

Homework 5 - Detailed Solutions

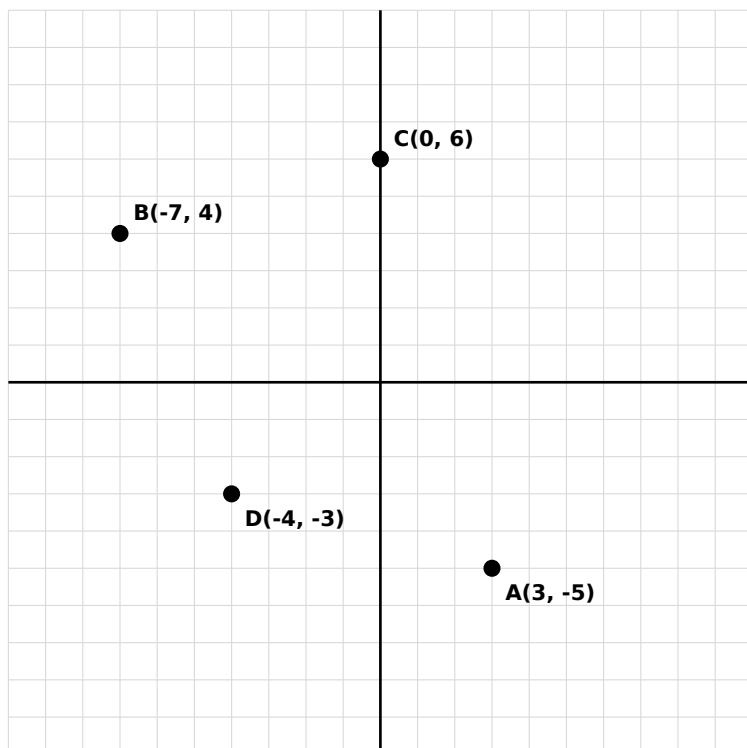
MATH 1215: Coordinate Geometry
Distance, Midpoint, and Circles

Distance Formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint Formula	$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$
Standard Form of a Circle	$(x - h)^2 + (y - k)^2 = r^2$

Part I: Plotting Points

1. Plot and label A(3, -5), B(-7, 4), C(0, 6), and D(-4, -3).

For an ordered pair (x, y), move horizontally according to x and vertically according to y. The correctly plotted and labeled points are shown below.



Answer: The four points are plotted and labeled above.

Part II: Distance Formula

2. Find the distance between each pair of points.

(a) (2, 2) and (5, -2)

$$\begin{aligned}d &= \sqrt{(5 - 2)^2 + (-2 - 2)^2} \\&= \sqrt{3^2 + (-4)^2} \\&= \sqrt{9 + 16} = \sqrt{25} = 5\end{aligned}$$

Answer: 5 units

(b) (0, 2) and (-2, 10)

$$\begin{aligned}d &= \sqrt{(-2 - 0)^2 + (10 - 2)^2} \\&= \sqrt{(-2)^2 + 8^2} \\&= \sqrt{4 + 64} = \sqrt{68} \\&= \sqrt{4 \cdot 17} = 2\sqrt{17}\end{aligned}$$

Answer: $2\sqrt{17}$ units

(c) (-3, -4) and (5, 2)

$$\begin{aligned}d &= \sqrt{(5 - (-3))^2 + (2 - (-4))^2} \\&= \sqrt{8^2 + 6^2} \\&= \sqrt{64 + 36} = \sqrt{100} = 10\end{aligned}$$

Answer: 10 units

Part III: Midpoint Formula

3. Find the midpoint of each line segment.

(a) (3, -2) and (-5, 6)

$$M = ((3 + (-5))/2, (-2 + 6)/2) = (-2/2, 4/2) = (-1, 2)$$

Answer: (-1, 2)

(b) (-2, 7) and (5, -7)

$$M = ((-2 + 5)/2, (7 + (-7))/2) = (3/2, 0/2) = (3/2, 0)$$

Answer: (3/2, 0)

(c) (-8, -3) and (4, 9)

$$M = ((-8 + 4)/2, (-3 + 9)/2) = (-4/2, 6/2) = (-2, 3)$$

Answer: (-2, 3)

Part IV: Midpoint and Distance from a Graph

4. The endpoints are (-4, -2) and (5, 3).

(a) Midpoint

$$M = ((-4 + 5)/2, (-2 + 3)/2) = (1/2, 1/2)$$

Answer: (1/2, 1/2)

(b) Distance

$$\begin{aligned} d &= \sqrt{(5 - (-4))^2 + (3 - (-2))^2} \\ &= \sqrt{9^2 + 5^2} \\ &= \sqrt{81 + 25} = \sqrt{106} \end{aligned}$$

Since $106 = 2 \cdot 53$ has no perfect-square factor greater than 1, $\sqrt{106}$ is already simplified.

Answer: $\sqrt{106}$ units

Part V: Equations of Circles

5. Write the equation of each circle.

Use the standard form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius.

(a) Center $(5, 3)$, radius $r = 3$

$$(x - 5)^2 + (y - 3)^2 = 3^2$$

Answer: $(x - 5)^2 + (y - 3)^2 = 9$

(b) Center $(-2, -4)$, radius $r = 5$

$$(x - (-2))^2 + (y - (-4))^2 = 5^2$$

$$(x + 2)^2 + (y + 4)^2 = 25$$

Answer: $(x + 2)^2 + (y + 4)^2 = 25$

(c) Center $(0, 0)$, radius $r = 7$

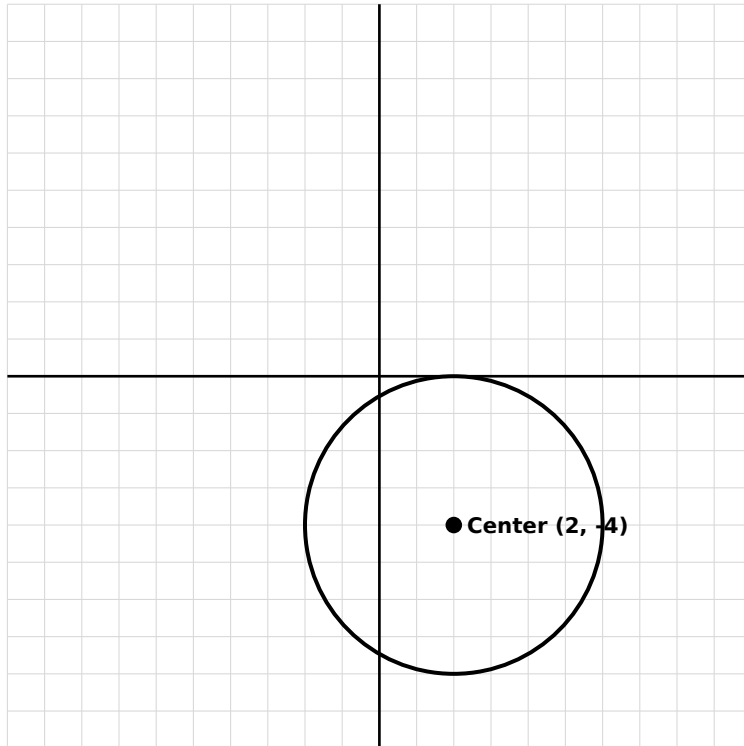
$$(x - 0)^2 + (y - 0)^2 = 7^2$$

Answer: $x^2 + y^2 = 49$

Part VI: Equation of a Circle from a Graph

6. Determine the center and radius, then write the equation of the circle.

From the graph, the center is 2 units to the right of the y-axis and 4 units below the x-axis, so the center is $(2, -4)$. The circle reaches the x-axis directly above the center at $(2, 0)$, so the radius is 4.



Center: (2, -4) Radius: 4

$$(x - 2)^2 + (y - (-4))^2 = 4^2$$

Answer: $(x - 2)^2 + (y + 4)^2 = 16$

Part VII: Applications

7. A park is located at (2, 3) and a school is located at (10, 9). Find the distance.

$$\begin{aligned} d &= \sqrt{(10 - 2)^2 + (9 - 3)^2} \\ &= \sqrt{8^2 + 6^2} \\ &= \sqrt{64 + 36} = \sqrt{100} = 10 \end{aligned}$$

Answer: The park and school are 10 units apart.

8. A circular fountain has center (1, -2) and radius 6 feet.

(a) Write the equation of the fountain.

$$(x - 1)^2 + (y - (-2))^2 = 6^2$$

Answer: $(x - 1)^2 + (y + 2)^2 = 36$

(b) Is the point (7, -2) on the circle? Explain.

Substitute $x = 7$ and $y = -2$ into the equation:

$$(7 - 1)^2 + (-2 + 2)^2 = 6^2 + 0^2 = 36.$$

The left side equals 36, so the point satisfies the equation of the circle.

Answer: Yes. The point (7, -2) is on the circle.