ECE 523/421 - Analog Electronics:
University of New Mexico
Solutions Homework 2

Problem 5.49
The PMOS transistor in the circuit of Fig. P5.49 has $V_t = -0.5$ V, $\mu_p C_{ox} = 100\ \mu A/V^2$, L = 0.18 μm, and λ=0. Find the values required for W and R in order to establish a drain current of 180 μA and a voltage V_D of 1 V.

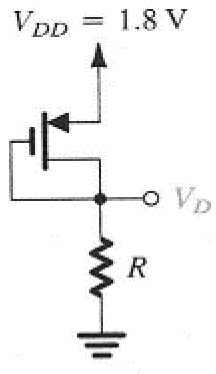


Figure P5.49

For having a drain current of 180 μA and a voltage V_D of 1 V.

$$R = \frac{V_D}{I_D} = \frac{1}{180x10^{-6}} = 5.55\ k\Omega$$

Using the equation for the drain current.

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

And:

$$v_{GS} = V_D - V_{DD} = 1 - 1.8 = -0.8\ V$$

We obtain:

$$180x10^{-6} = \frac{1}{2}(100x10^{-6} A/V^2)\left(\frac{W}{0.18x10^{-6}}\right)(-0.8 - (-0.5))^2$$

$$W = 7.2\ \mu m$$

Problem 5.51

The NMOS transistors in the circuit of Fig. P5.51 have V_t = 0.5 V, $\mu_n C_{ox}$ = 90 μA/V², λ = 0, and L1 = L2 = L3 = 0.5 μm. Find the required values of gate width for each of Q1, Q2, and Q3 to obtain the voltage and current values indicated.

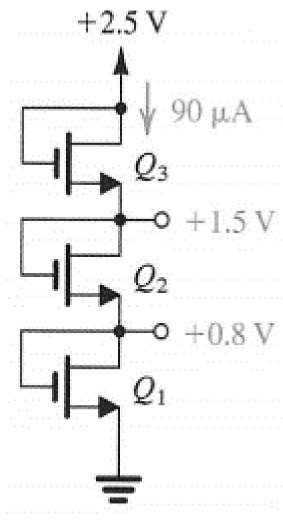


Figure P5.51

For Q3

- V_D = 2.5 V
- V_S = 1.5 V
- V_D = V_G. Transistor operates in the saturation region.

The expression for the drain current in saturation is:

$$i_D = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

Substituting the values in the equation:

$$90x10^{-6} = \frac{1}{2}(90x10^{-6})\left(\frac{W}{0.5x10^{-6}}\right)((2.5 - 1.5) - 0.5)^2$$

$$W = 4\ \mu m$$

For Q2

- V_D = 1.5 V
- V_S = 0.8 V
- V_D = V_G. Transistor operates in the saturation region.

The expression for the drain current in saturation is:

$$i_D = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

Substituting the values in the equation:

$$90x10^{-6} = \frac{1}{2}(90x10^{-6})\left(\frac{W}{0.5x10^{-6}}\right)((1.5 - 0.8) - 0.5)^2$$

$$W = 25\ \mu m$$

For Q1

- V_D = 0.8 V
- V_S = 0 V
- V_D = V_G. Transistor operates in the saturation region.

The expression for the drain current in saturation is:

$$i_D = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

Substituting the values in the equation:

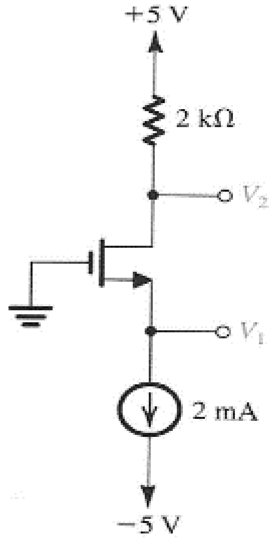
$$90x10^{-6} = \frac{1}{2}(90x10^{-6})\left(\frac{W}{0.5x10^{-6}}\right)((0.8 - 0) - 0.5)^2$$

$$W = 11.1\ \mu m$$

Problem 5.55

In the circuits shown in Fig. P5.55, transistors are characterized by $|V_t|$ =1 V, k'W/L = 4 mA/V^2, and $\lambda = 0$.

- **Find the labeled voltages V_1 through V_7.**
- **In each of the circuits, replace the current source with a resistor. Select the resistor value to yield a current as close to that of the current source as possible, while using resistors specified in the 1% table provided in Appendix J.**



(a)

For Circuit a:

$$V_2 = 5 - 2x2 = 1V$$

Assuming that the transistors are in saturation:

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

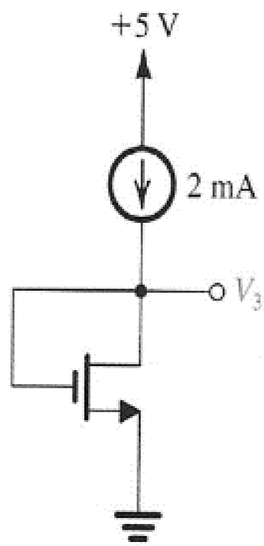
$$2x10^{-3} = \frac{1}{2}(4x10^{-3})(v_{GS} - 1)^2$$

$$V_{GS} = 2V$$

$$V_1 = -2V$$

Our assumption was correct:

$$V_{DS} = 3V > V_{GS} - V_t$$



(b)

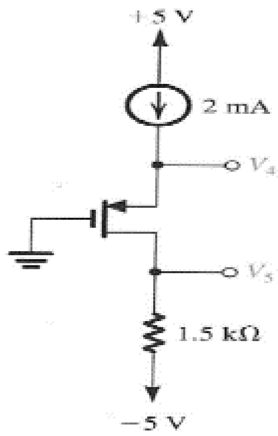
For circuit b:

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$2x10^{-3} = \frac{1}{2}(4x10^{-3})(v_{GS} - 1)^2$$

$$V_{GS} = 2V$$

$$V_3 = 2V$$

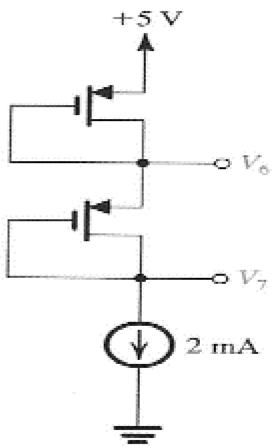


(c)

For circuit c:

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$
$$2x10^{-3} = \frac{1}{2}(4x10^{-3})(v_{GS} - 1)^2$$
$$V_{GS} = -2V$$
$$V_4 = 2V$$

$$V_5 = -5 + 1.5x2 = -2V$$



(d)

For circuit d:

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$
$$2x10^{-3} = \frac{1}{2}(4x10^{-3})(v_{GS} - 1)^2$$
$$V_{GS} = -2V$$
$$V_6 = 5 - 2 = 3V$$

$$V_7 = V_6 - 2 = 1V$$

Replace the current source with a resistor for each circuit:

- Circuit a:

$$R = \frac{V_1 - (-5)}{2mA} = 1.5k\Omega \cong 1.499k\Omega$$

- Circuit b:

$$R = \frac{5 - V_3}{2mA} = 1.5k\Omega \cong 1.499k\Omega$$

- Circuit c:

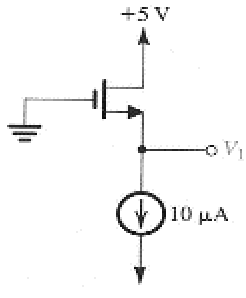
$$R = \frac{5 - V_4}{2mA} = 1.5k\Omega \cong 1.499k\Omega$$

- Circuit d:

$$R = \frac{V_7}{2mA} = 0.5k\Omega \cong 0.499k\Omega$$

Problem 5.56

For each of the circuits in Fig. P5.56, find the labeled node voltages. For all transistors, $k_n'(W/L)$ = 0.5 mA/V2, V_t = 0.8 V, and λ = 0.



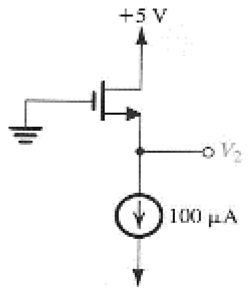
(a)

The drain voltage when λ = 0 is

$$i_D = \frac{1}{2}(k_n')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$10x10^{-6} = \frac{1}{2}(0.5x10^{-3})(0 - V_1 - 0.8)^2$$

$$V_1 = -1V$$



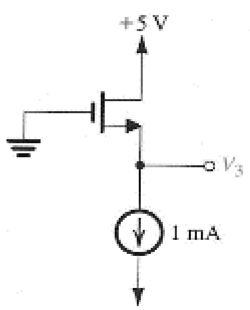
(b)

The drain voltage when λ = 0 is

$$i_D = \frac{1}{2}(k_n')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$100x10^{-6} = \frac{1}{2}(0.5x10^{-3})(0 - V_2 - 0.8)^2$$

$$V_2 = -1.43V$$



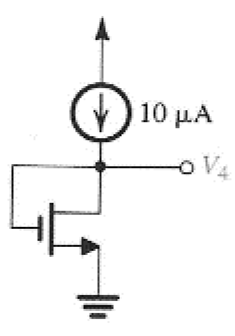
(c)

The drain voltage when λ = 0 is

$$i_D = \frac{1}{2}(k_n')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$1x10^{-3} = \frac{1}{2}(0.5x10^{-3})(0 - V_3 - 0.8)^2$$

$$V_3 = -2.8V$$



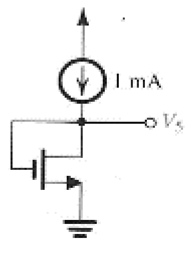
(d)

The drain voltage when λ = 0 is

$$i_D = \frac{1}{2}(k_n')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$10x10^{-6} = \frac{1}{2}(0.5x10^{-3})(V_4 - 0.8)^2$$

$$V_4 = 1V$$



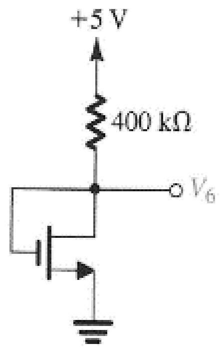
(e)

The drain voltage when λ = 0 is

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$1x10^{-3} = \frac{1}{2}(0.5x10^{-3})(V_5 - 0.8)^2$$

$$V_5 = 2.8V$$



(f)

$$i_D = \frac{5 - V_6}{400k\Omega}$$

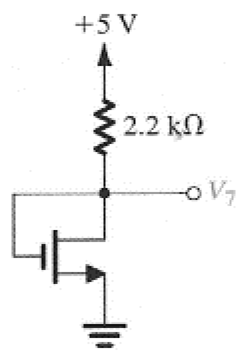
The drain voltage when λ = 0 is

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$\frac{5 - V_6}{400k\Omega} = \frac{1}{2}(0.5x10^{-3})(V_6 - 0.8)^2$$

$$0.01(5 - V_6) = (V_6 - 0.8)^2$$

$$V_6 = 1V$$



(g)

$$i_D = \frac{5 - V_7}{2.2k\Omega}$$

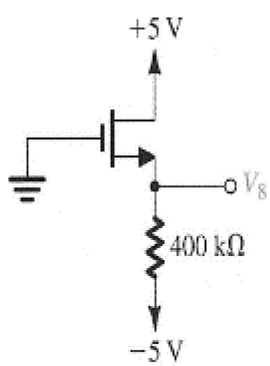
The drain voltage when λ = 0 is

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$\frac{5 - V_7}{2.2k\Omega} = \frac{1}{2}(0.5x10^{-3})(V_7 - 0.8)^2$$

$$1.82(5 - V_7) = (V_7 - 0.8)^2$$

$$V_7 = 2.8V$$



(h)

$$i_D = \frac{V_8 - (-5)}{400k\Omega}$$

The drain voltage when λ = 0 is

$$i_D = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

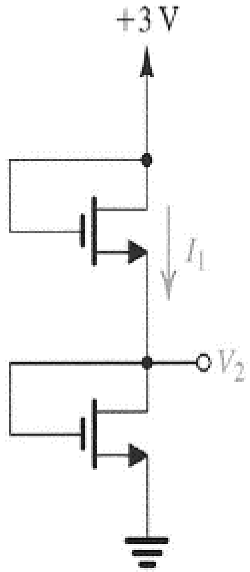
$$\frac{V_8 + 5}{400k\Omega} = \frac{1}{2}(0.5x10^{-3})(V_8 - 0.8)^2$$

$$0.01(V_8 + 5) = (V_8 - 0.8)^2$$

$$V_8 = -1V$$

Problem 5.59

For the circuits in Fig. P5.59, $\mu_n C_{ox} = 3\,\mu_p C_{ox} = 270\ \mu A/V^2$, $|V_t| = 0.5$ V, $\lambda = 0$, L = 1 μm, and W = 3 μm, unless otherwise specified. Find the labeled currents and voltages.



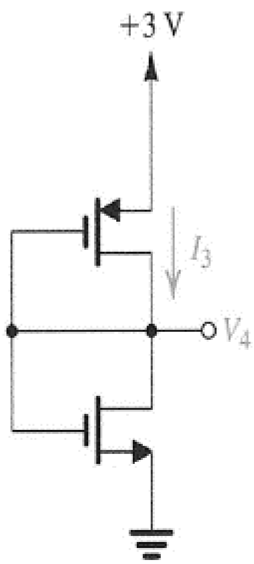
(a)

The transistors are operating in saturation:

$$i_{D1} = i_{D2}$$
$$v_{GS1} = v_{GS2}$$
$$3 = v_{GS1} + v_{GS2}$$
$$v_{GS1} = v_{GS2} = 1.5V$$
$$V_2 = 1.5\ V$$

The expression for the drain current in saturation is:

$$i_1 = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$
$$i_1 = \frac{1}{2}(270)\left(\frac{3}{1}\right)(1.5 - 0.5)^2$$
$$i_1 = 0.405\ mA$$



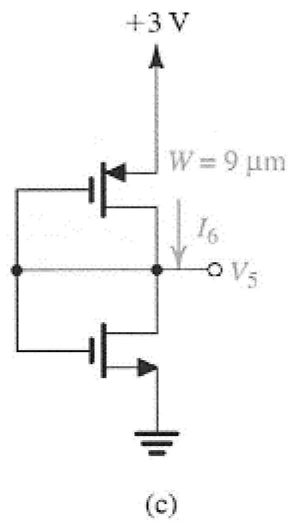
(b)

The transistors are operating in saturation:

$$V_D = V_G$$
$$i_{D1} = i_{D2}$$
$$\frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(v_{GS} - V_t)^2 = \frac{1}{2}(\mu_p C_{ox})\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$
$$3(v_4 - 0.5)^2 = (3 - v_4 - 0.5)^2$$
$$V_4 = 1.233V$$

The expression for the drain current in saturation is:

$$i_3 = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$
$$i_3 = \frac{1}{2}(270)\left(\frac{3}{1}\right)(1.233 - 0.5)^2$$
$$i_3 = 0.218\ mA$$



Using the equation for the drain current:

$$i_{D1} = i_{D2}$$

$$\frac{1}{2}(\mu_n C_{ox})\left(\frac{W_1}{L}\right)(v_{GS} - V_t)^2 = \frac{1}{2}(\mu_p C_{ox})\left(\frac{W_2}{L}\right)(v_{GS} - V_t)^2$$

$$\left(\frac{W_1}{L}\right) = \frac{9\mu m}{1\mu m} = 9$$

$$\left(\frac{W_2}{L}\right) = \frac{3\mu m}{1\mu m} = 3$$

$$\left(\frac{W_1}{L}\right) \Big/ \left(\frac{W_2}{L}\right) = 3$$

Since:

$$(\mu_n C_{ox})\left(\frac{W_2}{L}\right) = (\mu_p C_{ox})\left(\frac{W_1}{L}\right)$$

$$(\mu_n C_{ox}) = 3(\mu_p C_{ox})$$

Both are equal so:

$$v_{GS1} = v_{GS2}$$

$$3 = v_{GS1} + v_{GS2}$$

$$v_{GS1} = v_{GS2} = 1.5V$$

$$V_5 = 1.5\ V$$

Now using the equation for the drain current

$$i_6 = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$

$$i_6 = \frac{1}{2}(270)\left(\frac{3}{1}\right)(1.5 - 0.5)^2$$

$$i_6 = 0.405\ mA$$

Problem 5.60

For the devices in the circuit of Fig. P5.60, $|V_t|$ = 1 V, λ = 0, $\mu_n C_{ox}$ = 50 µA/V^2, L = 1 µm, and W = 10 µm. Find V_2 and I_2. How do these values change if Q3 and Q4 are made to have W = 100 µm?

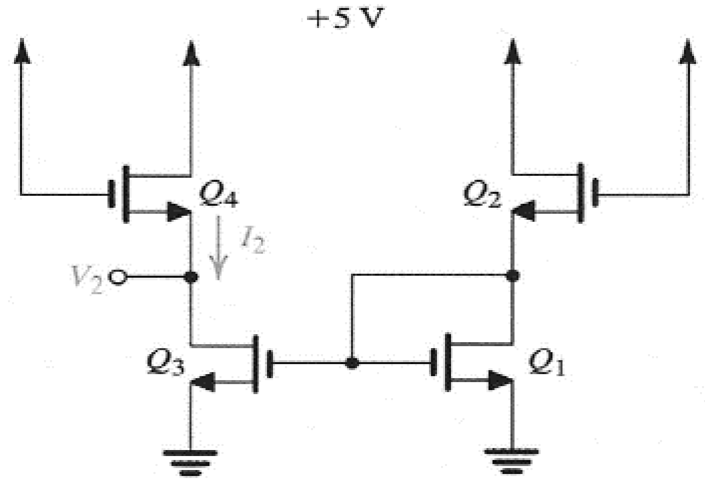


Figure P5.60

Transistor Q1 is on saturation because the gate ad drain terminals are short circuited. Also the drain currents of both transistors Q1 and Q2 are the same. So the V_{GS} for both transistors are the same.

$$V_{GS1} = V_{GS2}$$

Applying KVL.

$$-5 + V_{GS1} + V_{GS2} = 0$$
$$V_{GS1} = V_{GS2} = 2.5V$$

Using the equation for the drain current for transistor Q1:

$$I_{D1} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS1} - V_t)^2$$
$$I_{D1} = \frac{1}{2}(50x10^{-6})\left(\frac{10}{1}\right)(2.5 - 1)^2$$
$$I_{D1} = 562.5\ \mu A$$
$$I_{D1} = I_{D2} = 562.5\ \mu A$$

Also from the circuit:

$$V_{GS1} = V_{GS3} = 2.5V$$

Q1 and Q3 are in a current mirror configuration:

$$I_{D1} = I_{D3} = 562.5\ \mu A$$

And also from the circuit:

$$I_{D3} = I_2 = 562.5\ \mu A$$

$$I_2 = 562.5\ \mu A$$

Also from the circuit:

$$I_{D3} = I_2 = I_{D4} = 562.5\ \mu A$$
$$V_{GS4} = V_{GS1}$$

Applying KVL.

$$-5 + V_{GS4} + V_2 = 0$$
$$-5 + 2.5 + V_2 = 0$$
$$V_2 = 2.5V$$

If we change the width of Q3 and Q4 W = 100 um

$$I_{D1} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS1} - V_t)^2$$
$$I_{D1} = \frac{1}{2}(50x10^{-6})\left(\frac{100}{1}\right)(2.5 - 1)^2$$
$$I_{D1} = 5.625\ mA$$
$$I_{D1} = I_{D2} = 5.625\ mA$$

From the same circuit,

$$I_{D1} = I_{D3} = I_2 = 5.625\ mA$$
$$I_2 = 5.625\ mA$$

From the same circuit,

$$-5 + V_{GS4} + V_2 = 0$$
$$-5 + 2.5 + V_2 = 0$$
$$V_2 = 2.5V$$

Problem 5.61

In the circuit of Fig. P5.61, transistors Q1 and Q2 have V_t =0.7 V, and the process transconductance parameter k_n' = 125 μA/V2. Find V_1, V_2, and V_3 for each of the following cases:

a) $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 20$

b) $\left(\frac{W}{L}\right)_1 = 1.5\left(\frac{W}{L}\right)_2 = 20$

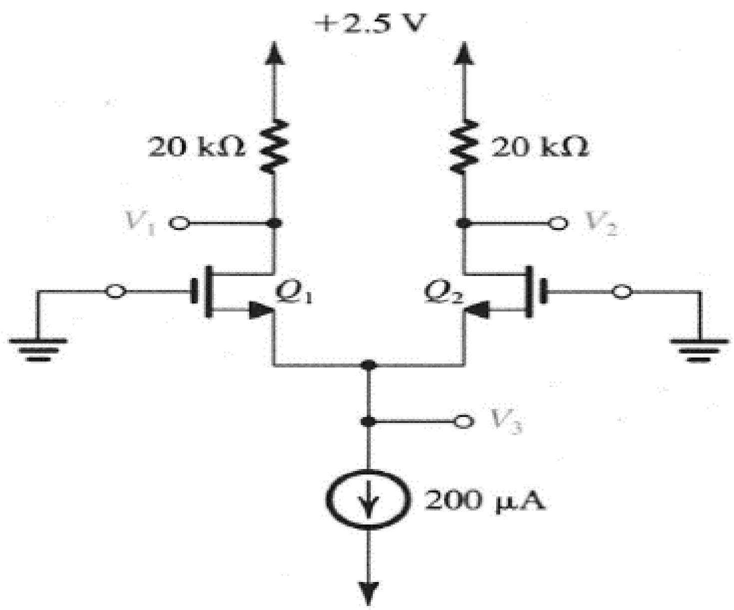


Figure P5.61

For part a:

The drain currents for transistors Q1 and Q2 are:

$$I_{D1} = \frac{2.5 - V_1}{20k\Omega}$$

$$I_{D2} = \frac{2.5 - V_2}{20k\Omega}$$

Using the equation for the drain currents for the NMOS:

$$i_{D1} = \frac{1}{2}(k_n{'})\left(\frac{W}{L}\right)_1 (v_{GS} - V_t)^2$$

$$i_{D1} = \frac{1}{2}(125x10^{-6})(20)(0 - V_3 - 0.7)^2$$

$$i_{D1} = 1.25(V_3 + 0.7)^2\ mA$$

Similarly for i_{D2}

$$i_{D2} = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)_2(v_{GS} - V_t)^2$$

$$i_{D2} = \frac{1}{2}(125x10^{-6})(20)(0 - V_3 - 0.7)^2$$

$$i_{D2} = 1.25(V_3 + 0.7)^2\ mA$$

From the circuit we can see that $i_{D1} + \ i_{D2}\ = 200\ \mu A$

$$i_{D1} = \ i_{D2}\ = 100\ \mu A$$

From the equations of drain currents for transistors Q1 and Q2

$$100\ \mu A = \frac{2.5 - V_1}{20k\Omega}$$

$$V_1\ = 0.5V$$

Similarly for i_{D2}

$$100\ \mu A = \frac{2.5 - V_2}{20k\Omega}$$

$$V_2\ = 0.5V$$

From the equation for the drain currents for the NMOS

$$i_{D1} = 1.25(V_3 + 0.7)^2\ mA$$

$$V_3 = -0.983V$$

For part b:

The drain currents for transistors Q1 and Q2 are:

$$I_{D1} = \frac{2.5 - V_1}{20k\Omega}$$

$$I_{D2} = \frac{2.5 - V_2}{20k\Omega}$$

Using the equation for the drain currents for the NMOS:

$$i_{D1} = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)_1 (v_{GS} - V_t)^2$$

$$i_{D1} = \frac{1}{2}(125x10^{-6})(20)(0 - V_3 - 0.7)^2$$

$$i_{D1} = 1.25(V_3 + 0.7)^2\ mA$$

Similarly for i_{D2}

$$i_{D2} = \frac{1}{2}(k_n{}')\left(\frac{W}{L}\right)_2 (v_{GS} - V_t)^2$$

$$i_{D2} = \frac{1}{2}(125x10^{-6})(13.33)(0 - V_3 - 0.7)^2$$

$$i_{D2} = 0.833(V_3 + 0.7)^2\ mA$$

Diving the two equations to find the relation.

$$\frac{i_{D1}}{i_{D2}} = \frac{1.25(V_3 + 0.7)^2}{0.833(V_3 + 0.7)^2}$$

$$\frac{i_{D1}}{i_{D2}} = 1.5$$

From the circuit we can see that $i_{D1} + \ i_{D2} \ = 200\ \mu A$

$$1.5\ i_{D2} + \ i_{D2} \ = 200\ \mu A$$

$$i_{D2} = 80\ mA$$

$$i_{D1} = 120\ mA$$

From the equations of drain currents for transistors Q1 and Q2

$$120\ \mu A = \frac{2.5 - V_1}{20k\Omega}$$

$$V_1 = 0.1V$$

Similarly for i_{D2}

$$80\ \mu A = \frac{2.5 - V_2}{20k\Omega}$$
$$V_2 = 0.9V$$

From the equation for the drain currents for the NMOS

$$i_{D1} = 1.25(V_3 + 0.7)^2\ mA$$
$$120\ \mu A = 1.25(V_3 + 0.7)^2\ mA$$
$$V_3 = -1.01V$$