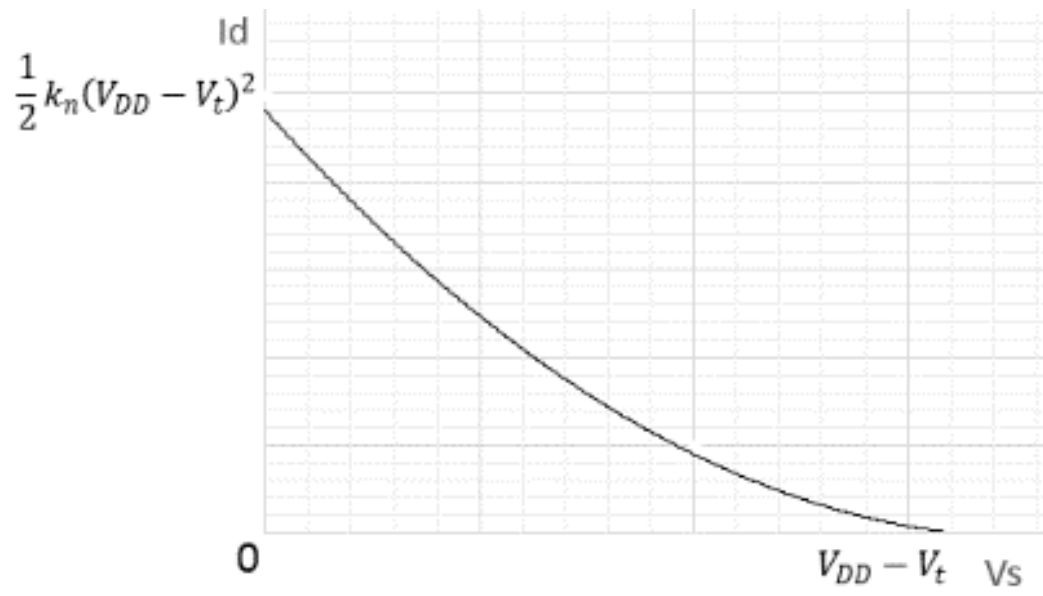
For  $v_S > V_{DD} - V_t$  the drain current is zero, so we only need to plot in the region:

$$0 \leq v_S \leq V_{DD} - V_t$$

When  $v_S = 0$ ,  $i_D$  is:

$$i_D = \frac{1}{2} k_n [(V_{DD} - V_t)^2 - 2(0)(V_{DD} - V_t) + 0^2]$$

$$i_D = \frac{1}{2} k_n (V_{DD} - V_t)^2$$



Plot  $I_D$  vs.  $V_S$

### Problem 5.32

In a particular IC design in which the standard channel length is  $1\text{ }\mu\text{m}$ , an NMOS device with  $W/L$  of 10 operating at  $200\text{ }\mu\text{A}$  is found to have an output resistance of  $100\text{ k}\Omega$ , about  $1/5$  of that needed. What dimensional change can be made to solve the problem? What is the new device length? The new device width? The new  $W/L$  ratio? What is  $V_A$  for the standard device in this IC? The new device?

Using the device parameter  $V_A$

$$V_A = V_A' L$$

And  $r_o$

$$r_o = \frac{V_A}{I_D'} = \frac{V_A' L}{I_D'}$$

To obtain the output resistance needed that is  $500\text{ k}\Omega$  ( $100 \times 5$ ) we need to increase the channel length  $L$  5 times, so the new channel length will be  $5\text{ }\mu\text{m}$ .

Also to keep the operating current unchanged, the ratio  $\left(\frac{W}{L}\right)$  has to be constant and therefore the width  $W$  will be  $50\text{ }\mu\text{m}$ . With this value of  $W$ , we assure that the ratio is the same.

$V_A$  for the standard device is:

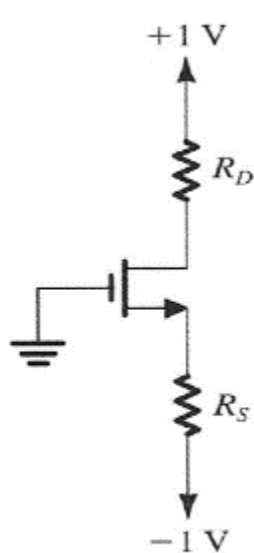
$$V_A = r_o I_D' = (100\text{ k}\Omega)(200\text{ }\mu\text{A}) = 20\text{ V}$$

$V_A$  for the new device is:

$$V_A = 5r_o I_D' = (500\text{ k}\Omega)(200\text{ }\mu\text{A}) = 100\text{ V}$$

### Problem 5.45

The NMOS transistor in the circuit of Fig. P5.44 has  $V_t = 0.4 \text{ V}$  and  $k_n = 4 \text{ mA/V}^2$ . The voltages at the source and the drain are measured and found to be  $-0.6 \text{ V}$  and  $+0.2 \text{ V}$ , respectively. What current  $I_D$  is flowing, and what must the values of  $R_D$  and  $R_S$  be? What is the largest value for  $R_D$  for which  $I_D$  remains unchanged from the value found?



$$V_G = 0 \text{ V}$$

$$V_t = 0.4 \text{ V}$$

$$v_s = -0.6 \text{ V}$$

$$v_d = 0.2 \text{ V}$$

Reading the values from the circuit.

$$v_{DS} = 0.8 \text{ V}$$

$$v_{GS} = 0.6 \text{ V}$$

Checking that the transistor is operating in saturation mode.

$$v_{DS} > (v_{GS} - V_t)$$

$$0.8 > (0.6 - 0.4)$$

Figure P5.44

The NMOS is indeed in saturation mode.

The drain current for the NMOS in saturation mode is:

$$i_D = \frac{1}{2} k_n (v_{GS} - V_t)^2$$

$$i_D = \frac{1}{2} 4 \text{ mA/V}^2 (0.6 - 0.4)^2$$

$$i_D = 0.08 \text{ mA}$$

Determining  $R_D$

$$R_D = \frac{V_{DD} - V_D}{I_D}$$

$$R_D = \frac{1 - 0.2}{0.08 \text{ mA}}$$

$$R_D = 10 \text{ k}\Omega$$

Determining  $R_S$

$$R_S = \frac{V_S - V_{SS}}{I_D}$$

$$R_S = \frac{-0.6 - (-1)}{0.08mA}$$

$$R_S = 5k\Omega$$

**Problem 5.47**

The transistor in the circuit of Fig. P5.47 has  $k_n' = 0.4 \text{ mA/V}^2$ ,  $V_t = 0.4 \text{ V}$ , and  $\lambda = 0$ . Show that operation at the edge of saturation is obtained when the following condition is satisfied:

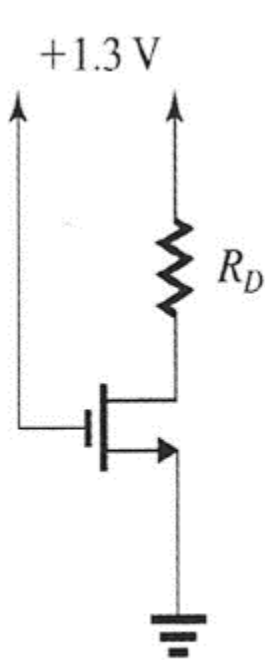


Figure P5.47

$$\left(\frac{W}{L}\right) R_D \cong 2.5 k\Omega$$

$$V_t = 0.4 \text{ V}$$

$$k_n' = 0.4 \text{ mA/V}^2$$

$$V_S = 0 \text{ V}$$

$$V_G = 1.3 \text{ V}$$

When  $\lambda = 0$ , the drain current  $i_D$  is:

$$i_D = \frac{1}{2} (k_n') \left(\frac{W}{L}\right) (v_{GS} - V_t)^2$$

$$i_D = \frac{1}{2} (0.4 \text{ mA/V}^2) \left(\frac{W}{L}\right) (1.3 - 0 - 0.4)^2$$

$$i_D = \frac{1}{2} (0.4 \text{ mA/V}^2) \left(\frac{W}{L}\right) (1.3 - 0 - 0.4)^2$$

$$i_D = 1.62 \times 10^{-4} \left(\frac{W}{L}\right) \text{ A}$$

At the edge of saturation

$$v_{DS} = (v_{GS} - V_t)$$

Since  $v_S = 0$

$$v_D - 0 = (1.3 - 0 - 0.4)$$

$$v_D = 0.9 \text{ V}$$

$$R_D = \frac{V_{DD} - V_D}{I_D}$$

$$R_D = \frac{1.3 - 0.9}{1.62 \times 10^{-4} \left(\frac{W}{L}\right)}$$

$$R_D \left(\frac{W}{L}\right) = \frac{1.3 - 0.9}{1.62 \times 10^{-4}}$$

$$R_D\left(\frac{W}{L}\right) = 2469\,\Omega \cong 2.5k\Omega$$