

MATH 1215 - Homework 6 Solutions

Polynomials, Rational Expressions, and Scientific Notation

Show all work clearly. Simplify all answers completely and use proper exponent rules.

Part I: Simplify

1. $(10xy^2)(3x^2y)$

$$(10xy^2)(3x^2y) = (10)(3)x^{1+2}y^{2+1} \quad \text{Product rule: } a^m a^n = a^{m+n}$$

$$= 30x^3y^3.$$

Final Answer: $30x^3y^3$

2. $\frac{18x^3y^2}{36xy^5}$

$$\frac{18x^3y^2}{36xy^5} = \frac{18}{36}x^{3-1}y^{2-5} \quad \text{Quotient rule: } \frac{a^m}{a^n} = a^{m-n}$$

$$= \frac{1}{2}x^2y^{-3} = \frac{x^2}{2y^3}.$$

Final Answer: $\frac{x^2}{2y^3}$

3. $\frac{24m^2n^6}{6m^3n^2}$

$$\frac{24m^2n^6}{6m^3n^2} = 4m^{2-3}n^{6-2} = 4m^{-1}n^4 = \frac{4n^4}{m}.$$

Final Answer: $\frac{4n^4}{m}$

4. $\frac{12x^5}{4x^2}$

$$\frac{12x^5}{4x^2} = 3x^{5-2} = 3x^3.$$

Final Answer: $3x^3$

5. $(3a^2b)(-2ab^3)$

$$(3a^2b)(-2ab^3) = (3)(-2)a^{2+1}b^{1+3} = -6a^3b^4.$$

Final Answer: $-6a^3b^4$

6. $\frac{15p^4q^2}{5p^2q}$

$$\frac{15p^4q^2}{5p^2q} = 3p^{4-2}q^{2-1} = 3p^2q.$$

Final Answer: $3p^2q$

Part II: Scientific Notation

7. Write each number in scientific notation.

(a) 62,000

$$62,000 = 6.2 \times 10^4$$

The decimal point moves 4 places to the left, so the exponent is 4.

Final Answer: 6.2×10^4

(b) 0.000071

$$0.000071 = 7.1 \times 10^{-5}$$

The decimal point moves 5 places to the right, so the exponent is -5 .

Final Answer: 7.1×10^{-5}

(c) 5,420,000

$$5,420,000 = 5.42 \times 10^6$$

Final Answer: 5.42×10^6

8. Write each number in standard form.

(a) 4.2×10^5

$$4.2 \times 10^5 = 420,000$$

Final Answer: 420,000

(b) 7.8×10^{-4}

$$7.8 \times 10^{-4} = 0.00078$$

Final Answer: 0.00078

(c) 9.15×10^3

$$9.15 \times 10^3 = 9,150$$

Final Answer: 9,150

9. Simplify and write the answer in both scientific notation and standard form.

(a) $(3 \times 10^4)(2 \times 10^3)$

$$\begin{aligned}(3 \times 10^4)(2 \times 10^3) &= (3)(2) \times 10^{4+3} \\ &= 6 \times 10^7 \\ &= 60,000,000.\end{aligned}$$

Final Answer: $6 \times 10^7 = 60,000,000$

(b) $\frac{8 \times 10^7}{2 \times 10^2}$

$$\frac{8 \times 10^7}{2 \times 10^2} = \frac{8}{2} \times 10^{7-2} = 4 \times 10^5 = 400,000.$$

Final Answer: $4 \times 10^5 = 400,000$

(c) $(5 \times 10^{-3})(4 \times 10^5)$

$$(5 \times 10^{-3})(4 \times 10^5) = 20 \times 10^2$$

$$= 2 \times 10^3 \quad \text{write the coefficient between 1 and 10}$$

$$= 2,000.$$

Final Answer: $2 \times 10^3 = 2,000$

Part III: Simplifying Polynomial Expressions

10. Simplify each expression by combining like terms.

(a) $j - 4k + 7 + 2j - 3k$

$$\begin{aligned} j - 4k + 7 + 2j - 3k &= (j + 2j) + (-4k - 3k) + 7 \\ &= 3j - 7k + 7. \end{aligned}$$

Final Answer: $3j - 7k + 7$

(b) $(2x^2 + 3x - 1) + (7x^2 - x + 4)$

$$\begin{aligned} &= (2x^2 + 7x^2) + (3x - x) + (-1 + 4) \\ &= 9x^2 + 2x + 3. \end{aligned}$$

Final Answer: $9x^2 + 2x + 3$

(c) $(3x^2 - 7x + 2) - (x^2 + 2x - 5)$

$$\begin{aligned} &= 3x^2 - 7x + 2 - x^2 - 2x + 5 && \text{distribute the negative sign} \\ &= (3x^2 - x^2) + (-7x - 2x) + (2 + 5) \\ &= 2x^2 - 9x + 7. \end{aligned}$$

Final Answer: $2x^2 - 9x + 7$

(d) $(10n^3 + 3n^2 - n) - (2n^3 - 4n^2 + 5)$

$$\begin{aligned} &= 10n^3 + 3n^2 - n - 2n^3 + 4n^2 - 5 \\ &= (10n^3 - 2n^3) + (3n^2 + 4n^2) - n - 5 \\ &= 8n^3 + 7n^2 - n - 5. \end{aligned}$$

Final Answer: $8n^3 + 7n^2 - n - 5$

(e) $(-2m^2 - 6m + 3) + (-3m^2 + m - 1)$

$$\begin{aligned} &= (-2m^2 - 3m^2) + (-6m + m) + (3 - 1) \\ &= -5m^2 - 5m + 2. \end{aligned}$$

Final Answer: $-5m^2 - 5m + 2$

(f) $(9a^2 - 2a + 2) + (5a^2 - 5a + 7)$

$$\begin{aligned} &= (9a^2 + 5a^2) + (-2a - 5a) + (2 + 7) \\ &= 14a^2 - 7a + 9. \end{aligned}$$

Final Answer: $14a^2 - 7a + 9$

(g) $(4y^2 - 3y + 8) - (2y^2 + y - 5)$

$$\begin{aligned} &= 4y^2 - 3y + 8 - 2y^2 - y + 5 \\ &= (4y^2 - 2y^2) + (-3y - y) + (8 + 5) \\ &= 2y^2 - 4y + 13. \end{aligned}$$

Final Answer: $2y^2 - 4y + 13$

(h) $(-5p^2 + 2p - 7) + (3p^2 - 9p + 4)$

$$\begin{aligned} &= (-5p^2 + 3p^2) + (2p - 9p) + (-7 + 4) \\ &= -2p^2 - 7p - 3. \end{aligned}$$

Final Answer: $-2p^2 - 7p - 3$

Part IV: Applications

11. A bacteria population grows from 3×10^4 bacteria to 9×10^6 bacteria. How many times larger is the final population than the initial population? Write the answer in scientific notation.

To find the multiplicative growth factor, divide the final population by the initial population:

$$\begin{aligned}\frac{9 \times 10^6}{3 \times 10^4} &= \frac{9}{3} \times 10^{6-4} \\ &= 3 \times 10^2.\end{aligned}$$

Thus, the final population is $3 \times 10^2 = 300$ times the initial population.

Final Answer: 3×10^2 times

12. The area of a rectangle is represented by $(2x + 3)(x + 5)$. Expand and simplify the polynomial expression.

$$\begin{aligned}(2x + 3)(x + 5) &= (2x)(x) + (2x)(5) + 3(x) + 3(5) \quad \text{distributive property} \\ &= 2x^2 + 10x + 3x + 15 \\ &= 2x^2 + 13x + 15.\end{aligned}$$

Final Answer: $2x^2 + 13x + 15$