ECE 523/421 - Analog Electronics
University of New Mexico
Solutions Homework 4

Problem 6.11

Show that for a transistor with α close to unity, if α changes by a small per-unit amount (∆α/α), the corresponding per-unit change in β is given approximately by

$$\boldsymbol{\frac{\Delta\beta}{\beta} \cong \beta\left(\frac{\Delta\alpha}{\alpha}\right)}$$

Now, for a transistor whose nominal β is 100, find the percentage change in its α value corresponding to a drop in its β of 10%.

The relationship between α and β is

$$\beta = \frac{\alpha}{1-\alpha}$$

$$\alpha \to \alpha + \Delta\alpha$$

$$\beta \to \beta + \Delta\beta$$

$$\beta + \Delta\beta = \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha}$$

Subtracting $\beta + \Delta\beta$, β we obtain $\Delta\beta$

$$\Delta\beta = \frac{\alpha + \Delta\alpha}{1 - \alpha - \Delta\alpha} - \frac{\alpha}{1-\alpha}$$

$$\Delta\beta = \frac{\Delta\alpha}{(1 - \alpha - \Delta\alpha)\,(1-\alpha)}$$

Dividing this by β

$$\frac{\Delta\beta}{\beta} = \left(\frac{1}{1 - \alpha - \Delta\alpha}\right)\left(\frac{\Delta\alpha}{\alpha}\right)$$

For $\Delta\alpha \ll 1$ The first part is approximately β

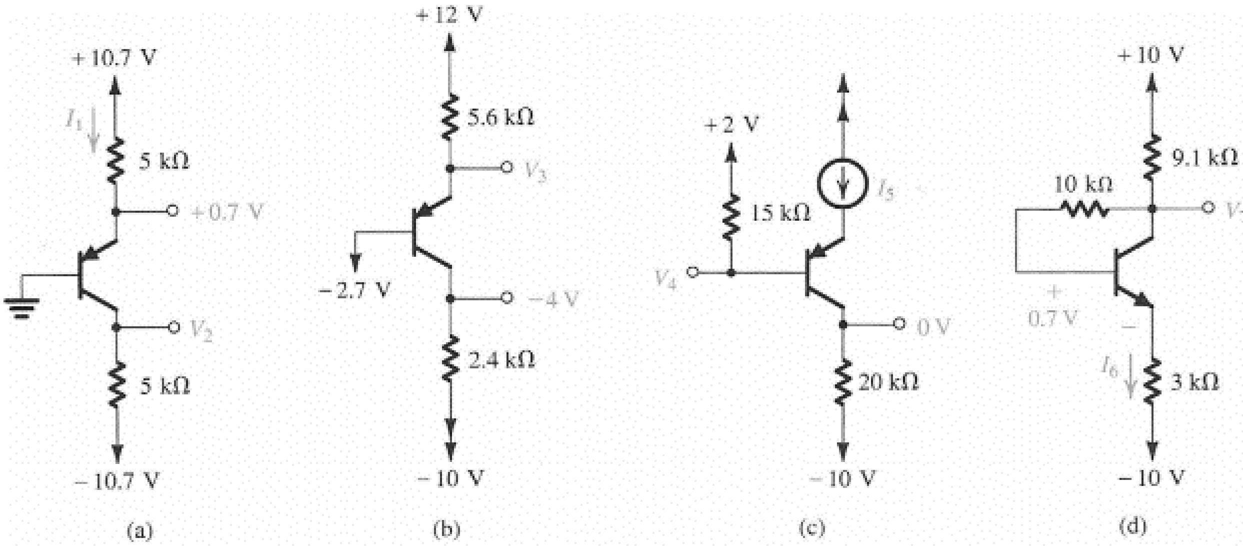
$$\frac{\Delta\beta}{\beta} \cong \beta\left(\frac{\Delta\alpha}{\alpha}\right)$$

To a drop in its β of 10% and β is 100 using the equation,

$$\left(\frac{\Delta\alpha}{\alpha}\right) \cong \frac{-10\%}{100} = -0.1$$

Problem 6.28

For the circuits in Fig. P6.28, assume that the transistors have very large β. Some measurements have been made on these circuits, with the results indicated in the figure. Find the values of the other labeled voltages and currents.



- For circuit a

$$I_1 = \frac{10.7 - 0.7}{5\ k\Omega} = 2\ mA$$
$$I_1 = 2\ mA$$

$$I_1 = I_C$$
$$V_2 - (-10.7) = 2\ mA\ x\ 5\ k\Omega$$
$$V_2 = -0.7V$$

- For circuit b

$$I_C = \frac{-4 - (-10)}{2.4\ k\Omega} = 2.5\ mA$$
$$I_E = \frac{I_C}{\alpha} \cong 2.5\ mA$$
$$\frac{12 - V_3}{5.6\ k\Omega} = 2.5\ mA$$
$$V_3 = -2V$$

- For circuit c

Due to I_B is approximately equal to 0

$$V_4 \cong 2V$$

$$I_C = \frac{0-(-10)}{20\ k\Omega} = 0.5\ mA$$

$$I_5 = I_E = \frac{I_C}{\alpha} \cong 0.5\ mA$$

- For circuit d

Due to I_B is approximately equal to 0

$$V_7 = V_B$$

$$V_E = V_B - 0.7$$

$$I_6 = \frac{V_E-(-10)}{3\ k\Omega} = \frac{V_7 - 0.7-(-10)}{3\ k\Omega}$$

$$I_6 = \frac{V_7 + 9.3}{3\ k\Omega}$$

Also due to I_B is approximately equal to 0

$$I_C = \frac{10 - V_7}{9.1\ k\Omega}$$

Since $\alpha \cong 1$

$$I_6 = I_E \cong \frac{I_C}{\alpha}$$

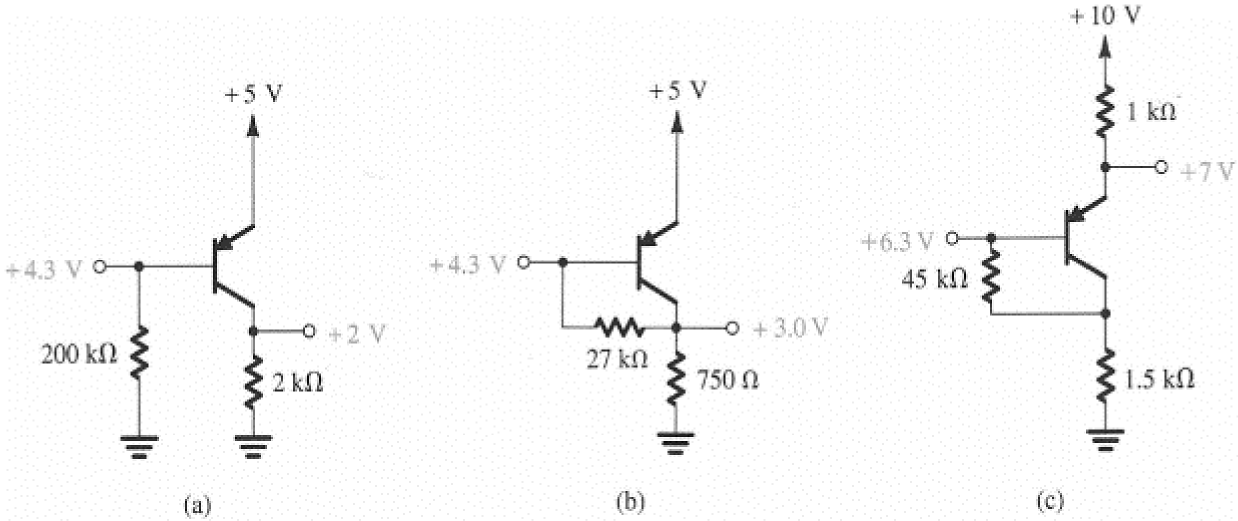
$$\frac{V_7 + 9.3}{3\ k\Omega} = \frac{10 - V_7}{9.1\ k\Omega}$$

$$V_7 = -4.5V$$

$$I_6 = 1.6\ mA$$

Problem 6.29

Measurements on the circuits of Fig. P6.29 produce labeled voltages as indicated. Find the value of β for each transistor.



- For circuit a

Calculating base and collector currents

$$I_B = \frac{4.3}{200k\Omega} = 0.0215\ mA$$

$$I_C = \frac{2}{2k\Omega} = 1\ mA$$

$$\beta = \frac{I_C}{I_B} = 46.5$$

- For circuit b

Calculating base and collector currents

$$I_B = \frac{4.3 - 3}{27k\Omega} = 0.048\ mA$$

$$I_x = \frac{3}{0.75k\Omega} = 4\ mA$$

$$I_C = 4 - 0.048 = 3.952\ mA$$

$$\beta = \frac{I_C}{I_B} = 82.3$$

- For circuit c

Calculating base and emitter currents

$$I_E = \frac{10-7}{1k\Omega} = 3\ mA$$

$$V_C = 3mA\ x\ 1.5k\Omega$$

$$V_C = 4.5V$$

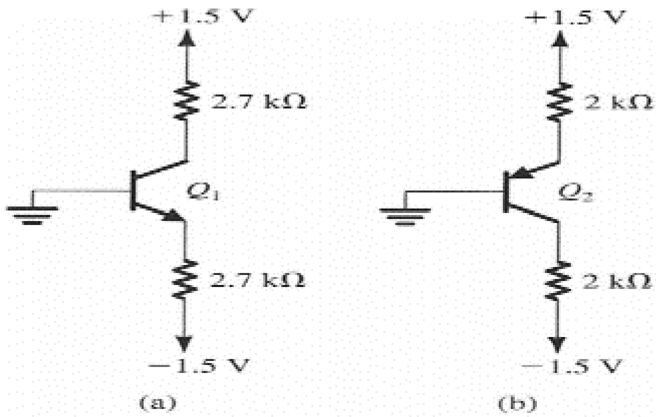
$$I_B = \frac{6.3-4.5}{45k\Omega} = 0.04\ mA$$

$$I_C = I_E - I_B = 3 - 0.04 = 2.96\ mA$$

$$\beta = \frac{I_C}{I_B} = 74$$

Problem 6.35

For each of the circuits shown in Fig. P6.35, find the emitter, base, and collector voltages and currents. Use β=50, but assume $|V_{BE}|$ = 0.8 V independent of current level.



For circuit a

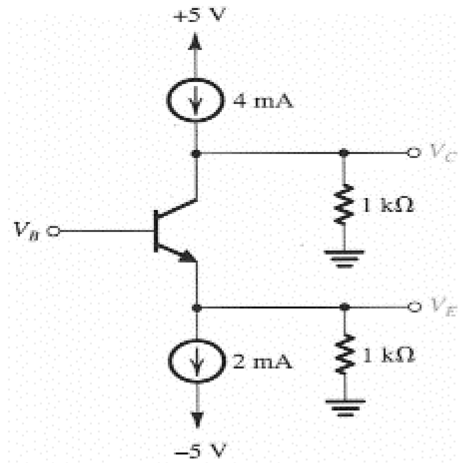
$$V_{BE} = 0.8V$$
$$V_E = -0.8V$$
$$I_E = \frac{-0.8 - -1.5}{2.7k\Omega} = 0.26\ mA$$
$$\alpha = \frac{\beta}{\beta + 1}$$
$$I_C = \alpha I_E = 0.255\ mA$$
$$V_C = 1.5 - I_C x 2.7$$
$$V_C = 0.81V$$
$$I_B = I_E - I_C = 5\ \mu A$$

For circuit b

$$V_{BE} = 0.8V$$
$$V_E = 0.8V$$
$$I_E = \frac{1.5 - 0.8}{2k\Omega} = 0.35\ mA$$
$$\alpha = \frac{\beta}{\beta + 1}$$
$$I_C = \alpha I_E = 0.343\ mA$$
$$V_C = -1.5 + I_C x 2$$
$$V_C = -0.81V$$
$$I_B = I_E - I_C = 7\ \mu A$$

Problem 6.54

For the transistor shown in Fig. P6.54, assume α ≅ 1 and V_{BE} = 0.5 V at the edge of conduction. What are the values of V_E and V_C for V_B = 0 V? For what value of V_B does the transistor cut off? Saturate? In each case, what values of V_E and V_C result?



For V_B = 0 V

$$V_E = -0.7V$$

$$I_X = \frac{-0.7}{1k\Omega} = -0.7\ mA$$

$$I = 2 - 0.7 = 1.3\ mA$$

$$I_{CC} = 1.3\ mA$$

$$I_C = 4 - 1.3 = 2.7\ mA$$

$$V_C = 2.7m\ x\ 1k$$

$$V_C = 2.7V$$

For cut off the two currents in the collector and emitter are zero

$$I_E = I_C = 0$$

$$V_E = 2m\ x\ 1k$$

$$V_E = -2V$$

$$V_B = V_E + 0.5$$

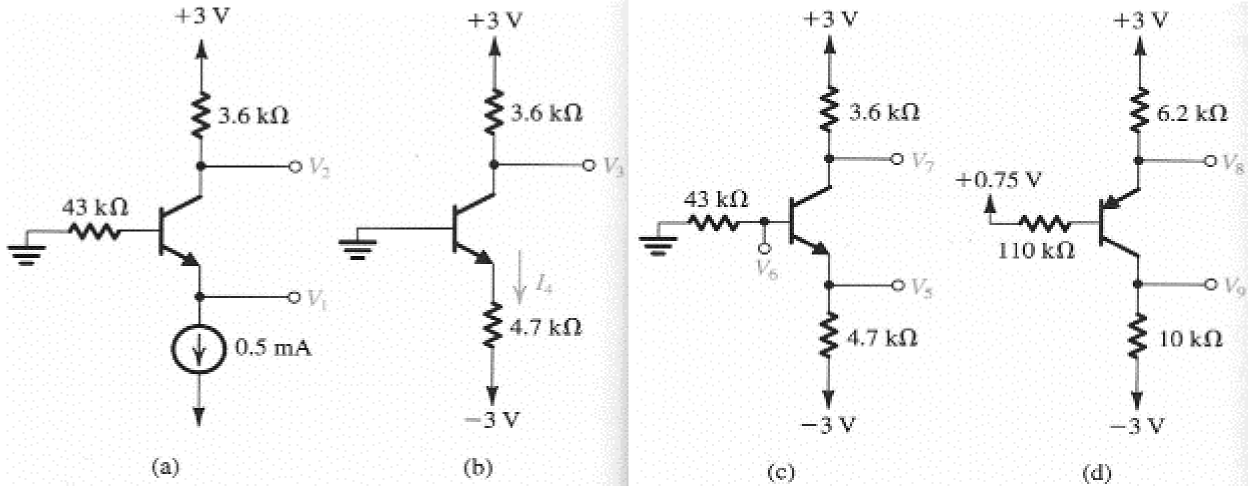
$$V_B = -1.5V$$

$$V_C = 4m\ x\ 1k$$

$$V_C = 4V$$

Problem 6.62

Repeat the analysis of the circuits in Problem 6.61 using β=100. Find all the labeled node voltages and branch currents.



- For circuit a

$$I_E = 0.5\ mA$$
$$I_C = \alpha I_E = 0.99\ x\ 0.5$$
$$I_C = 495\ mA$$
$$I_B = \frac{I_E}{\beta+1} = \frac{0.5}{101}$$
$$I_B = -0.005\ mA$$
$$V_B = I_B\ x\ R_B = -0.005\ x\ 43$$
$$V_B = -0.215\ V$$
$$V_1 = -0.215 - 0.7$$
$$V_1 = -0.915\ V$$
$$V_2 = 3 - 0.495\ x\ 3.6$$
$$V_2 = 1.218\ V$$

- For circuit b

$$V_E = -0.7V$$
$$I_4 = \frac{-0.7-(-3)}{4.7k\Omega}$$
$$I_E = I_4 = 0.5\ mA$$
$$I_B = \frac{I_E}{\beta+1} = \frac{0.5}{101}$$
$$I_B = 0.005\ mA$$

$$I_C = \alpha I_E = 0.99 \; x \; 0.5$$
$$I_C = 495 \; mA$$
$$V_3 = 3 - 0.495 \; x \; 3.6$$
$$V_3 = 1.218 \; V$$

- For circuit c

Applying KVL

$$43I_B + 0.7 + 4.7I_E = 3$$
$$I_B = \frac{I_E}{\beta + 1}$$
$$I_E = 0.449 \; mA$$
$$V_5 = -3 + 0.449 \; x \; 4.7$$
$$V_5 = -0.9 \; V$$
$$V_6 = -0.9 + 0.7$$
$$V_6 = -0.2 \; V$$
$$I_C = \alpha I_E = 0.99 \; x \; 0.449$$
$$I_C = 0.444 \; mA$$
$$V_7 = 3 - 0.444 \; x \; 3.6$$
$$V_7 = 1.4 \; V$$

- For circuit d

Applying KVL

$$0.75 + 110I_B + 0.7 + 6.2I_E = 3$$
$$I_B = \frac{I_E}{\beta + 1}$$
$$I_E = 0.213 \; mA$$
$$V_8 = 3 - 0.213 \; x \; 6.2$$
$$V_8 = 1.7 \; V$$
$$I_C = \alpha I_E = 0.99 \; x \; 0.213$$
$$I_C = 0.21 \; mA$$
$$V_7 = -3 + 0.21 \; x \; 10$$
$$V_7 = -0.9 \; V$$

Problem 6.65

Consider the circuit shown in Fig. P6.65. It resembles that in Fig. 6.30 but includes other features. First, note diodes D1 and D2 are included to make design (and analysis) easier and to provide temperature compensation for the emitter–base voltages of Q1 and Q2. Second, note resistor R, whose purpose is to provide negative feedback (more on this later in the book!). Using $|V_{BE}|$ and VD = 0.7 V independent of current, and β=∞, find the voltages VB1, VE1, VC1, VB2, VE2, and VC2, initially with R open-circuited and then with R connected. Repeat for β=100, with R open-circuited initially, then connected.

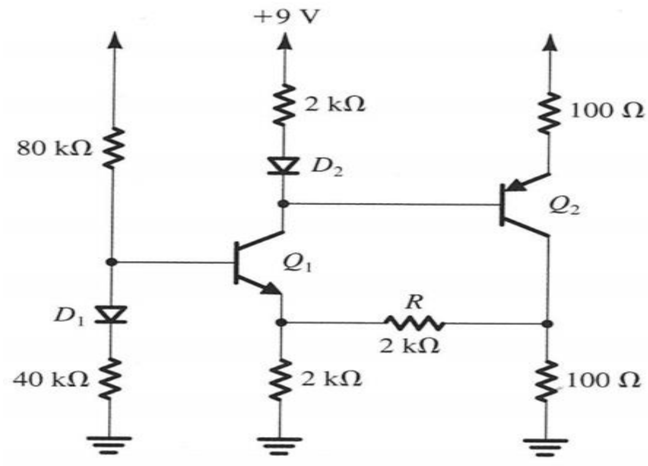


Figure P6.65

For β=∞ and R open-circuited

$$V_{D1} = V_{BE1}$$
$$V_1 = V_{E1}$$
$$40I_{D1} = 2I_{E1}$$
$$I_{D1} = \frac{9-0.7}{80+40} = 0.07\,mA$$
$$I_{E1} = 1.4\,mA$$
$$V_{E1} = 2I_{E1} = 2.8V$$
$$V_{B1} = V_{E1} + 0.7 = 3.5V$$
$$I_{C1} = I_{E1} = 1.4\,mA$$
$$V_2 = 9 - 2I_{C1} = 6.2V$$
$$V_{C1} = V_2 - V_{D2} = 5.5V$$
$$V_{E2} = V_2 = 6.2V$$
$$I_{E2} = \frac{9-6.2}{100} = 28\,mA$$
$$I_{C2} = I_E = 28\,mA$$
$$V_{C2} = 28\,x\,0.1 = 2.8V$$

For β=∞ and R = 2kΩ
There will be no current through R, so all the voltages and currents are the same

For β=100 and R open-circuited

$$V_1 = V_{E1}$$
$$40I_{D1} = 2I_{E1}$$
$$I_{D1} = 0.05I_{E1}$$
$$I_{B1} = \frac{I_{E1}}{101} = 0.01I_{E1}$$

The current through the 80kΩ resistor is

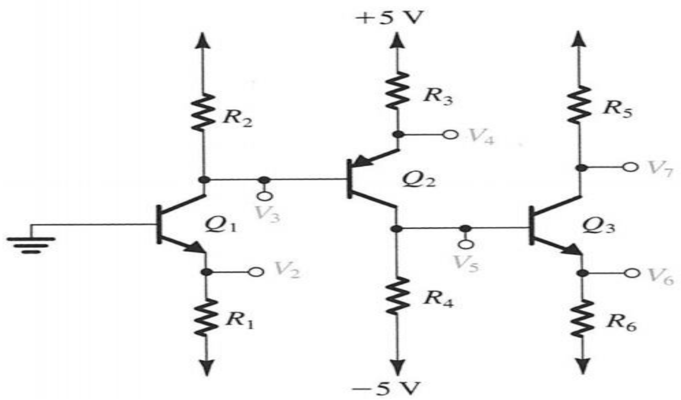
$$I_{D1} + I_{B1} = 0.06I_{E1}$$
$$V_{B1} = V_{E1} + 0.7 = 2I_{E1} + 0.7$$
$$0.06I_{E1} = \frac{9 - V_{B1}}{80}$$
$$I_{E1} = 1.22\ mA$$
$$V_{E1} = 2I_{E1} = 2.44V$$
$$V_{B1} = V_{E1} + 0.7 = 3.14V$$
$$I_{C1} = \alpha I_{E1} = 1.21\ mA$$

Voltage across the 2kΩ resistor is equal to voltage in 100Ω resistor

$$2I_{D2} = 0.1I_{E2}$$
$$I_{D2} = 0.05I_{E2}$$
$$I_{D2} = I_{C1} - 0.01I_{E2}$$
$$I_{E2} = \frac{I_{C1}}{0.06} = 20.13\ mA$$
$$I_{D2} = 1\ mA$$
$$V_{C1} = 9 - 2I_{D2} = 6.3V$$
$$V_{E2} = V_{C1} + 0.7 = 7V$$
$$I_{C2} = \alpha I_{E2} = 20\ mA$$
$$V_{C2} = 20\ x\ 0.1 = 2V$$

Problem 6.67

Using β=∞, design the circuit shown in Fig. P6.67 so that the emitter currents of Q1, Q2, and Q3 are 0.5 mA, 0.5 mA, and 1 mA, respectively, and V3 = 0, V5 =−2 V, and V7 =1 V. For each resistor, select the nearest standard value utilizing the table of standard values for 5% resistors in Appendix J. Now, for β=100, find the values of V3, V4, V5, V6, and V7.



For β=∞, and calculating the values of the resistors

$$V_2 = -0.7V$$

$$R_1 = \frac{V_2 - (-5)}{0.5\ mA}$$

$$R_1 = 8.6\ k\Omega$$

$$R_2 = \frac{(5) - V_3}{0.5\ mA}$$

$$R_2 = 10\ k\Omega$$

$$V_4 = 0 + 0.7 = 0.7V$$

$$R_3 = \frac{(5) - V_4}{0.5\ mA}$$

$$R_3 = 8.6\ k\Omega$$

$$R_4 = \frac{V_5 - (-5)}{0.5\ mA}$$

$$R_4 = 6\ k\Omega$$

$$V_6 = V_5 - 0.7 = -2.7V$$

$$R_6 = \frac{V_6 - (-5)}{1\ mA}$$

$$R_4 = 2.3\ k\Omega$$

$$R_5 = \frac{(5) - V_7}{1\ mA}$$

$$R_5 = 4\ k\Omega$$

Using the table for 5% resistors

$$R_1 = 8.2\ k\Omega$$
$$R_2 = 10\ k\Omega$$
$$R_3 = 10\ k\Omega$$
$$R_4 = 6.2\ k\Omega$$
$$R_5 = 3.9\ k\Omega$$
$$R_6 = 2.4\ k\Omega$$

Then for β=100

$$V_2 = -0.7V$$
$$I_{E1} = \frac{V_2 - (-5)}{8.2k\Omega}$$
$$I_{E1} = 0.524\ mA$$
$$I_{C1} = \alpha I_{E1} = 0.99\ x\ 0.524$$
$$I_{C1} = 0.52\ mA$$

The current I_1 through $10\ k\Omega$

$$I_1 = I_{C1} - I_{B1} = I_{C1} - \frac{I_{E2}}{101}$$

Also the voltage through $10\ k\Omega$

$$I_{E2}\ x\ 8.2\ k + 0.7$$

Using the voltages

$$10I_1 = 8.2\ I_{E2} + 0.7$$
$$10\left(I_{C1} - \frac{I_{E2}}{101}\right) = 8.2\ I_{E2} + 0.7$$
$$I_{E2} = 0.542\ mA$$

$$V_4 = 5 - 0.542\ x\ 8.2 = 0.56V$$
$$V_3 = 0.56 - 0.7 = -0.14V$$

$$I_{C2} = \alpha I_{E2} = 0.99\ x\ 0.542$$
$$I_{C2} = 0.537\ mA$$

The current I_2 through $6.2\ k\Omega$

$$I_2 = I_{C2} - I_{B3} = 0.537 - \frac{I_{E3}}{101}$$

Also the voltage through $6.2\ k\Omega$

$$I_{E2}\ x\ 2.4\ k + 0.7$$

Using the voltages

$$6.2 I_2 = 2.4 I_{E3} + 0.7$$

$$6.2\left(0.537 - \frac{I_{E3}}{101}\right) = 2.4 I_{E3} + 0.7$$

$$I_{E3} = 1.07\ mA$$

$$V_6 = -5 + 1.07\ x\ 2.4 = -2.43\ V$$

$$V_5 = V_6 + 0.7 = -1.73\ V$$

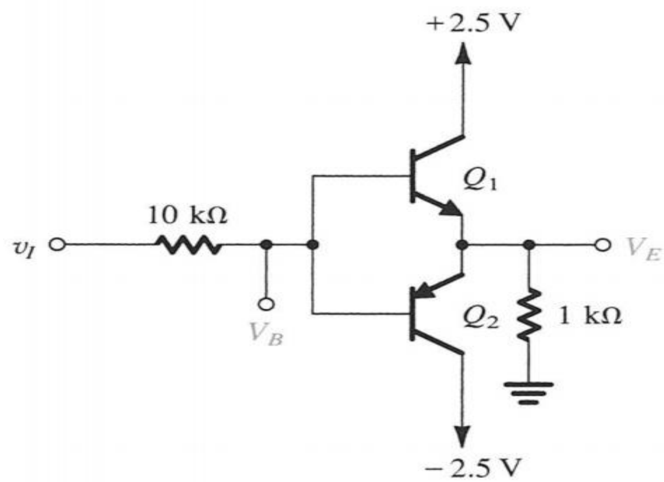
$$I_{C3} = \alpha I_{E3} = 0.99\ x\ 1.O7$$

$$I_{C3} = 1.06\ mA$$

$$V_7 = -3.9\ x\ 1.06 = 0.87\ V$$

Problem 6.68

For the circuit in Fig. P6.68, find VB and VE for vI = 0 V, +2 V, –2.5 V, and –5 V. The BJTs have β=50.



For V_I = 0 V both transistors are cut off and all the currents are zero

$$V_B = V_E = 0$$

For V_I = 2 V Q1 will be conducting and Q2 will be cut off.

$$I_E = \frac{2 - 0.7}{1 + \frac{10}{51}}$$

$$I_E = 1.1\ mA$$

$$V_E = 1.1\ V$$

$$V_B = 1.8\ V$$

For V_I = -2.5 V Q1 will be cut off and Q2 will be conducing.

$$I_E = \frac{2.5 - 0.7}{1 + \frac{10}{51}}$$

$$I_E = 1.5\ mA$$

$$V_E = -I_E x1 = -1.5\ V$$

$$V_B = -2.2\ V$$