

MATH 1215 - Homework 2 Solutions

Solving Linear Equations and Inequalities

Complete worked solutions with algebraic steps, interval notation, and inequality graphs

Important algebra principle. Perform the same operation on both sides of an equation. For an inequality, multiplying or dividing by a **negative** number reverses the inequality symbol.

Part I: Solve the Linear Equations

1. Solve $-5x = 7x + 1$.

$$-5x = 7x + 1$$

$$-12x = 1 \quad \text{Subtract } 7x \text{ from both sides.}$$

$$x = -\frac{1}{12} \quad \text{Divide both sides by } -12.$$

Answer: $x = -\frac{1}{12}$

2. Solve $5 - 5x = x - 2(2 + x)$.

$$5 - 5x = x - 2(2 + x)$$

$$5 - 5x = x - 4 - 2x \quad \text{Distribute } -2.$$

$$5 - 5x = -x - 4 \quad \text{Combine like terms.}$$

$$5 = 4x - 4 \quad \text{Add } 5x \text{ to both sides.}$$

$$9 = 4x \quad \text{Add } 4 \text{ to both sides.}$$

$$x = \frac{9}{4} \quad \text{Divide both sides by } 4.$$

Answer: $x = \frac{9}{4}$

3. Solve $\frac{1}{2}(x - 3) = 4(x - 12)$.

$$\frac{1}{2}(x - 3) = 4(x - 12)$$

$$x - 3 = 8(x - 12) \quad \text{Multiply both sides by } 2.$$

$$x - 3 = 8x - 96 \quad \text{Distribute } 8.$$

$$93 = 7x \quad \text{Add } 96 \text{ and subtract } x.$$

$$x = \frac{93}{7} \quad \text{Divide both sides by } 7.$$

Answer: $x = \frac{93}{7}$

4. Solve $3(x + 4) - 1 = 3x - 2$.

$$3(x + 4) - 1 = 3x - 2$$

$$3x + 12 - 1 = 3x - 2 \quad \text{Distribute } 3.$$

$$3x + 11 = 3x - 2 \quad \text{Combine like terms.}$$

$$11 = -2 \quad \text{Subtract } 3x \text{ from both sides.}$$

The statement $11 = -2$ is false, so no value of x can satisfy the original equation.

Answer: No solution, $\emptyset$

5. Solve $2(r + 10) = -2(15r + 10)$.

$$2(r + 10) = -2(15r + 10)$$

$$2r + 20 = -30r - 20 \quad \text{Distribute on both sides.}$$

$$32r + 20 = -20 \quad \text{Add } 30r \text{ to both sides.}$$

$$32r = -40 \quad \text{Subtract 20 from both sides.}$$

$$r = -\frac{40}{32} = -\frac{5}{4} \quad \text{Divide by 32 and simplify.}$$

Answer: $r = -\frac{5}{4}$

6. Solve $3(x + 2) - 5 = 2(x - 3) - 7$.

$$3(x + 2) - 5 = 2(x - 3) - 7$$

$$3x + 6 - 5 = 2x - 6 - 7 \quad \text{Distribute.}$$

$$3x + 1 = 2x - 13 \quad \text{Combine like terms.}$$

$$x + 1 = -13 \quad \text{Subtract } 2x \text{ from both sides.}$$

$$x = -14 \quad \text{Subtract 1 from both sides.}$$

Answer: $x = -14$

Part II: Solve and Graph the Inequalities

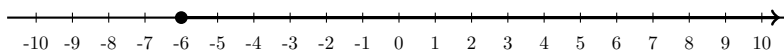
7. Solve $-\frac{2}{3}n \leq 4$.

$$-\frac{2}{3}n \leq 4$$

$$n \geq 4 \left(-\frac{3}{2}\right) \quad \text{Multiply by } -\frac{3}{2}; \text{ reverse the inequality.}$$

$$n \geq -6$$

Answer: $n \geq -6$; interval notation: $[-6, \infty)$



8. Solve $2 \leq -\frac{3}{5}w + 1$.

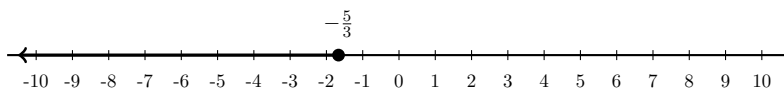
$$2 \leq -\frac{3}{5}w + 1$$

$$1 \leq -\frac{3}{5}w \quad \text{Subtract 1 from both sides.}$$

$$-\frac{5}{3} \geq w \quad \text{Multiply by } -\frac{5}{3}; \text{ reverse the inequality.}$$

$$w \leq -\frac{5}{3}$$

Answer: $w \leq -\frac{5}{3}$; interval notation: $(-\infty, -\frac{5}{3}]$



9. Solve $4x - 3 > 2(x - 1)$.

$$4x - 3 > 2(x - 1)$$

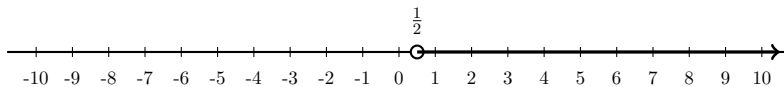
$$4x - 3 > 2x - 2 \quad \text{Distribute 2.}$$

$$2x - 3 > -2 \quad \text{Subtract } 2x.$$

$$2x > 1 \quad \text{Add 3.}$$

$$x > \frac{1}{2} \quad \text{Divide by 2.}$$

Answer: $x > \frac{1}{2}$; interval notation: $(\frac{1}{2}, \infty)$



10. Solve $7x - 4 \leq 3(x - 4)$.

$$7x - 4 \leq 3(x - 4)$$

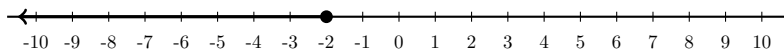
$$7x - 4 \leq 3x - 12 \quad \text{Distribute 3.}$$

$$4x - 4 \leq -12 \quad \text{Subtract } 3x.$$

$$4x \leq -8 \quad \text{Add 4.}$$

$$x \leq -2 \quad \text{Divide by 4.}$$

Answer: $x \leq -2$; interval notation: $(-\infty, -2]$



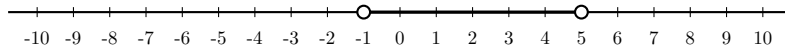
11. Solve $-2 < 3x + 1 < 16$.

$$-2 < 3x + 1 < 16$$

$$-3 < 3x < 15 \quad \text{Subtract 1 from all three parts.}$$

$$-1 < x < 5 \quad \text{Divide all three parts by 3.}$$

Answer: $-1 < x < 5$; interval notation: $(-1, 5)$



12. Solve $-2x + 7 > 3$ or $3x - 4 \geq 5$.

Solve each inequality separately.

$$-2x + 7 > 3$$

$$-2x > -4 \quad \text{Subtract 7.}$$

$$x < 2 \quad \text{Divide by } -2; \text{ reverse the inequality.}$$

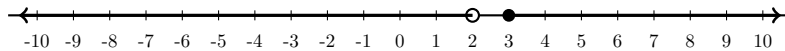
$$3x - 4 \geq 5$$

$$3x \geq 9 \quad \text{Add 4.}$$

$$x \geq 3 \quad \text{Divide by 3.}$$

Because the connector is **or**, take the union of the two solution sets.

Answer: $x < 2$ or $x \geq 3$; interval notation: $(-\infty, 2) \cup [3, \infty)$



Part III: Applications

13. Cell phone plan. A company charges a monthly fee of \$25 plus \$8 per gigabyte. The bill is \$81. Find the number of gigabytes used.

Let g be the number of gigabytes of data used. Then

$$25 + 8g = 81 \quad \text{monthly fee} + \text{data charge} = \text{total bill}$$

$$8g = 56 \quad \text{Subtract 25 from both sides.}$$

$$g = 7 \quad \text{Divide both sides by 8.}$$

Check: $25 + 8(7) = 25 + 56 = 81$.

Answer: The customer used **7 gigabytes** of data.

14. Taxi fare. A taxi charges a \$4 starting fee and \$3 per mile. Determine how many miles a customer can travel while spending no more than \$28.

Let m be the number of miles traveled. The phrase “no more than \$28” means the total cost must be less than or equal to 28:

$$4 + 3m \leq 28$$

$$3m \leq 24 \quad \text{Subtract 4 from both sides.}$$

$$m \leq 8 \quad \text{Divide both sides by 3.}$$

Since distance cannot be negative, the practical solution is $0 \leq m \leq 8$.

Answer: The customer can travel **at most 8 miles**.