

Homework 4 Solutions

Systems of Linear Equations

MATH 1215 - Complete worked solutions

Goal. Solve each system carefully, verify the solution, and interpret the intersection point as the ordered-pair solution of the system.

Part I: Solve Graphically

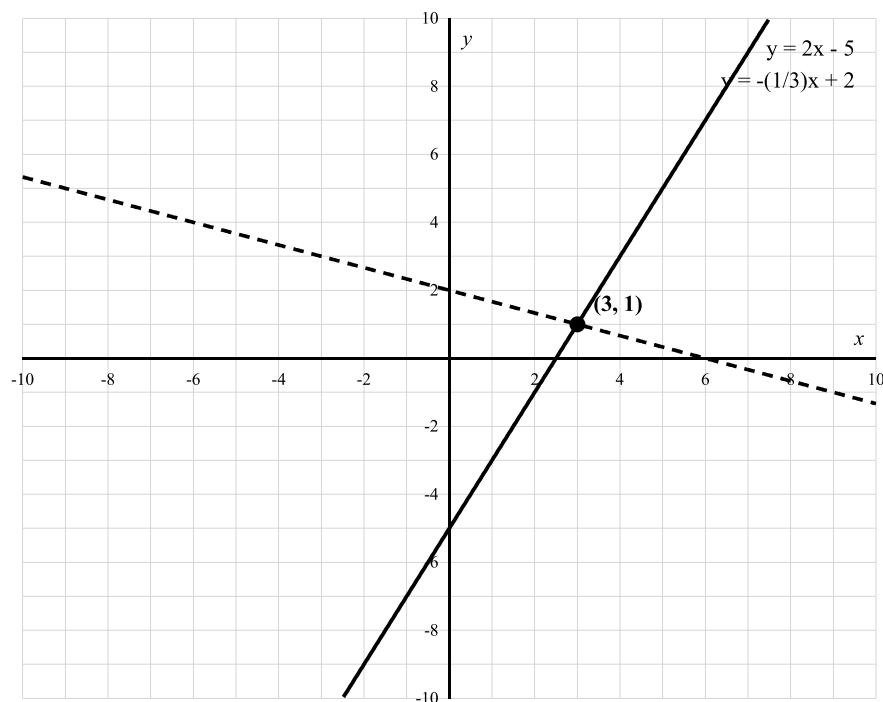
1. Solve the system graphically.

$$y = 2x - 5$$

$$y = -(1/3)x + 2$$

The first line has slope 2 and y-intercept -5. The second has slope -1/3 and y-intercept 2.

The solution is the point where the two lines intersect.



Solution:

(3, 1)

Part I: Solve Graphically (continued)

2. Solve graphically and by elimination.

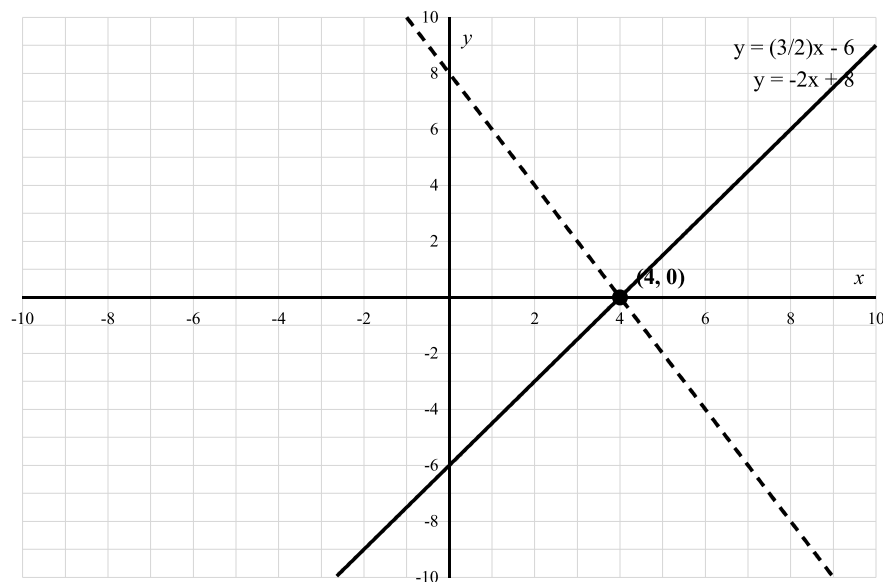
$$12x - 8y = 48$$

$$y - 4 = -2(x - 2)$$

For graphing, write both equations in slope-intercept form:

$$12x - 8y = 48 \Rightarrow y = (3/2)x - 6$$

$$y - 4 = -2x + 4 \Rightarrow y = -2x + 8$$



Elimination check:

Rewrite $y = -2x + 8$ as $2x + y = 8$, then multiply that equation by 8.

$$12x - 8y = 48$$

$$16x + 8y = 64$$

$$28x = 112 \Rightarrow x = 4$$

$$2(4) + y = 8 \Rightarrow y = 0$$

Solution:

(4, 0)

Part II: Solve Algebraically

Solve each system using substitution or elimination.

(a)

$$12x - 6y = -6$$

$$(3/2)x + (5/3)y = 5$$

Clear the fractions in the second equation by multiplying by 6:

$$9x + 10y = 30$$

Eliminate x: multiply the first equation by 3 and the second equation by -4.

$$36x - 18y = -18$$

$$-36x - 40y = -120$$

$$-58y = -138 \Rightarrow y = 69/29$$

$$12x - 6(69/29) = -6 \Rightarrow x = 20/29$$

Solution:

(20/29, 69/29)

(b)

$$16x - 8y = 40$$

$$2x + y = 7$$

Multiply the second equation by 8 and add:

$$16x - 8y = 40$$

$$16x + 8y = 56$$

$$32x = 96 \Rightarrow x = 3$$

$$2(3) + y = 7 \Rightarrow y = 1$$

Solution:

(3, 1)

Part II: Solve Algebraically (continued)

(c)

$$3x + 2y = 12$$

$$x - y = 1$$

Use substitution. From $x - y = 1$, solve for x :

$$x = y + 1$$

$$3(y + 1) + 2y = 12$$

$$3y + 3 + 2y = 12$$

$$5y = 9 \Rightarrow y = 9/5$$

$$x = 9/5 + 1 = 14/5$$

Solution:

(14/5, 9/5)

Part III: Applications

(a) Valentine Dance tickets

Let s = number of single-person tickets and c = number of couple tickets.

The attendance equation counts people; the revenue equation counts dollars.

$$s + 2c = 180$$

$$3s + 5c = 475$$

Multiply the first equation by -3 and add:

$$-3s - 6c = -540$$

$$3s + 5c = 475$$

$$-c = -65 \Rightarrow c = 65$$

Check: $s = 180 - 2(65) = 50$, and $3(50) + 5(65) = 475$.

Answer:

65 couples

Part III: Applications (continued)

(b) The sum and difference of two numbers

Let x be the larger number and y be the smaller number.

$$x + y = 200$$

$$x - y = 28$$

Add the equations to eliminate y :

$$2x = 228 \Rightarrow x = 114$$

$$114 + y = 200 \Rightarrow y = 86$$

Check: $114 + 86 = 200$ and $114 - 86 = 28$.

Answer:

114 and 86

(c) Movie theater tickets

Let a = number of adult tickets and s = number of student tickets.

$$a + s = 250$$

$$12a + 8s = 2600$$

Multiply the first equation by -8 and add:

$$-8a - 8s = -2000$$

$$12a + 8s = 2600$$

$$4a = 600 \Rightarrow a = 150$$

$$150 + s = 250 \Rightarrow s = 100$$

Check: $12(150) + 8(100) = 1800 + 800 = 2600$.

Answer:

150 adults; 100 students

End of Homework 4 Solutions