

Homework 1 Solutions

Real Numbers, Order of Operations, and Fractions

Course: MATH 1215

Intermediate Algebra

These solutions show one clear method for each problem. Equivalent correct methods are also acceptable. Final fractional answers are reduced to lowest terms.

A. Order of Operations and Evaluating Expressions

1. Evaluate $5 - 3(-7 - (-4 + 1))^2$.

$$\begin{aligned} 5 - 3(-7 - (-4 + 1))^2 &= 5 - 3(-7 - (-3))^2 \\ &= 5 - 3(-4)^2 \\ &= 5 - 3(16) \\ &= 5 - 48 \\ &= -43 \end{aligned}$$

Answer: $\boxed{-43}$

2. Evaluate $y^2 + \frac{\sqrt{x+7}}{2y}$ when $x = 9$ and $y = 3$.

$$\begin{aligned} y^2 + \frac{\sqrt{x+7}}{2y} &= 3^2 + \frac{\sqrt{9+7}}{2(3)} \\ &= 9 + \frac{\sqrt{16}}{6} \\ &= 9 + \frac{4}{6} \\ &= 9 + \frac{2}{3} \\ &= \frac{27}{3} + \frac{2}{3} = \frac{29}{3} \end{aligned}$$

Answer: $\boxed{\frac{29}{3}}$

3. Evaluate $-|-7| + 4^2 - 3(5 - 8)$.

$$\begin{aligned} -|-7| + 4^2 - 3(5 - 8) &= -7 + 16 - 3(-3) \\ &= -7 + 16 + 9 \\ &= 18 \end{aligned}$$

Answer: $\boxed{18}$

4. Evaluate $\frac{3}{4}(16 - 8) + \frac{5}{2}$.

$$\begin{aligned}
 \frac{3}{4}(16 - 8) + \frac{5}{2} &= \frac{3}{4}(8) + \frac{5}{2} \\
 &= 6 + \frac{5}{2} \\
 &= \frac{12}{2} + \frac{5}{2} \\
 &= \frac{17}{2}
 \end{aligned}$$

Answer: $\boxed{\frac{17}{2}}$

B. Sets, Inequalities, and Number Lines

5. List the natural numbers between 7 and 14.

The numbers strictly greater than 7 and strictly less than 14 are

$$\{8, 9, 10, 11, 12, 13\}.$$

Answer: $\boxed{\{8, 9, 10, 11, 12, 13\}}$

6. Graph the real numbers less than 4.

“Less than 4” means $x < 4$. Use an **open circle** at 4 because 4 is not included, and shade to the left.



Answer: $\boxed{x < 4}$

7. Write the inequality represented by the graph.

The graph has an **open circle** at -2 and is shaded to the right. The endpoint is not included and the values are greater than -2 .



Answer: $\boxed{x > -2}$

C. Reducing Fractions and Mixed Numbers

8. Reduce each fraction to lowest terms.

$$(a) \quad \frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \boxed{\frac{3}{4}}$$

$$(b) \quad \frac{45}{60} = \frac{45 \div 15}{60 \div 15} = \boxed{\frac{3}{4}}$$

$$(c) \quad \frac{72}{90} = \frac{72 \div 18}{90 \div 18} = \boxed{\frac{4}{5}}$$

$$(d) \quad \frac{-36}{48} = -\frac{36 \div 12}{48 \div 12} = \boxed{-\frac{3}{4}}$$

$$(e) \quad \frac{56}{64} = \frac{56 \div 8}{64 \div 8} = \boxed{\frac{7}{8}}$$

$$(f) \quad \frac{81}{108} = \frac{81 \div 27}{108 \div 27} = \boxed{\frac{3}{4}}$$

9. Convert each mixed number to an improper fraction.

Use $a\frac{b}{c} = \frac{ac + b}{c}$. For a negative mixed number, keep the negative sign on the entire value.

$$(a) \quad 2\frac{1}{3} = \frac{2(3) + 1}{3} = \boxed{\frac{7}{3}}$$

$$(b) \quad 4\frac{5}{6} = \frac{4(6) + 5}{6} = \boxed{\frac{29}{6}}$$

$$(c) \quad -3\frac{2}{5} = -\frac{3(5) + 2}{5} = \boxed{-\frac{17}{5}}$$

$$(d) \quad 7\frac{4}{8} = \frac{7(8) + 4}{8} = \frac{60}{8} = \boxed{\frac{15}{2}}$$

$$(e) \quad 1\frac{9}{12} = \frac{1(12) + 9}{12} = \frac{21}{12} = \boxed{\frac{7}{4}}$$

$$(f) \quad 5\frac{10}{15} = \frac{5(15) + 10}{15} = \frac{85}{15} = \boxed{\frac{17}{3}}$$

10. Write each improper fraction as a mixed number or whole number.

$$(a) \quad \frac{15}{4} = 3\frac{3}{4}$$

$$(b) \quad \frac{21}{7} = 3$$

$$(c) \quad \frac{33}{8} = 4\frac{1}{8}$$

$$(d) \quad \frac{48}{12} = 4$$

$$(e) \quad \frac{29}{5} = 5\frac{4}{5}$$

$$(f) \quad \frac{54}{9} = 6$$

D. Adding, Subtracting, Multiplying, and Dividing Fractions**11. Simplify. Leave each answer as an improper fraction in lowest terms.**

$$(a) \quad \frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \boxed{\frac{19}{12}}$$

$$(b) \quad \frac{7}{8} - \frac{1}{3} = \frac{21}{24} - \frac{8}{24} = \boxed{\frac{13}{24}}$$

$$(c) \quad \frac{2}{7} + \frac{5}{3} = \frac{6}{21} + \frac{35}{21} = \boxed{\frac{41}{21}}$$

$$(d) \quad -\frac{5}{8} - \frac{9}{2} = -\frac{5}{8} - \frac{36}{8} = \boxed{-\frac{41}{8}}$$

$$(e) \quad \frac{2}{5} \cdot \frac{11}{-4} = \frac{22}{-20} = \boxed{-\frac{11}{10}}$$

$$(f) \quad \frac{2}{11} \div \frac{5}{4} = \frac{2}{11} \cdot \frac{4}{5} = \boxed{\frac{8}{55}}$$

12. Simplify. Write final answers in lowest terms.

$$(a) \quad 3\frac{1}{2} + 1\frac{3}{4} = \frac{7}{2} + \frac{7}{4} = \frac{14}{4} + \frac{7}{4} = \frac{21}{4} = \boxed{5\frac{1}{4}}$$

$$(b) \quad 5\frac{2}{3} - 2\frac{5}{6} = \frac{17}{3} - \frac{17}{6} = \frac{34}{6} - \frac{17}{6} = \frac{17}{6} = \boxed{2\frac{5}{6}}$$

$$(c) \quad 2\frac{1}{4} \cdot \frac{8}{9} = \frac{9}{4} \cdot \frac{8}{9} = \frac{72}{36} = \boxed{2}$$

$$(d) \quad 4\frac{1}{2} \div 1\frac{1}{5} = \frac{9}{2} \div \frac{6}{5} = \frac{9}{2} \cdot \frac{5}{6} = \frac{45}{12} = \frac{15}{4} = \boxed{3\frac{3}{4}}$$

E. Fraction Word Problems

13. Flour for $2\frac{1}{2}$ batches.

Each batch uses $\frac{3}{4}$ cup. Multiply the amount per batch by the number of batches.

$$\begin{aligned} \frac{3}{4} \left(2\frac{1}{2} \right) &= \frac{3}{4} \cdot \frac{5}{2} \\ &= \frac{15}{8} \\ &= 1\frac{7}{8} \end{aligned}$$

Answer: $\boxed{1\frac{7}{8} \text{ cups}}$

14. Fraction of the assignment completed and remaining.

First add the portions completed on Monday and Tuesday.

$$\begin{aligned} \frac{5}{8} + \frac{1}{4} &= \frac{5}{8} + \frac{2}{8} \\ &= \frac{7}{8} \end{aligned}$$

So $\frac{7}{8}$ of the assignment is complete. The amount remaining is

$$1 - \frac{7}{8} = \frac{1}{8}.$$

Answer: $\boxed{\frac{7}{8} \text{ completed, } \frac{1}{8} \text{ remaining}}$

15. Cutting a $6\frac{1}{2}$ -foot board into $\frac{3}{4}$ -foot pieces.

Divide the total length by the length of each piece.

$$\begin{aligned} 6\frac{1}{2} \div \frac{3}{4} &= \frac{13}{2} \cdot \frac{4}{3} \\ &= \frac{52}{6} \\ &= \frac{26}{3} = 8\frac{2}{3} \end{aligned}$$

The quotient $8\frac{2}{3}$ means there are **8 complete pieces**. Eight pieces use

$$8\left(\frac{3}{4}\right) = 6 \text{ feet.}$$

The wood left over is

$$6\frac{1}{2} - 6 = \frac{1}{2} \text{ foot.}$$

Answer: 8 complete pieces and $\frac{1}{2}$ foot left over

F. Mixed Review

16. Simplify each expression completely.

$$(a) \quad |-12| - |5 - 9| = 12 - |-4| = 12 - 4 = \boxed{8}$$

$$(b) \quad \frac{6}{10} + \frac{9}{15} = \frac{3}{5} + \frac{3}{5} = \boxed{\frac{6}{5}}$$

$$(c) \quad 3\left(\frac{2}{5}\right) - \frac{1}{10} = \frac{6}{5} - \frac{1}{10} = \frac{12}{10} - \frac{1}{10} = \boxed{\frac{11}{10}}$$

$$(d) \quad \frac{5}{6} \div \left(-\frac{10}{9}\right) = \frac{5}{6} \cdot \left(-\frac{9}{10}\right) = -\frac{45}{60} = \boxed{-\frac{3}{4}}$$