

Simplify the following rational expressions.

$$1. \frac{x-y}{y-x} = \frac{x-y}{-(x-y)} = \boxed{-1}$$

$$2. \frac{4x+8}{x+2} = \frac{4(x+2)}{x+2} = \boxed{4}$$

$$3. \frac{x+5}{x^2-25} = \frac{x+5}{(x-5)(x+5)} = \boxed{\frac{1}{x-5}}$$

$$4. \frac{x^2-2x-15}{x^2+10x+21} = \frac{(x-5)(x+3)}{(x+3)(x+7)} = \boxed{\frac{x-5}{x+7}}$$

$$5. \frac{x^2-36}{x^2+12x+36} = \frac{(x-6)(x+6)}{(x+6)(x+6)} = \boxed{\frac{x-6}{x+6}}$$

$$6. \frac{6x^2-5x-1}{10x^2+9x-19} = \frac{(6x+1)(x-1)}{(10x+19)(x-1)} = \boxed{\frac{6x+1}{10x+19}}$$

$$7. \frac{6x-12}{x+3} \cdot \frac{4x+12}{3x-6} = \frac{6(x-2)}{x+3} \cdot \frac{4(x+3)}{3(x-2)} = \frac{6 \cdot 4}{3} = \frac{24}{3} = \boxed{8}$$

$$8. \frac{x^2+x-6}{x^2+3x-4} \cdot \frac{x^2-6x+5}{x^2-2x-15} = \frac{(x+3)(x-2)}{(x+4)(x-1)} \cdot \frac{(x-5)(x-1)}{(x-5)(x+3)} = \boxed{\frac{(x-2)}{(x+4)}}$$

$$9. \frac{x^2-x-2}{x^2+8x+15} \cdot \frac{x^2-x-12}{x^2-9x+14} = \frac{(x-2)(x+1)}{(x+5)(x+3)} \cdot \frac{(x-4)(x+3)}{(x-7)(x-2)} = \boxed{\frac{(x+1)(x-4)}{(x+5)(x-7)}}$$

$$10. \frac{4x^2-9}{x^2-10x+25} \div \frac{2x-3}{x-5} = \frac{(2x-3)(2x+3)}{(x-5)(x-5)} \cdot \frac{x-5}{2x-3} = \boxed{\frac{2x+3}{x-5}}$$

$$11. \frac{x^2-3x-10}{x^2-3x-28} \div \frac{x^2-x-6}{x^2+x-12} = \frac{(x-5)(x+2)}{(x-7)(x+4)} \cdot \frac{(x-4)(x-3)}{(x-3)(x+2)} = \boxed{\frac{x-5}{x-7}}$$

$$12. \frac{x}{x+3} + \frac{4}{x-5} = \frac{x(x-5)}{(x+3)(x-5)} + \frac{4(x+3)}{(x-5)(x+3)} = \frac{x^2-5x+4x+12}{(x+3)(x-5)}$$

$$= \boxed{\frac{x^2-x+12}{(x+3)(x-5)}}$$

$$13. \frac{x-4}{x+1} - \frac{x-2}{x-1} = \frac{x-4}{x+1} - \frac{(x-2)(x+1)}{(x-1)(x+1)} = \frac{x^2-5x+4-x^2+x+2}{(x+1)(x-1)}$$

$$= \boxed{\frac{-4x+6}{(x+1)(x-1)}}$$

Solve:

14.  $\frac{5}{7} + \frac{9}{t+3} = 2 \Rightarrow \text{LCD} = 7(t+3)$

$t \neq -3$   
 $\Rightarrow 7(t+3) \frac{5}{7} + \frac{9}{t+3} 7(t+3) = 2 \cdot 7(t+3)$

$\Rightarrow 5t + 15 + 63 = 14t + 42 \Rightarrow 5t + 78 = 14t + 42$   
 $\quad \quad \quad -42 \quad \quad -42$

15.  $\frac{3}{x-1} + \frac{12}{5} = 3$   
 $\text{LCD} = 5(x-1)$

$\frac{3}{x-1} + \frac{12}{5} \cdot 5(x-1) = 3 \cdot 5(x-1) \Rightarrow 15 + 12(x-1) = 15(x-1)$

$\Rightarrow 15 + 12x - 12 = 15x - 15$   
 $\Rightarrow 3 + 12x = 15x - 15$   
 $\quad \quad -3 \quad \quad -3$

$12x = 15x - 18 \Rightarrow -3x = -18$   
 $\quad \quad -15x \quad -15x \quad \quad \quad -3x = -18$   
 $\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad x = \frac{-18}{-3} = 6$

$\Rightarrow 5t + 36 = 14t$   
 $\Rightarrow -5t + 36 = 9t$   
 $t = \frac{36}{9} = 4$

16.  $\frac{7}{9} - \frac{x+8}{x+1} = -1$

$x \neq -1$   
 $\text{LCD} = 9(x+1)$

$\Rightarrow \frac{7}{9} \cdot 9(x+1) - \frac{(x+8) \cdot 9(x+1)}{(x+1)} = -1 \cdot 9(x+1)$

$\Rightarrow 7(x+1) - 9(x+8) = -9(x+1)$

$\Rightarrow 7x + 7 - 9x - 72 = -9x - 9$

$7x - 65 = -9 \Rightarrow 7x = -9 + 65$   
 $7x = 56$   
 $x = \frac{56}{7} = 8$

