Homework 13: Properties of Logarithms

MATH 1215

Name: _____ **Section:** _____

Instructor: _____ **Date:** _____

Directions. Show all work clearly. Use logarithm properties carefully, keep exact answers unless a decimal is requested, and check all logarithmic equations for domain restrictions.

A. Expand each logarithm

Write each expression as a sum, difference, and/or constant multiple of logarithms. Assume all variables represent positive numbers.

1. $\log_5\left(\dfrac{x^3 y}{z^2}\right)$

2. $\log_2(8x^4\sqrt{y})$

3. $\ln\left(\dfrac{a\sqrt[3]{b}}{c^5}\right)$

4. $\log\left(\dfrac{100m^2}{\sqrt{n}}\right)$

B. Combine into one logarithm

Use the product, quotient, and power properties to write each expression as a single logarithm. Simplify constants when possible.

1. $\log_7 x + \log_7 y + \log_7 z$

2. $\log_6 4 + \log_6 9$

3. $\log_8 x + \log_8 2y$

4. $\log_6 144 - \log_6 4$

5. $5\log_8 x - 3\log_8(2x)$

6. $2\ln(a+1) - \ln b + \ln 3$

C. Simplify using inverse properties

Simplify each expression completely.

1. $\log_8\left(8^{3x+1}\right)$

2. $2^{\log_2 27}$

3. $\log_4\left(\sqrt{4^x}\right)$

4. $10^{\log(3x-1)}$

D. Solve exponential equations

Solve exactly. Rewrite both sides with a common base when possible.

1. $3^{4x} = 27$

2. $8^x = 2$

3. $5^{x-2} = 125$

4. $4^{2x+1} = \dfrac{1}{32}$

5. $9^x = 3^{x+4}$

6. $6^x = \dfrac{1}{36}$

E. Solve logarithmic equations

Solve each equation and check that the solution is in the domain of the original logarithms.

1. $\log_3(x-5)=2$

2. $\log_4(5x)-\log_4 3=1$

3. $\log_4(x^2)=2$

4. $\log_2(x+3)+\log_2(x-2)=3$

5. $\log_5(2x-1)=\log_5(x+7)$

6. $\ln(x)-\ln(4)=\ln(3)$

F. Mixed practice

Use any appropriate logarithm or exponent rule. Show the main property used.

1. Expand and simplify: $\log_3\left(\dfrac{27x^2}{\sqrt{y}}\right)$.

2. Combine and simplify: $\dfrac{1}{2}\log_5 x+3\log_5 y-\log_5 10$.

G. Applications of logarithms

Write an equation first, then solve. Round only at the final step when a decimal is requested.

1. **Earthquake magnitude.** The Richter magnitude difference between two earthquakes is modeled by

$$M_1 - M_2 = \log\left(\frac{I_1}{I_2}\right),$$

where I_1 and I_2 are the intensities. An earthquake has intensity 1,000 times the intensity of another earthquake. How many units greater is its Richter magnitude?

2. **Acidity and pH.** The pH of a solution is modeled by

$$\text{pH} = -\log[H^+],$$

where $[H^+]$ is the hydrogen ion concentration in moles per liter. If $[H^+] = 10^{-5}$, find the pH.

3. **Sound intensity.** The decibel level of a sound is modeled by

$$D = 10\log\left(\frac{I}{I_0}\right),$$

where I_0 is the threshold intensity. If a sound has intensity $10^6 I_0$, find its decibel level.

Final check before submitting: Did you combine logarithms into one logarithm where requested? Did you check restrictions for logarithmic equations? Did you include units for application problems?