a complete set of momentum dyadics $|p\rangle\langle p|$ and doing the resulting Fourier transform.

Example 16.1 (The Bohm-Aharonov Effect) From our formula (11.311) for the action of a relativistic particle of mass m and charge q, we infer (exercise 16.7) that the action a nonrelativistic particle in an electromagnetic field with no scalar potential is

$$S = \int_{\boldsymbol{x}_1}^{\boldsymbol{x}_2} \left[\tfrac{1}{2} m \boldsymbol{v} + q\,\boldsymbol{A}\right] \cdot \boldsymbol{dx}\,. \tag{16.55}$$

Now imagine that we shoot a beam of such particles past but not through a narrow cylinder in which a magnetic field is confined. The particles can go either way around the cylinder of area S but cannot enter the region of the magnetic field. The difference in the phases of the amplitudes is the loop integral from the source to the detector and back to the source

$$\frac{\Delta S}{\hbar} = \oint \left[\frac{m\boldsymbol{v}}{2} + q\,\boldsymbol{A}\right] \cdot \frac{\boldsymbol{dx}}{\hbar} = \oint \frac{m\boldsymbol{v}\cdot\boldsymbol{dx}}{2\hbar} + \frac{q}{\hbar}\int_S \boldsymbol{B}\cdot\boldsymbol{dS} = \oint \frac{m\boldsymbol{v}\cdot\boldsymbol{dx}}{2\hbar} + \frac{q\Phi}{\hbar} \tag{16.56}$$

in which Φ is the magnetic flux through the cylinder. □

16.6 Free Particle in Imaginary Time

If we mimic the steps of the preceding section (16.5) in which the hamiltonian is $H = \boldsymbol{p}^2/2m$, set $\beta = it/\hbar = 1/kT$, and use Dirac's delta function

$$\delta^3(\boldsymbol{q}) = \lim_{t\to 0}\left(\frac{m}{2\pi\hbar t}\right)^{3/2} e^{-m\boldsymbol{q}^2/2\hbar t} \tag{16.57}$$

then we get

$$\langle \boldsymbol{q}|e^{-\beta H}|\boldsymbol{0}\rangle = \left(\frac{m}{2\pi\hbar^2\beta}\right)^{3/2} \exp\left[-\frac{m\boldsymbol{q}^2}{2\hbar^2\beta}\right] = \left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} e^{-mkT\boldsymbol{q}^2/2\hbar^2}. \tag{16.58}$$

To study the ground state of the system, we set $\beta = t/\hbar$ and let $t \to \infty$ in

$$\langle \boldsymbol{q}|e^{-tH/\hbar}|\boldsymbol{0}\rangle = \left(\frac{m}{2\pi\hbar t}\right)^{3/2} \exp\left[-\frac{m}{2}\frac{\boldsymbol{q}^2}{\hbar t}\right] \tag{16.59}$$

which for $D = \hbar/(2m)$ is the solution (3.200 & 13.107) of the diffusion equation.