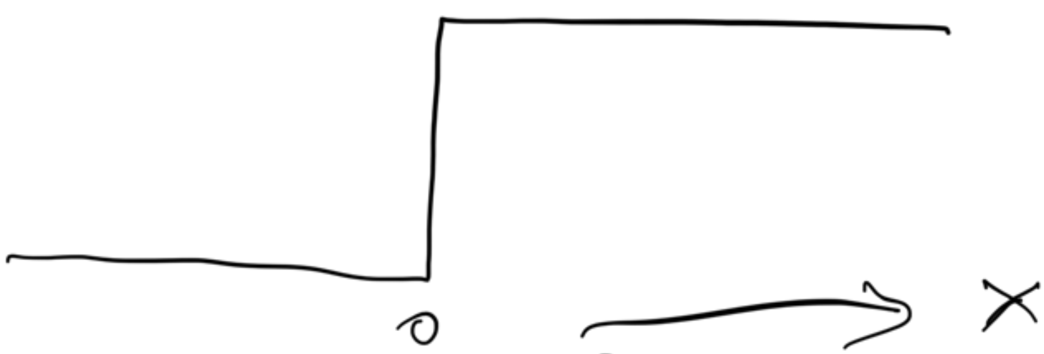
$$p = w\rho$$

$$\frac{d\rho}{da} = -3\frac{(\rho + p)}{a}$$

$$= -3\frac{(\rho + w\rho)}{a} = -\frac{3(1+w)\rho}{a}$$

$$\frac{d\rho}{\rho} = -3(1+w)\frac{da}{a}$$



$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

$$\frac{d}{dx}\theta(x) = f(x)$$

$$\int_a^x k(x,y)\,u(y)\,dy = \lambda u(x)$$

$$\left(\frac{x-a}{N}\right) K(x_i, y_j)\, u(y_j) = \lambda u(x_i)$$

$$\Leftrightarrow K u = \lambda u$$

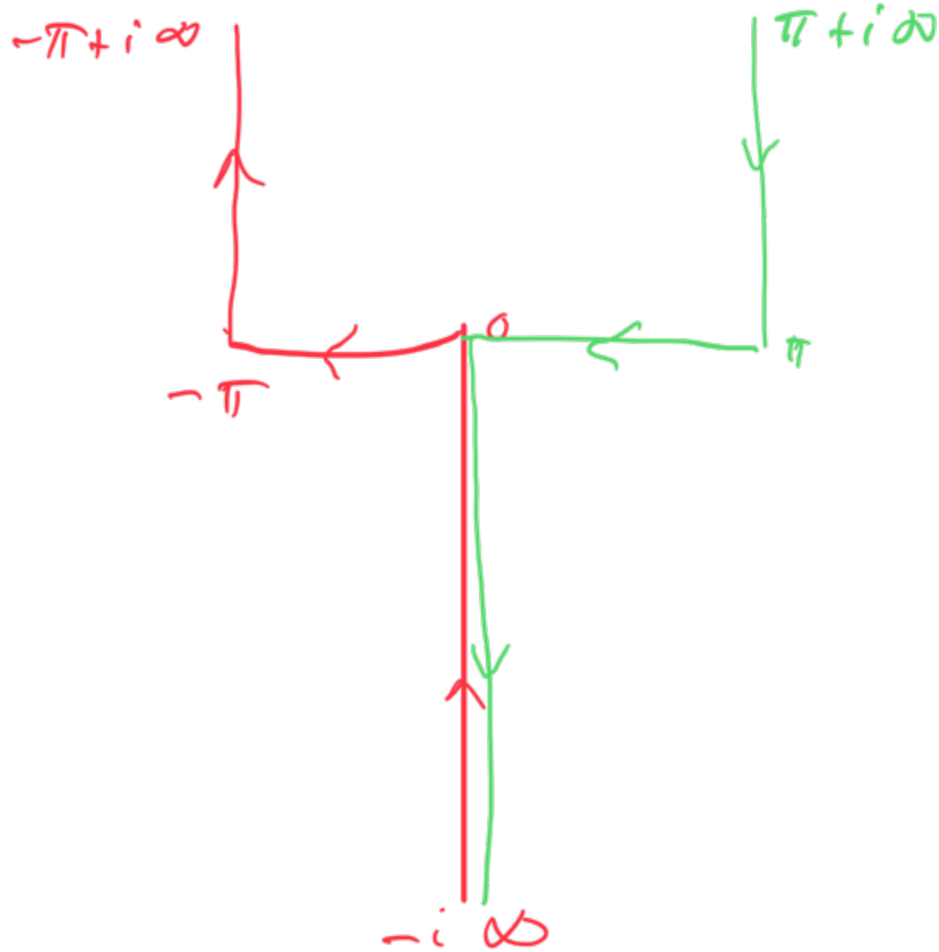
$$K u = f$$

$$u = K^{-1} f$$

$$K u = 0$$

$$f(z) = \oint \frac{f(w)\,dw}{2\pi i\,(w-z)}$$

$(w-z)$



$$ct = x^0 \qquad -ct = +x_0 = -x^0$$

$$x = x^1 = x_1$$

$$y = x^2 = x_2$$

$$z = x^3 = x_3$$

$$\partial_a = \frac{\partial}{\partial x^a} \qquad \partial^a = \frac{\partial}{\partial x_a}$$

$$\partial_t = \frac{\partial}{\partial t} = \frac{\partial}{\partial x^0} \equiv \partial_0 = -\partial^0$$

$$\partial_x = \frac{\partial}{\partial x} = \frac{\partial}{\partial x^1} = \partial_1 = \partial^1$$

$$\partial_y = \frac{\partial}{\partial y} = \frac{\partial}{\partial x^2} = \partial_2 = \partial^2$$

$$\partial_z = \frac{\partial}{\partial z} = \frac{\partial}{\partial x^3} = \partial_3 = \partial^3$$

$$x_a = \eta_{ab} x^b \equiv \sum_{b=0}^{3} \eta_{ab} x^b$$

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

$$p^2 = -p^{0\,2} + \vec{p} \cdot \vec{p}$$

$$= p_a p^a = \sum_{a=0}^{3} p_a p^a$$

$$= -p^{0\,2} + (p^1)^2 + (p^2)^2 + (p^3)^2$$

$$= -E^2 + \vec{p}^{\,2} = -m^2$$

$$x \cdot y = \vec{x} \cdot \vec{y} - x^0 \cdot y^0$$

$$= x^a \eta_{ab} y^b$$

$$g_{ab}$$

$$ds^2 = \sum_{a,b=0}^{3} g_{ab} (x^a - y^a)(x^b - y^b)$$

ordinary variables

x^a is usual variable

no hidden minus signs.

$\partial_a = \frac{\partial}{\partial x^a}$ ordinary derivative

no hidden minus signs.

$$\sum_{a=0}^{3} \frac{\partial}{\partial x^a}\left(\frac{\partial L}{\partial\left(\frac{\partial\phi}{x^a}\right)}\right) = \frac{\partial L}{\partial \phi}$$

$$\partial_a = \frac{\partial}{\partial x^a}$$

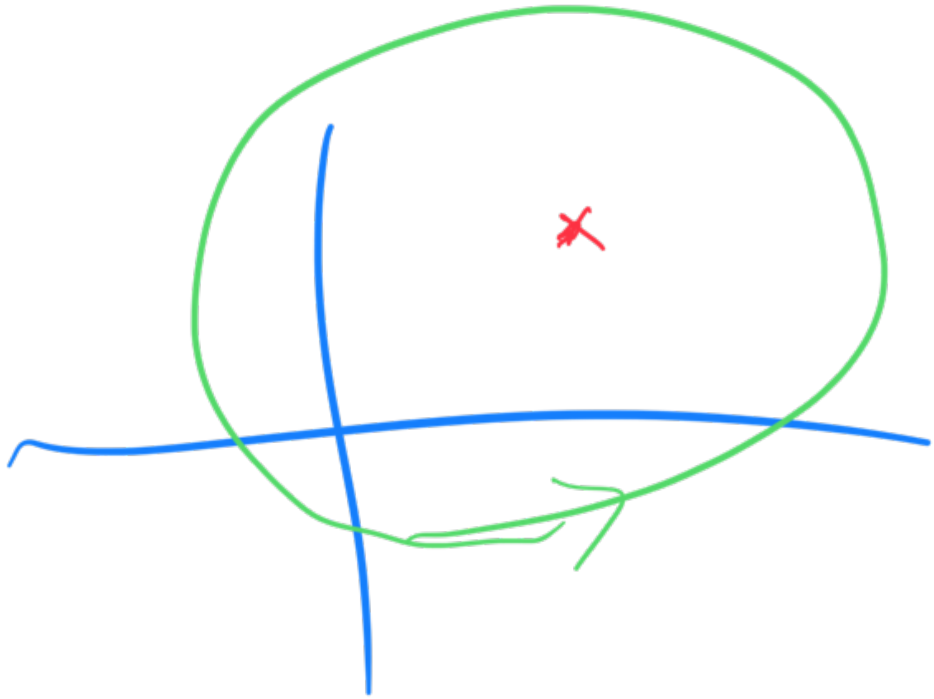
$$g(t,x) = \frac{1}{\sqrt{1-2tx+t^2}}$$

$$P_n(x) = \frac{1}{2\pi i}\oint \frac{P_n(z)}{(z-x)}\,dz$$

$$= \frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n (x^2-1)^n$$

$$= \frac{1}{2\pi i\, 2^n n!}\left(\frac{d}{dx}\right)^n \oint \frac{(z^2-1)^n}{(z-x)}\,dz$$

$$= \frac{1}{2^n 2\pi i} \oint \frac{\quad}{(z-x)^{n+1}} dz$$



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10 Nov. 2020

$$e^{i(px-\omega t)}$$

$$e^{iP\vec{x}} \qquad e^{i\sum_i p_i x_i}$$

pand3.phys.unm.edu/~kevin/

466-201

$$Lu = -(xu')' + \frac{n^2}{x}u = \lambda x u$$

$u(0)=0 \qquad u(1)=0$ for $n>0$

$$\boxed{z! = \Gamma(z+1)}$$

$$J_n(z) = \left(\frac{z}{2}\right)^n \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,(m+n)!}\left(\frac{z}{2}\right)^{2m}$$

$n \to \nu$ complex

$$J_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\nu+1)}\left(\frac{z}{2}\right)^{2m}$$

$$e^{-i\frac{z}{2i}(e^{iw}-e^{-iw}) + i\lambda w}$$

$$e^{-\frac{z}{2}(e^{iw}-e^{-iw}) + i\lambda w}$$

$-\pi + i\infty \sim w$

$$e^{-\frac{z}{2}\left(\underset{\overset{\parallel}{0}}{e^{-\infty}} - \underset{\overset{\parallel}{-\infty}}{e^{+\infty}}\right) + i\lambda w}$$

$$e^{-\frac{z}{2}\left(e^{i(-\pi+i\infty)} - e^{-i(-\pi+i\infty)}\right) + i\lambda}$$

$$e^{\frac{z}{2}\left(e^{-\infty} - e^{+\infty}\right) + i\lambda w}$$

$\mathrm{Re}(z) > 0 \qquad z = x + iy, \quad x > 0$

$$e^{-e^{+\infty}} \to 0$$

$-\pi + i\infty$

CW

$$\rho_0 =$$

$$r^4 = \frac{32\pi G \rho_0}{3}$$

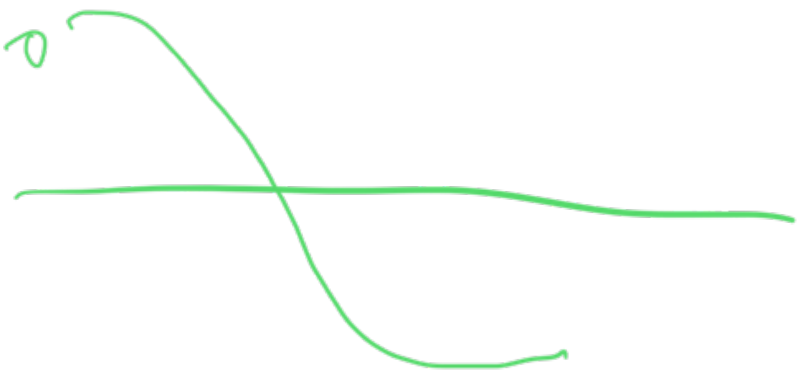
$$\dot{a} = \frac{q}{a} = \frac{da}{dt} \qquad k=0$$

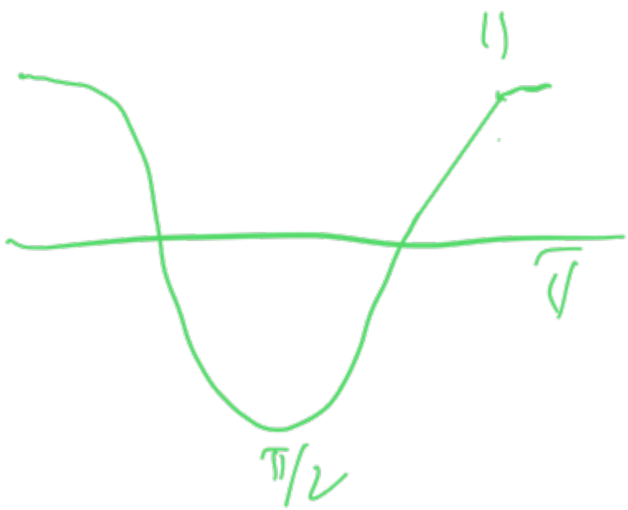
$$dt = \frac{1}{q}\frac{da}{a}$$

$$t - t_1 = \frac{1}{q}\ln\frac{a}{a_1}$$

$$\dot{a} = \sqrt{\frac{b}{a^2} - \frac{c^2 h}{L^2}}$$

$$\frac{dq}{\sqrt{\frac{k}{a^2} - \frac{c^2 h}{L^2}}} = dt$$





$\cos(2\theta)$

$z! = \Gamma(z+1)$

defined except at $n = -1, -2, -3, \ldots$

$$\cos\left(z - \frac{n\pi}{2} - \frac{\pi}{4}\right) = 0$$

$$z - n\frac{\pi}{2} - \frac{\pi}{4} = (2m+1)\frac{\pi}{2}$$

$$z_{n,m} = (2m+n+1)\frac{\pi}{2} + \frac{\pi}{4}$$

$$z_{n,m} \sim m\pi \to \infty$$

as $m \to \infty$

$$f(x) = x^{\alpha}\int_0^b \left[\sum_{k=1}^{\infty} \frac{2}{b^2 J_{n+1}^2(z_{n,k})} y^{1-\alpha} J_n\left(\frac{z_{n,k}y}{b}\right) J_n\left(\frac{z_{n,k}x}{b}\right)\right] d$$

$\times f(y)$

$$= \int_0^b \delta(x-y) f(y)\, dy = f(x)$$

∞

$$\sum_{n=1}^{\infty} \langle x|n\rangle\langle n|y\rangle = \delta(x-y)$$

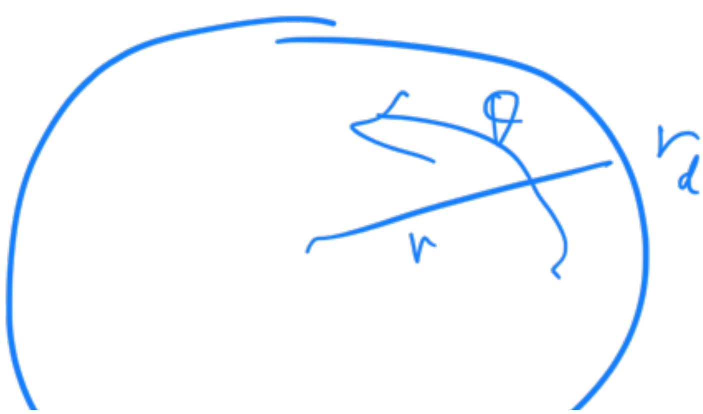
$$\sqrt{dx^2+dh^2} = \sqrt{1+h_x^2}\;dx$$

$$h_x\,dx = \frac{dh}{dx}\,dx = dh$$

$$\nabla v = -\omega^2 \frac{\mu}{\sigma} v$$

$$\lambda = \omega^2 \frac{\mu}{\sigma} \qquad \omega^2 = \frac{\sigma\lambda}{\mu}$$

$$\omega = \sqrt{\frac{\sigma\lambda}{\mu}}$$



$$S = \int \frac{\mu}{2}\left(\frac{dh}{dt}\right)^2 - \frac{\sigma}{2}\left(\left(\frac{dh}{dx}\right)^2 + \left(\frac{dh}{dy}\right)^2\right) dx\,dy\,dt$$

action

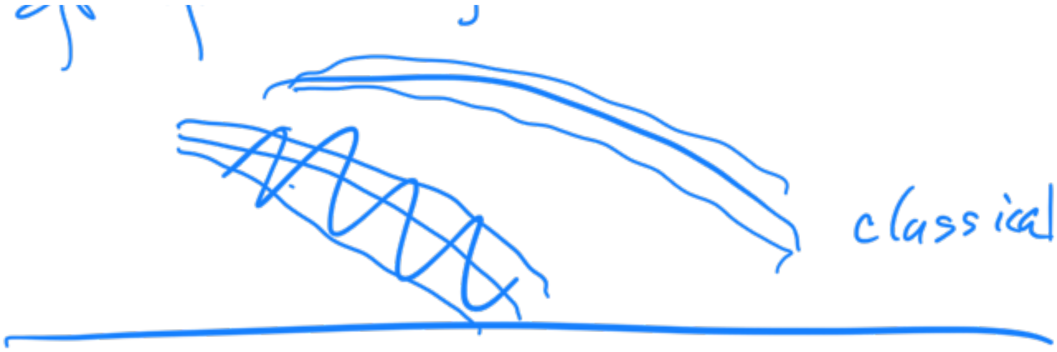
$0 = \delta S$ get the classical equations of motion.

to first order in δh_t, δh_x, δh_y

δh is ARBITRARY

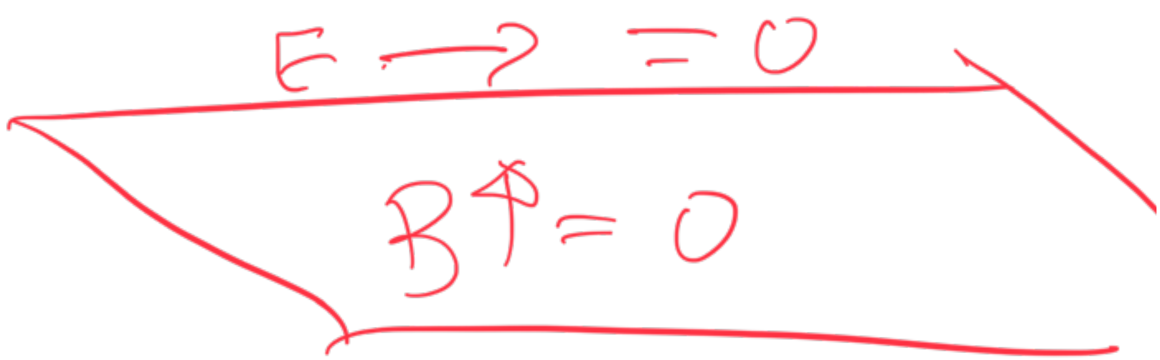
$$\delta h \sim \epsilon\,\delta(x - x_0)\,\delta(y - y_0)\,\delta(t - t_0)$$

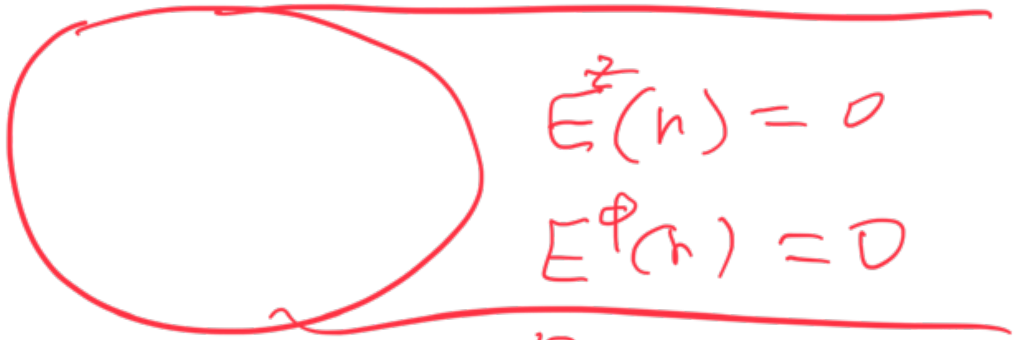
$$\langle \;|\; \rangle = \int e^{iS/\hbar}\, DP$$



$$-\Delta E = \frac{\omega^2}{c^2}\ddot{E}$$

$$-\Delta B = \frac{\omega^2}{c^2}\ddot{B}$$

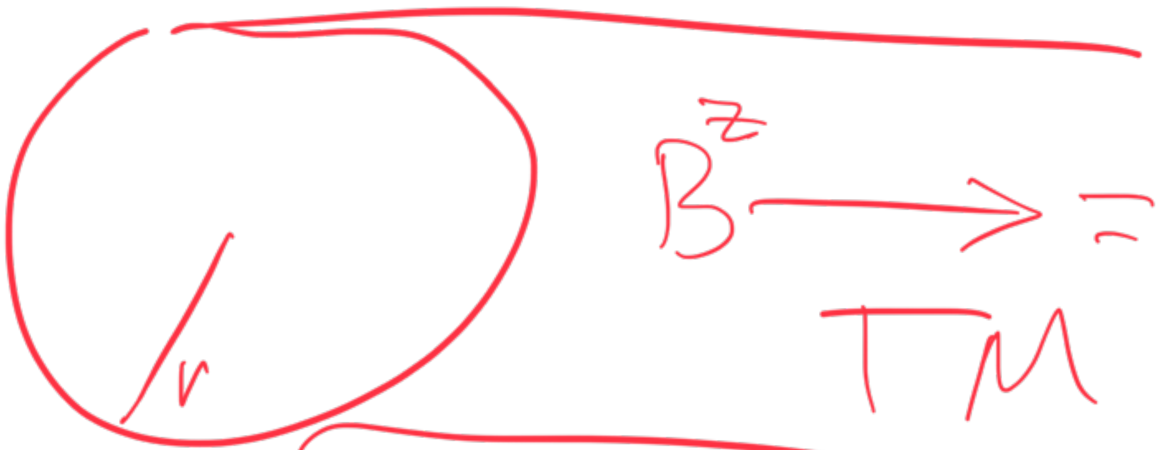




$E^z(r) = 0$

$E^\phi(r) = 0$

$B^r(r) = 0$



$B^z \longrightarrow = 0$

TM

$B \perp \hat{z}$

$B \cdot \hat{z} = 0 \quad B^z = 0$

$E^r E^\phi = 0$

$$0 \leq \alpha \leq 1$$

$$\delta(x-y) = \rho^{\alpha}(x)\,\rho^{1-\alpha}(y)\sum_{i=0}^{\infty} u_i^*(x)\,u_i(y)$$

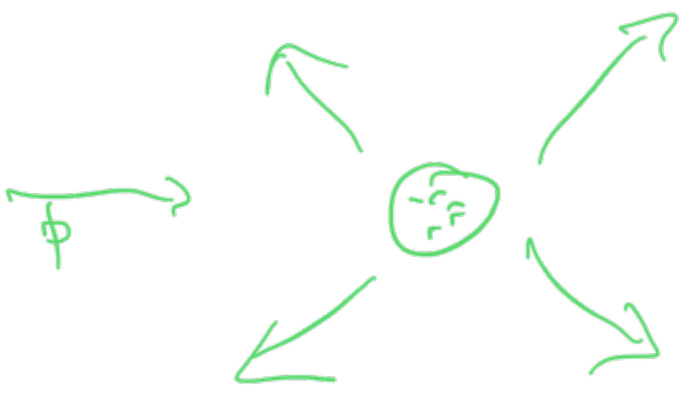
$$\longrightarrow \int_a^b u_i^*(x)\,u_j(x)\,\rho(x)\,dx = \delta_{ij}$$

$$j_\ell\left(z_{\ell,m}\frac{r}{a}\right) = 0 \quad \text{for } r = a$$

$$e^{i\vec{k}\cdot\vec{x}} = e^{ikr\cos\theta}$$

$$H_0 \to H = \frac{\vec{p}^{\,2}}{2m} + V(r)$$

$$\frac{e^{i\vec{k}'\cdot\vec{r}'}}{(2\pi)^{3/2}} = \sum_{lm}\int \langle \vec{r}|klm\rangle\langle klm|\vec{r}'\rangle\, dk$$



nuclear forces are of short range

$$\frac{e^{-\mu r}}{r}$$ mass of exchanged boson

μ 105 MeV

(π^+, π^0, π^-) 140 MeV

πr_0^2

$\sigma = 4\pi r_0^2$

$\delta_0 \sim k r_0$ $\qquad$ $\delta_1 \sim (k r_0)^3$

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n (x+1)^n (x-1)^n}{dx^n}$$

$x \simeq 1$

$$P_n(1) \simeq \frac{1}{2^n n!} (x+1)^n \frac{d^n}{dx^n} (x-1)^n$$

$$1 \quad 2^n \quad d^n \quad (x-1)^n$$

$$\approx \frac{}{2^m m!} \quad \frac{d}{dx^m}$$

$$\approx \frac{1}{m!} m(m-1)\cdots 2 \frac{d}{dx}(x-1)$$

$$= \frac{m!}{m!} 1 = 1 .$$

$y = J_m(ax)$ 10.11

$$(xy')' + (xa^2 - \frac{m^2}{x})y = 0$$

$$xy'(xy')' + xy'(xa^2 - \frac{m^2}{x})y = 0$$

$$\frac{1}{2}\left[(xy')^2\right]' + \underbrace{xy'(xa^2 - \frac{m^2}{x})y}_{} = 0$$

$J_m(z_{m,m}) = 0$

$m \; J_m(0) = 0$

$\left[0, b = \frac{z_{m,m}}{a}\right]$

$$-\frac{m^2 x}{x} y' y = -m^2 yy'$$

$$= -\frac{1}{2} m^2 (y^2)'$$

$$-\frac{1}{2}\left[xJ_m'(ax)\right]^2\Big|_0^b$$

$\Big|^b$

$$+ \frac{n^2}{2} J_n^2(ax)\Big|_0$$

$$= a^2 \int_0^b x^2 J_n'(ax) J_n(ax)\, dx$$

$$= a^2 \int_0^b x^2 y' y\, dx = \frac{a^2}{2} \int x^2 (y^2)' dx$$

$$n J_n(0) = 0 \qquad J_n(ab) = 0$$

$$-\frac{1}{2} z_{n,m}^2 J_n'^2(z_{n,m}) = -a^2 \int_0^b x J_n^2(ax)\, dx$$

10.17

$$E_z = E_z(\rho, \phi, z, t)$$

$$= J_n\left(\sqrt{\frac{\omega^2}{c^2} - h^2}\, \rho\right) e^{in\phi} e^{i(hz-\omega t)}$$

$$-\Delta E_z = -\frac{\ddot{E}_z}{c^2} = \frac{\omega^2 E_z}{c^2}$$

$$\frac{d^2}{dt^2} e^{i(hz-\omega t)} = -\omega^2 e^{i(hz-\omega t)}$$

$2 \qquad 2\int$

$$-\frac{1}{c^2}\frac{d^2}{dt^2}E_z = \omega\frac{E_z}{c^2}$$

$$-\Delta E_z = \omega^2\frac{E_z}{c^2} \qquad (2.32)$$

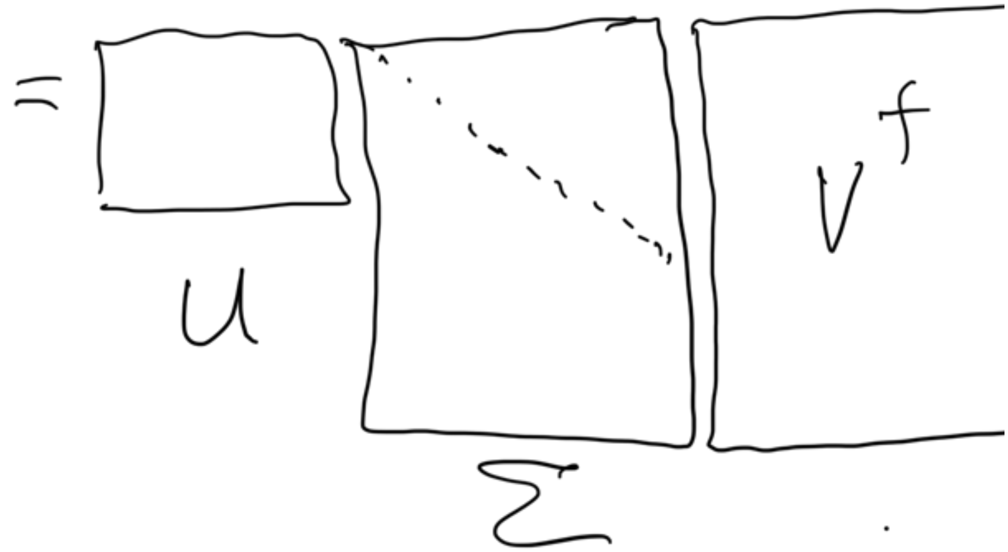
$$-\Delta E_z = -\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right) - \frac{m^2 E_z}{\rho^2} - k^2 E_z\right.$$

$$-\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right) + \frac{m^2}{\rho^2}E_z = \left(\frac{\omega^2}{c^2} - k^2\right)E_z$$

~10.11 $\quad J_m(a\rho) \quad a^2 = \frac{\omega^2}{c^2} - k^2.$

10.9 $$e^{\frac{z}{2}\left(u-\frac{1}{u}\right)} = \sum_{n=-\infty}^{\infty} u^n J_n(z)$$

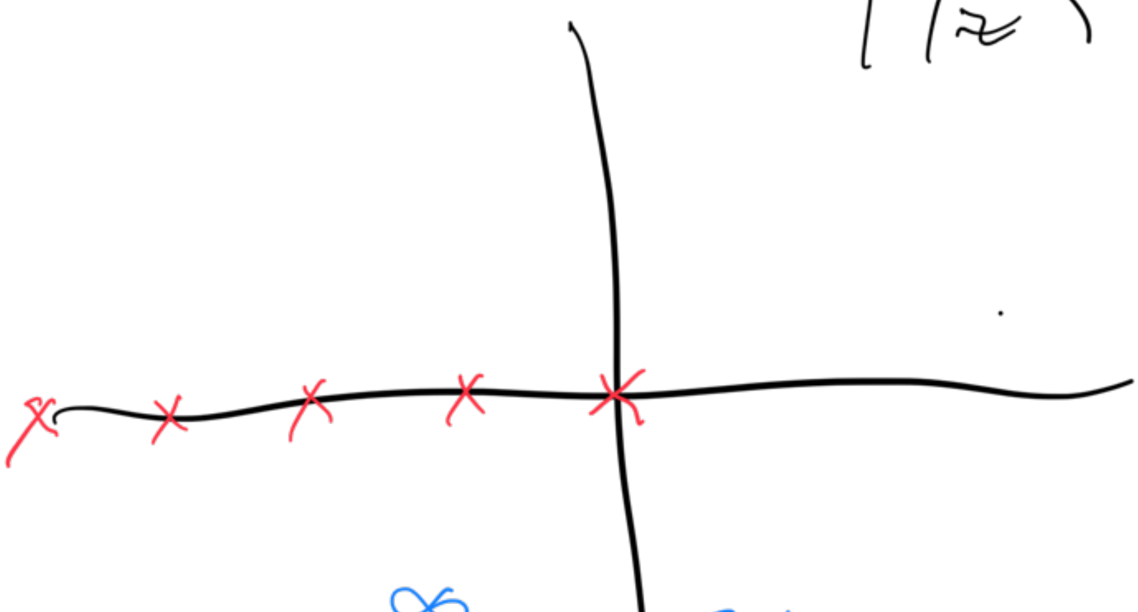
$$\boxed{A} = U\Sigma V^\dagger$$



$$e^{i\frac{2\pi n}{L}x} \qquad e^{i\frac{2\pi(n+1)}{L}x}$$

$$e^{i\frac{2\pi}{L}x} \simeq 1 + i\frac{2\pi}{L}x$$

$\Gamma(z)$



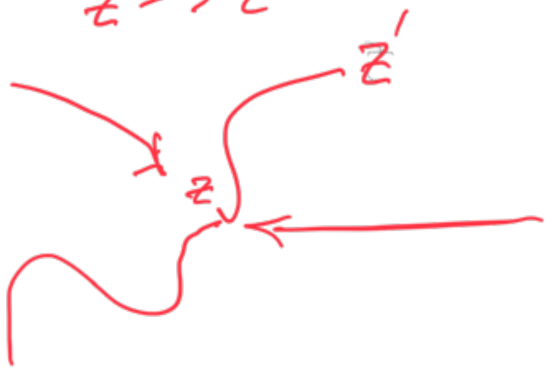
$$\int^{\infty} e^{-t}\, t^{z-1}\, dt = (z-1)!$$

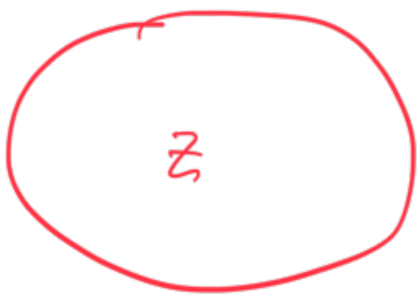
$$\Gamma(z) = \int_0 e$$

$$z = x + iy$$

$$f(z) = f(x,y) = f(x+iy)$$

$$f'(z) = \lim_{z' \to z} \frac{f(z') - f(z)}{z' - z}$$





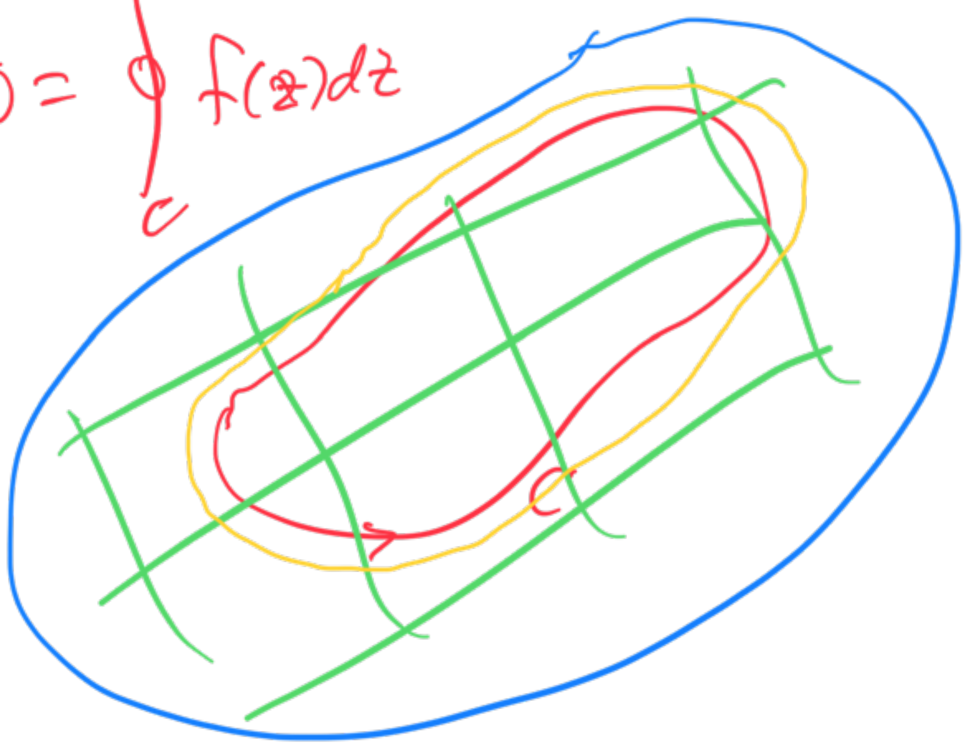
$$f(x,y) = u(x,y) + i v(x,y)$$

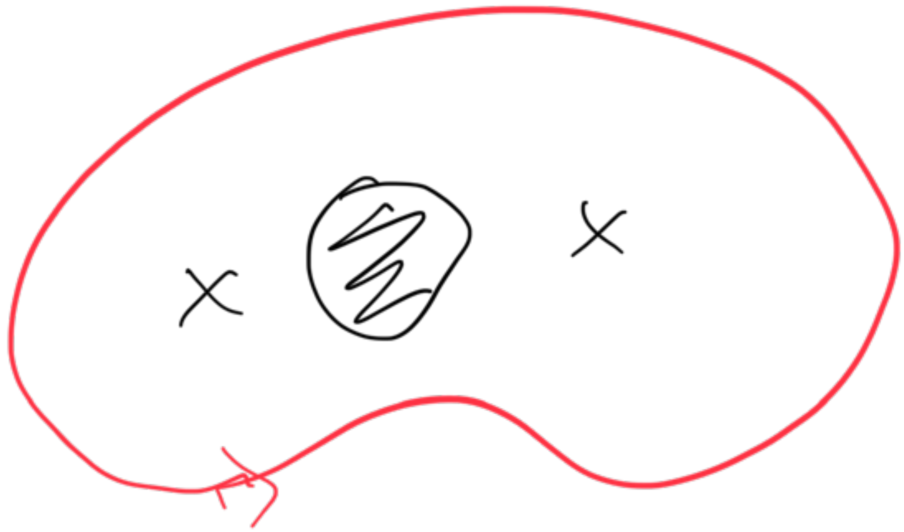
$$u_x = \frac{\partial u}{\partial x} = v_y = \frac{\partial v}{\partial y} \qquad v_x = -u_y$$

$$\oint_C f(z)\,dz = 0$$

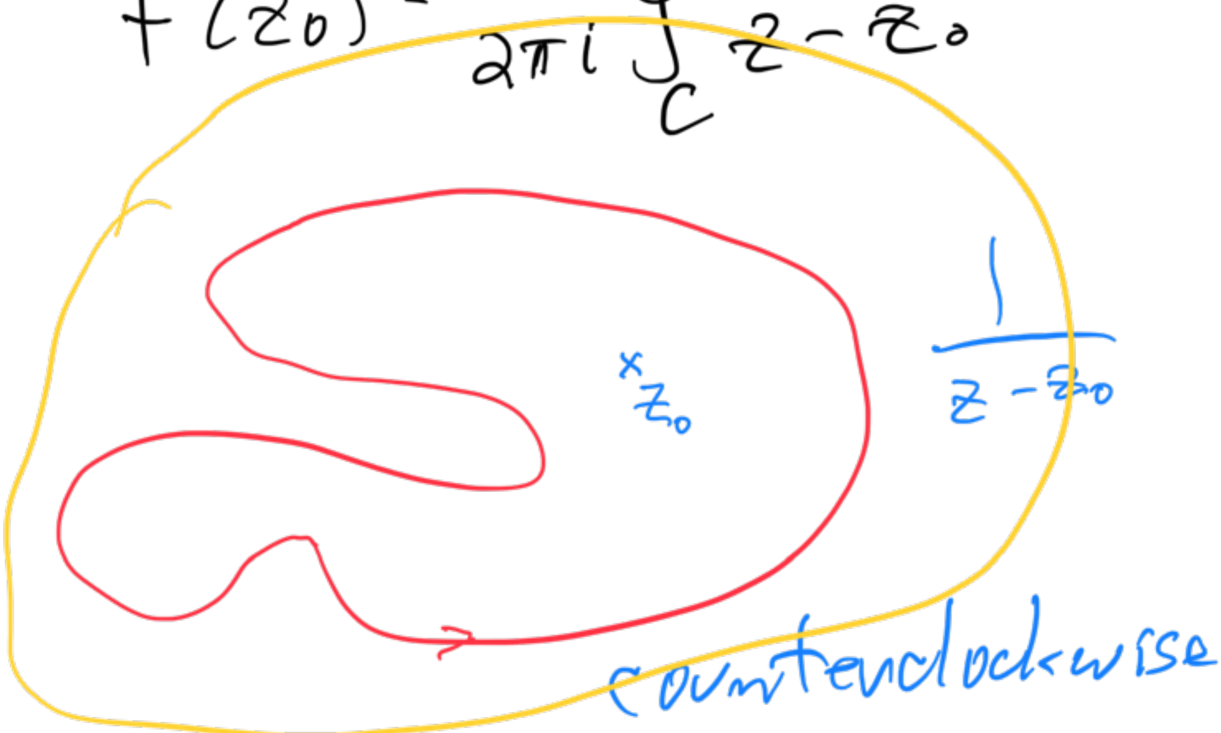
$$ax + bx^2$$

$$0 = \oint_C f(z)\,dz$$



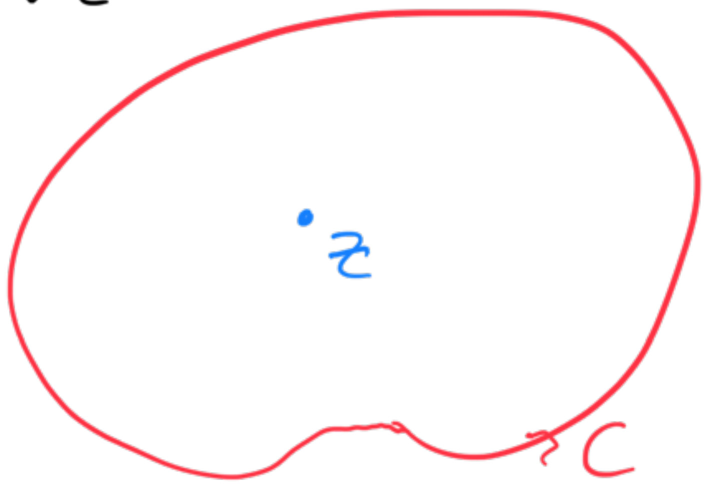


$$f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - z_0}\, dz$$



$$\frac{d^n}{dz^n} f(z) = f^{(n)}(z) = \frac{n!}{2\pi i} \oint_C \frac{f(z')\, dz'}{(z' - z)^{n+1}}$$

dt



$$\frac{1}{(z'-z)^{n+1}}$$

$$Y_\nu(z) = \frac{N_\nu(z)}{D_\nu(z)}$$

$$= \frac{0 + (\nu-n)N_n'(z)}{0 + (\nu-n)D_n'(z)} + \frac{(\nu-n)N_\nu'(z)}{(\nu-n)D_\nu'(z)}$$

$$N_\nu'(z) = \frac{dN_\nu(z)}{d\nu}$$

$$Y_\nu(z) = \frac{N_\nu'(z)}{D_\nu'(z)}$$

$$\nu \to n$$

$$Y_n(z) = \frac{N_n'(z)}{D_n'(z)}$$

$Y_n(z)$

$$N_\nu(z) = \cos(\pi\nu) J_\nu(z) - J_{-\nu}(z)$$

$$D_\nu(z) = \sin(\pi\nu)$$

$$j_\ell(x) = \sqrt{\frac{\pi}{2x}}\, J_{\ell+\frac{1}{2}}(x)$$

$$I = \int_a^b f(x)\,dx \longleftarrow$$

$$= \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

$$x_i = \frac{i}{b-a}$$

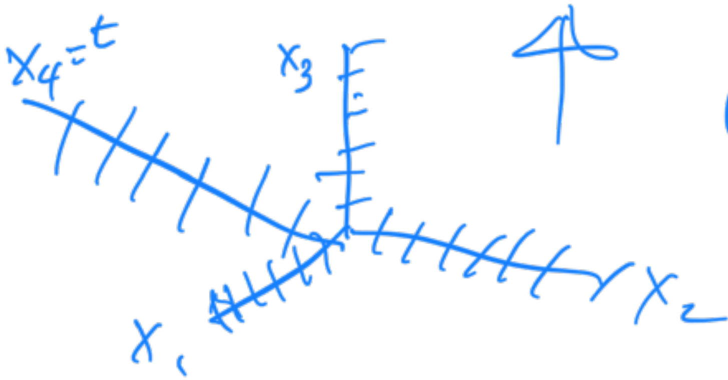
$$I = \int f(x_1, x_2, \ldots x_n)\, d^n x$$

$n = 10^4$ or 100^4

4

$(10^6)^{10^4}$

$\approx 10^{6\,10^4}$

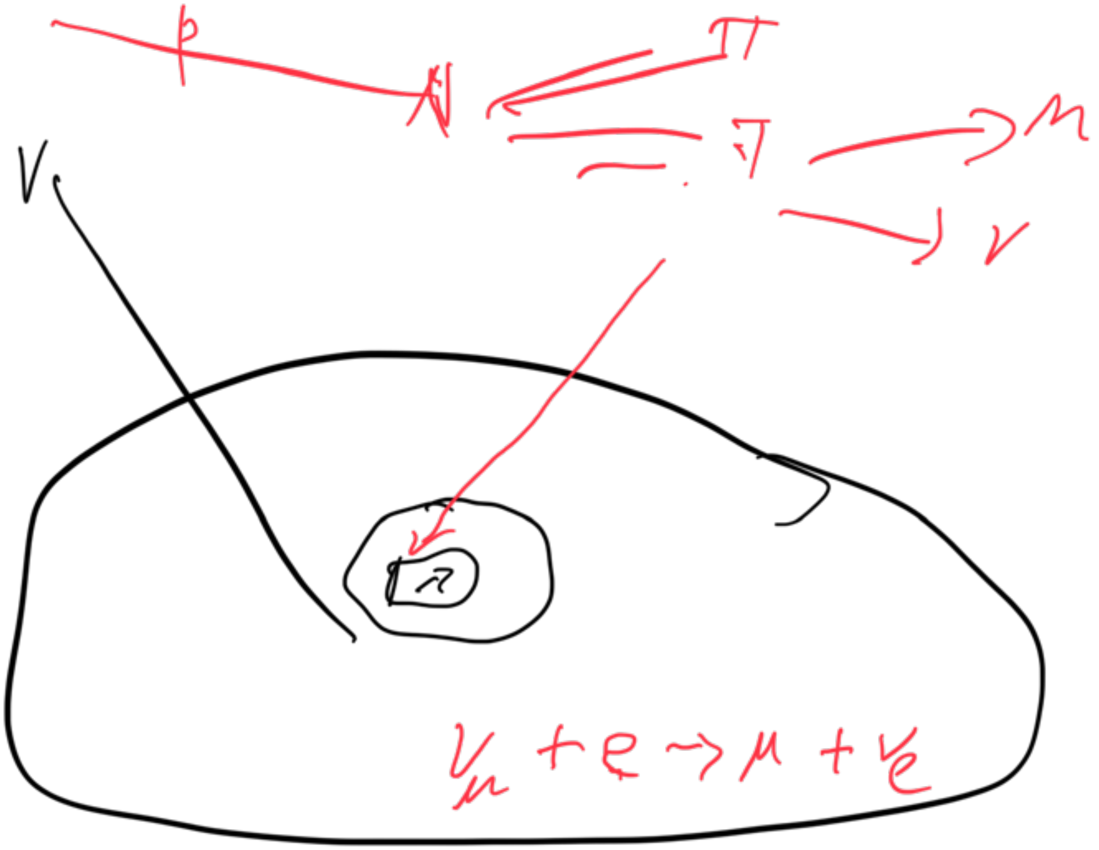


$$\int_R f(x)\, d^n x \cong \frac{L^n}{N} \sum_{k=1}^{N_R} f(x_k)$$

pseudorandom numbers

$\pi^+ \to \mu^+ + \nu_\mu$

$$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$$



$$P(n, \langle n \rangle) = \frac{\langle n \rangle^{n}}{\;} e^{-\langle n \rangle}$$