Chapter 4

Characterizing Data Numerically: Descriptive Statistics

While visual representations of data are very useful, they are only a beginning point from which we may gain more information. Numerical characterizations allow a more formal means of both describing a distribution and comparing two or more distributions. Numerical description ultimately allows us to make the inferences we desire. These numerical characterizations are termed either *statistics* or *parameters*. *Statistics are descriptions of the constructed characteristics of a sample, whereas parameters refer to the characteristics of a population*. A *population* is a group of data defined in time and space. Do not confuse this use of the term population with its use in biology and physical anthropology. Statistical populations are not things. They are not people, or animals, or rocks, or pots. They are data.

For example, the length of Folsom points manufactured during the Paleoindian occupation of the New World could be considered a population. The size of thimbles used in New York during the Historic period is another possible population. As you can see populations can vary in their scope from data about all of a class of objects ever in existence to data on a more limited subset of objects. We could even define a population at the scale of a site (e.g., the frequencies of each species of animals consumed at a site could be considered a population.)

In contrast a sample is a subset of a population that does not contain all of the population's members. For example, the animals reflected by bones recovered from a site is a sample of the population of all of the animals consumed at a settlement. A point to remember is that the archaeologist defines populations and samples. Samples and populations are not discovered. A particular group of data could be a sample or a population, depending on your question. For example the grade point average of undergraduate anthropology majors at a school could be considered a population, if the group of interest is anthropology majors from that school, or a sample, if you are interested in all undergraduates from the institution.

We generally analyze a sample to gain information about the population. We therefore often use sample statistics to make inferences about population parameters of interest. For example, we may numerically characterize the cranial capacity of a sample of australopithecine skulls. This characterization may be in terms of one or more statistics. We then use the statistics derived from a sample to infer information about the population of australopithecine skulls of interest.

It is important to remember that statistics may vary, as each is likely a description derived from a different subset of a population. Parameters, however, do not vary, because they are a description of a complete population bounded in space and time. They are unchanging, immutable.

Numerical descriptions of distributions can be categorized into two types: *measures of central tendency*, and *measures of dispersion*. Measures of central tendency, as the name suggests, provide a numerical account of the center, or midpoint of a distribution. Measures of dispersion provide information about the spread of data around the midpoint of the distribution. Both populations and samples can be characterized using measures of central tendency and dispersion. By convention, population parameters are denoted with Greek letters, and sample statistics by Roman letters.

MEASURES OF CENTRAL TENDENCY

Measures of central tendency are often referred to as well as *measures of location*. Measures of location may actually be the better term, in that these measures provide a numerical account of the location of the center of a distribution along the variable measured.

Three measures of central tendency are commonly used: the *median*, the *mean*, and the *mode*.

Median

The median of a set of data is the variate with the same number of observations of both greater and lesser value. Consider the following scaled list of variates:

14, 15, 16, 19, 23

The variate 16 is bounded on both sides by two variates. The median value is therefore 16. Consider an additional set of scaled variates that consist of an even number:

14, 15, 16, 19

Unlike the example above, no single number has an equal number of variates numerically greater and less than it. So how do we decide what the median is? Is it 15 or 16, or does this set of numbers simply have no median? In cases such as this, the median is determined by averaging the two values in the middle of the distribution. Here, the median is the average of 15 and 16, which is 15.5.

Mode

The mode is the most popular, or most abundant value, in a data set (i.e., the value with the highest frequency). An inspection of Figure 3.7 (Chapter 3) shows that the most popular value in the Gallina ceramic data is 5.0. It is possible, though, to have two or more modes, if the most popular classes have the exact same number of variates.

Mean

Everyone is familiar with this measure of central tendency, which is also called the *arithmetic mean*, or the *average*. *The mean is calculated by summing all of the values in the set of data, and then dividing this sum by the number of observations.*

To illustrate the calculation of the mean, consider the following set of data (Table 4.1) Jim O'Connell collected in an ethnoarchaeological study on the number of residents in Alyawara camps.

Table 4.1. Number of residents in Alyawara camps.

21	23
18	31
12	44
31	7

The mean is calculated by following the instructions in the following symbols presented in Equation 4.1.

Equation 4.1. Calculating the mean.

$$\mu = \frac{\sum_{i=1}^{i=n} Y_i}{n} \text{ or, more simply, } \frac{\sum Y}{n}$$

In equation 4.1, μ represents the population parameter of the mean of the variable Y. $\overline{Y}$ (vocalized as *Y-bar*) is the mean of a sample, and is an estimate of μ. In this case, $\overline{Y}$ refers to the mean of the number of residents of Alyawara camps. The Greek symbol $\sum$ should be familiar from Chapter 3 as the symbol for summation. What might not be familiar however, is the notation above and below $\sum$, and the subscript to the right of Y.

$\sum_{i=1}^{i=n} Y_i$ represents the following set of instructions: for the variable Y, beginning with the first value of Y (symbolized by i=1), sum all values continuing to the last value of Y (symbolized by i=n). Y_i simply refers to all values of Y, each of which can be numbered individually, as follows: $Y_1 = 21, Y_2 = 18, Y_3 = 12 ... Y_8 = 7$. For the set of instructions $\sum_{i=2}^{i=4} Y_i$ we begin summation with $Y_2 = 18$, and continue to sum through $Y_4 = 31$, or 18 + 12 + 31 = 61.

The left term of Equation 4.1 shows the full symbolism for the calculation of the mean. The term on the right means the same thing, all other symbolism associated with the term to the left is implied. Seldom is the full symbolism expressed for ease of application. So, when you encounter the symbol $\sum Y$, it can be assumed that the instructions are to sum all values of Y, beginning with the first, and continuing to the last.

The calculation of the mean in our Alyawara example is as follows.

$$\bar{Y} = \frac{21 + 18 + 12 + 31 + 23 + 31 + 44 + 7}{8}$$

$$\bar{Y} = \frac{187}{8}$$

$$\bar{Y} = 23.375$$

The mean, or average number of Alyawara's in camp is 23.375. The median is 22, and the mode, 31. Depending upon the situation, all three measures may be useful, but oftentimes, one may be preferred over the others. In the above example, the mean and the median provide values that are reasonably close to one another, and both constitute good measures of central tendency. The mode in this instance, however, is inaccurate as a measure of central tendency, probably because the sample size is so small.

When a distribution is perfectly symmetrical, the mode, mean, and median are identical. In general, when dealing with a symmetrical distribution, the mean is the most useful measure of central tendency, followed by the median, and then the mode. The mean is often the most useful primarily because of its utility in the analysis of variance and regression analysis, subjects of later chapters. The mean, however, can be innately influenced by extreme values, often called outliers. As a result, its applicability to heavily skewed distributions is questionable.

For example consider again the following values for the Alyawara data:

21, 18, 12, 31, 23, 31, 44, 7

Let us replace the variate 44 in the above data with 100, as illustrated in the following set of contrived data.

21, 18, 12, 31, 23, 31, 100, 7

In comparison with $\overline{Y} = 23.375$ for the original set of data, the recalculation off the mean with the extreme value of 100 provides us with a value of $\overline{Y} = 30.375$, a considerable increase. The median, however, is unaffected by the extreme value, and remains the same. It is in such cases as where extreme values exist, or where the data are heavily skewed, that the median becomes a better indicator of central tendency than the mean.

MEASURES OF DISPERSION

While measures of central tendency provide important information about the location of a distribution along a variable, they offer no information about the shape of that distribution. Two different distributions might have the same location, but not resemble each other at all in terms of shape, or dispersion (e.g., Figure 4.1). Three measures of dispersion are commonly used: the *range*, the *variance*, and the *standard deviation*.

Range

The range is the difference between the largest and smallest values in a set of data. For the Alyawara example:

Largest Value	44
Smallest Value	(-)7
Range	37

While the range allows a perspective on the dispersion of the distribution of a sample, the sample range almost always underestimates the population parameter. It is unlikely, after all, that both the absolute largest and absolute smallest variates in a population will be selected in most samples. Likewise, the range is greatly affected by outliers because it only takes into account two variates in the data set, the largest and smallest.

Despite the simplicity of the range, it is often a misunderstood statistic because of the differential use of the verb and noun forms of the word *range*. With the Alyawara data, it is appropriate to state that the data *range* (verb form) from 7 to 44. However, the range (noun form) is 37, not 7 to 44.

Interquartile Range

Interquartile range is a measure of variation that is closely related to the range, except that it attempts to measure the variation in variates towards the center of a distribution. It is calculated by subtracting the variate demarcating the lower 25% of a distribution from the variate demarcating the upper 25% of the distribution. This value in turn reflects the range of the middle 50% or the "body" of a distribution. To prevent confusion, the demarcation of the lower 25% of the distribution is called the *25th percentile* whereas the demarcation of the upper 25% is called the *75th percentile*.

Using the Alyawara data, 25% of the variates are equal to or less than 18 and 25% are equal to or greater than 31. Consequently the 25th percentile is 18 and the 75th percentile is 31. The interquartile range is 31 – 18 = 13. Thus, the middle 50% of the Alyawara data differ by no more than 13 people.

Variance and Standard Deviation

The variance and the standard deviation are related statistics used to describe, and ultimately compare, the shapes of distributions. Ideally, the value of every single variate should be considered when characterizing the shape of the distribution. Information about the shape of the distribution is contained in knowledge of the value, or location of *every* variate in space. A logical way to measure the distribution of variates to characterize the distribution's shape is to consider the distance, or deviation, of each value from the mean. For $\mathrm{Y}_1 = 21$, the distance or deviation from the mean is:

$$y = \mathrm{Y}_1 - \overline{Y}$$

$$y = 21 - 23.375$$

$$y = -2.375$$

Note that the lower case *y* is used as the symbol of this deviation, and by convention, the mean is subtracted from the variate in order to provide the measure of distance. Yet, this is only one value, and we are concerned with the shape of the complete distribution. It comes to mind that perhaps if we sum all of the deviations of all variates from the mean, and divide by the number of variates, we could create a kind of "average" deviation. Large values would indicate a broadly spread distribution, and small values a narrowly spread distribution. Unfortunately, this is not the case, as the sum of all deviations from the mean is equal to zero. Because of the way the mean is calculated, the amount of deviation is equal on either side and, therefore, the values greater and lesser than the mean wind up canceling each other out when summed. The problem then, is not with the magnitude of the deviation, but with its sign. All of the plusses and minuses cancel each other out. One way to get rid of the sign problem is to square each deviation, as the squaring process results in only positive numbers. This is precisely the solution we use when calculating variance and standard deviation. We therefore have the following calculation formulas for variance:

Equations 4.2

$$\sigma^2 = \frac{\sum y^2}{n} \text{ and } s^2 = \frac{\Sigma y^2}{n-1}$$

σ^2 is the symbol representing the population variance whereas s^2 represents the sample variance.

While the variance is useful for many purposes, remember that it is transformed by squaring the deviates before they were summed. By now taking the square root of the variance, we return those squared values to their original units, providing another measure of dispersion that makes more intuitive sense, the standard deviation σ. The sample standard deviation is represented by s. The standard deviation is calculated using the following formulas.

Equations 4.3

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum y^2}{n}} \text{ and } s = \sqrt{s^2} = \sqrt{\frac{\Sigma y^2}{n-1}}$$

You will notice that the sample variance and sample standard deviation are calculated by dividing the sum of squares by n-1, not n. Through experimentation, it has been determined that dividing by n in a sample tends to underestimate the true variance and standard deviation, but that dividing by n-1 provides a better estimate. Therefore, when calculating the population parameters of σ or σ^2, divide the sum of squares by n, but when calculating s or s^2, divide the sum of squares by n-1.

Table 4.2 presents in table form the computation of the sample variance and standard deviations for the Alyawara example.

Table 4.2. Computations of the Sample Variance and Standard Deviation for the Number of Residents in Alyawara Camps.

(1)	(2)	(3)	(4)	(5)
Y	*F*	$y = Y - \bar{Y}$	y^2	fy^2
21	1	-2.375	5.641	5.641
18	1	-5.375	28.891	28.891
12	1	-11.375	129.391	129.391
31	2	7.625	58.141	116.281
23	1	-.375	0.141	.141
44	1	20.625	425.391	425.391
7	1	-16.375	268.141	268.141
Sum=	8	0		973.877

$\bar{Y} = 23.375$

$$\sum y^2 = 973.877$$

$$s^2 = \frac{\Sigma y^2}{n-1} = \frac{973.877}{7} = 139.125$$

$$s = \sqrt{\frac{\Sigma y^2}{n-1}} = \sqrt{139.125} = 11.795$$

Column (1) of Table 4.2 presents each Y. Column (2) presents their frequency *f*. This column is necessary because the frequency of occurrence of a given Y may vary. If we counted each Y with multiple occurrences only once, we would either overestimate or underestimate the true variance. Column (3) presents *y*, $Y - \overline{Y}$, the deviation of the variate from the mean. Note, as discussed above, the sum of all values in column (3) is equal to zero. Column (4) provides a solution to the problem with signs by squaring the deviations created in column (3) as symbolized by y^2. Column (5) is the frequency of occurrence of Y as presented in Column (2) multiplied by the squared deviations calculated in column (4). This column is necessary in order to take into account the Y's with more than one observation. The sum of column (5) $\sum f y^2$=973.877 is also called the sum of squares. This value is then used in Equations 4.2 and 4.3.

The procedure illustrated in Table 4.2 is a useful approach to calculating the sample variance and standard deviation, but it does take some effort to produce. One of the most pleasant characteristics of statistics is that there are often simple ways to calculate otherwise complex calculations. Equation 4.4 offers an equivalent method of calculating the sum of squares that is less computationally intensive and time consuming than Table 4.2.

Equation 4.4. Calculation formula for the sum of squares.

$$\sum y^2 = \sum \mathrm{Y}^2 - \frac{(\sum \mathrm{Y})^2}{\mathrm{n}}$$

For the Alyawara example:

$$\sum y^2 = 5345 - \frac{(187)^2}{8} = 5345 - \frac{34969}{8} = 5345 - 4371.125 = 973.875$$

We have chosen to present all arithmetic operations above because of potential confusion in reading equation 4.4. The left hand term of the equation is an instruction to sum all Y's after squaring them, which sums to 5345. The numerator of the right hand term is an instruction to sum all Y's, then square them, which equals 187 squared, or 34969. The difference is subtle, but extremely important.

Please observe that the value of $\sum y^2 = 973.875$ is identical to the value previously computed using the more computationally intensive method. Now that we have the sum of squares, we can calculate the standard deviation s as follows:

$$\mathrm{s} = \sqrt{\frac{\sum \mathrm{y}^2}{\mathrm{n - 1}}}$$

$$s = \sqrt{\frac{973.875}{7}}$$

$$s = 11.795$$

With knowledge of the standard deviation, we now have obtained an extremely useful measure of dispersion. In addition, principles behind the calculation of the standard deviation provide extremely important conceptual tools for understanding many statistics we will be learning through the remainder of this book.

Calculating Estimates of the Mean and Standard Deviation

Occasionally, we are faced with situations where we wish to quickly gain information about the location and spread of a distribution; for example, at a professional presentation where we want to explore some relationship in data being presented. We may also wish to quickly check our mean and standard deviation calculations for possible errors without actually recomputing the values. Using the following procedures, we can calculate a quick estimate of both the mean and the standard deviation.

To estimate the mean, we can compute the *midrange*. The midrange is similar to the range except that the largest and smallest values are averaged rather than subtracted.

In the Alyawara example, the midrange is (44+7)/2 = 25.5. This value is fairly close to the computed mean of 23.375 and may serve as a reasonable estimate. If the midrange is very different, one may want to recheck the computation of the mean.

While estimates of the mean are easy to come by, a good estimate of the standard deviation is a bit more difficult. However, the standard deviation can be estimated by dividing the range, calculated as described previously, with the appropriate value from Table 4.3. Applying this method to the Alyawara example produces an estimate of (37/3) = 12.3. Once again, this value is not too far off from the computed value of s = 11.795.

Table 4.3. Denominators for the estimate of the standard deviation.

Sample Size	Divide Range By
5 to 29	3
30 to 99	4
100 to 499	5
500 to 999	6
1000+	6.5

Coefficients of Variation

Oftentimes we calculate standard deviations because our goal is to compare the spread of two or more distributions. In such cases distributions with larger variances and standard deviations relative to other distributions are thought to reflect populations with greater variation. This assumption is seems intuitively obvious but is in fact problematic because we must first recognize that the variance and standard deviation are strongly influenced by the size of the variable being measured. For example, Clovis projectile points tend to be much longer than projectile points used by those living in the proto-historic pueblos in the American Southwest. If the standard deviation were used to compare the amount of variation in the lengths of the two types of points, it would be calculated for each type using the sums of deviations of each variate from the group mean. The standard deviation for the proto-historic points consequently will always be smaller than the corresponding standard deviation for Clovis points because the proto-historic points are themselves smaller. Each proto-historic projectile point variate cannot differ at the same magnitude from its mean as a Clovis point variate can. As a result the standard deviation and variance are inappropriate for comparing the relative amount of variation within two or more groups *when group means are significantly different*, because they will tend to overestimate the amount of variation in large variables and underestimate the amount of variation in smaller variables relative to each other.

This problem is resolved by calculating the *coefficient of variation*. The coefficient of variation (CV) is an expression of the standard deviation as a percentage of the mean from the parent distribution. It standardizes the standard deviation so that the absolute size of the variable being measured is controlled. Instead of reflecting the absolute size of the variation from the mean, CVs reflect the *proportion* of variation from the mean. Thus, a Clovis point and a proto-historic projectile point that are two thirds of their respective mean lengths will demonstrate the same proportional variation as determined by the CV. The CV therefore allows the variation within distributions with significantly different means to be compared. The coefficient of variation is computed using equation 4.6.

Equation 4.6. The Coefficient of Variation

$$\mathrm{cv} = \frac{s*100}{\overline{Y}} \text{ for samples, or } \mathrm{cv} = \frac{\sigma*100}{\mu} \text{ for populations}$$

For the Alyawara example:

$$\mathrm{cv} = \frac{11.795*100}{23.375} = 50.46$$

The coefficient of variation has meaning only in a comparative sense to another coefficients of variation. For example, we might be interested in comparing the variation in camp sizes of the Alyawara with another group with $\overline{Y} = 30, s = 12, n = 8$. For this second group:

$$\mathrm{cv} = \frac{12*100}{30} = 40$$

Since 50.46 > 40, the Alyawara variation is greater than that of the second group.

It has been determined that the cv is a biased estimate of the population parameter in small samples. We therefore apply a correction term to eliminate the bias. The corrected coefficient of variation (corrected cv or cv*) is computed using Equation 4.7.

Equation 4.7. Correction formula for the coefficient of variation.

$$cv^* = (1 + \frac{1}{4n})\, cv$$

For the Alyawara example:

$$cv^* = (1 + \frac{1}{4(8)})50.46$$

cv* = 52.04

For the second group:

$$cv^* = (1 + \frac{1}{4(8)})40$$

cv* = 41.25

Our conclusions regarding the variation in our Alyawara example are the same, since 52.04 > 41.25, yet we now likely have better estimates of the parametric values.

Coefficients of variation are commonly used where comparisons of standard deviations are desired, particularly where means differ considerably. For example, to compare variation in the skeletal morphology of different primate groups who vary significantly in size, the comparison of coefficients of variation is necessary.

Another common application is where researchers are interested in examining specialized production of technology. Lower coefficients of variation suggest greater standardization of products, and may mean specialized production. Coefficients of variation have been employed by Crown (1995), Longacre et al (1988), and others (e.g., Arnold and Nieves 1992; Mills 1995) as a measure of standardization. Crown notes that with respect to the manufacture of ceramics, known specialist groups rarely produce ceramics with coefficients of variation above 10%. Her examination of variation in Salado Polychromes in the American Southwest allows her to conclude that there was little, if any, standardization and subsequently little, if any, specialized production.

Box Plots

Now that we have an understanding of the median and the range we can employ an addition means of visually characterizing data, box plots. Box plots are extremely useful for characterizing multiple distributions at the same time, although they can be used for even a single distribution, because they provide information about the variation and central tendencies of data in a very condensed manner (e.g., Figure 4.2).

Box plots reflect the median, the interquartile, and the range. They are created in three steps. First, calculate the distribution’s median and the quartiles. Second, plot the location of the median and the two quartiles. Draw a box using the quartiles as limits. This box reflects the distribution of the middle 50% of the data—the “body” of the data. Third, draw a line from the lower quartile to the value of the smallest variate and from the upper quartile to the value of the largest variate.

The utility of box plots can be demonstrated in Figure 4.2. This figure is the length of 4 sets of 12 flakes made as part of an experiment studying of the flaking characteristics of lithic raw materials (Table 4.XX). The box plots in Figure 4.2 provide a quick means of describing both the structure of each distribution and the differences between them.

Figure 4.2. Box plots of flake length data presented in Table 4.XX.

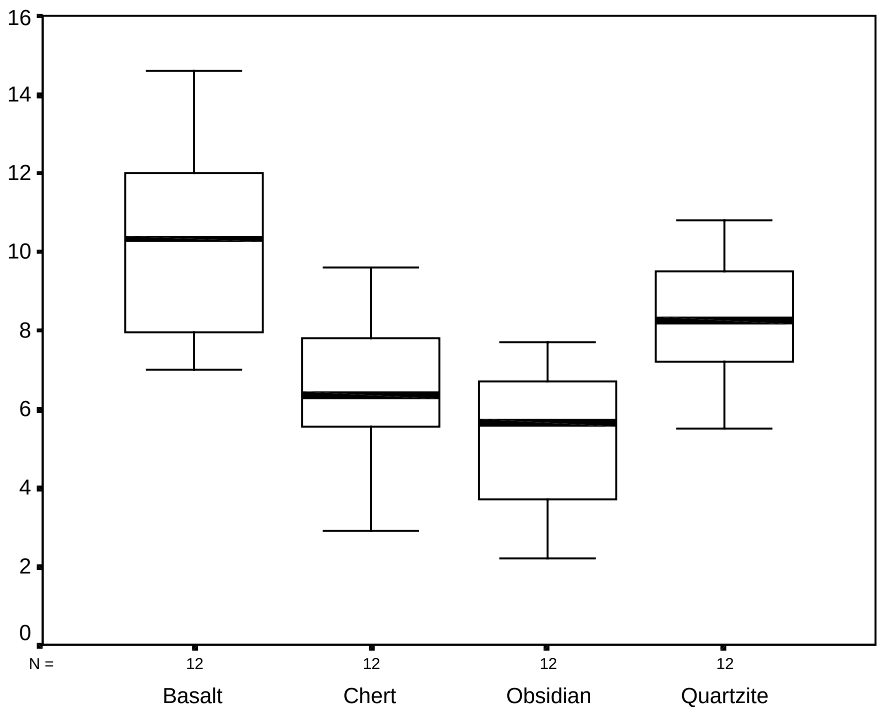
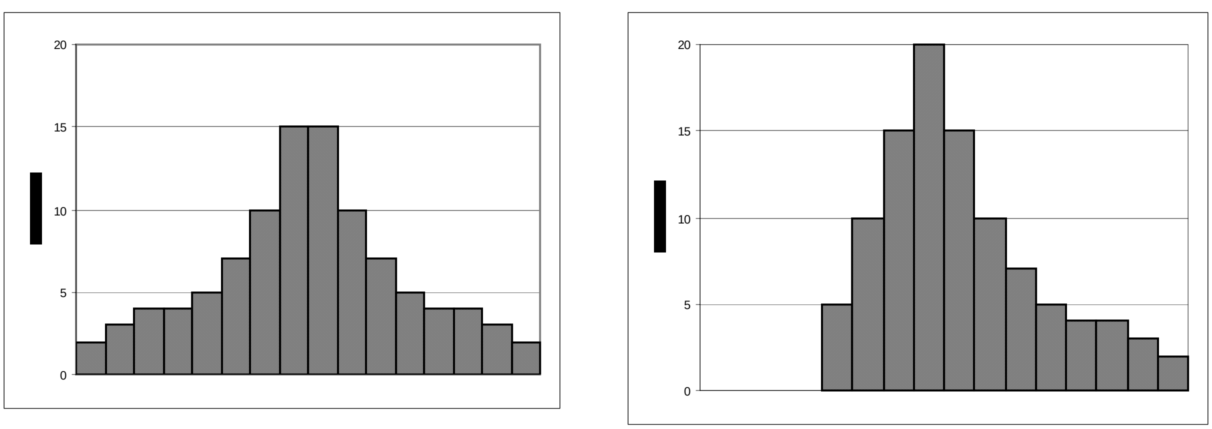


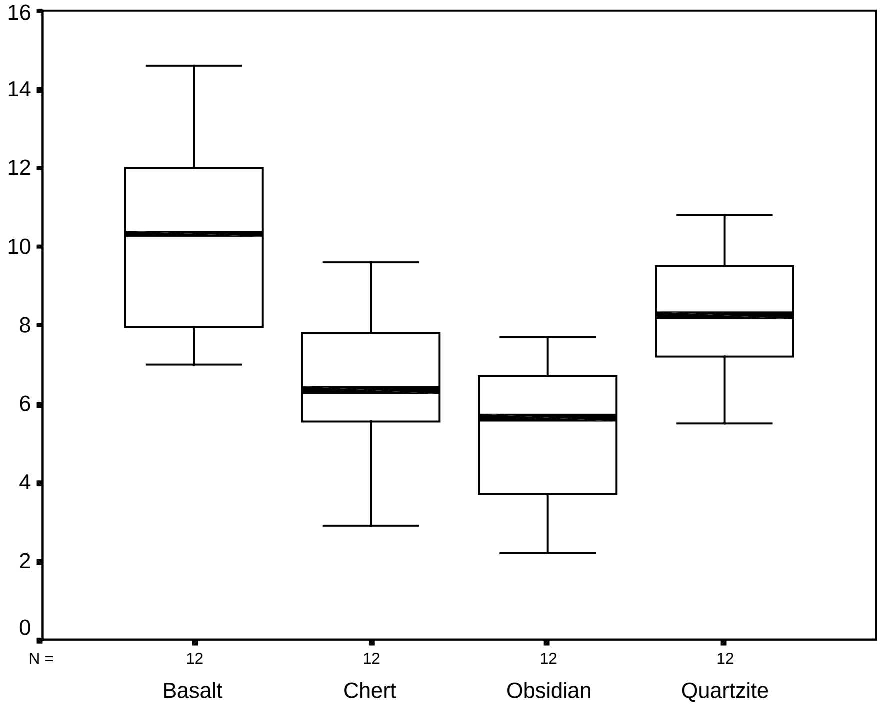
Table 4.XX. Flake length by raw material.

Basalt	Chert	Obsidian	Quartzite
7.0	2.9	2.2	5.5
7.0	4.8	2.4	5.5
7.7	5.3	3.1	7.0
8.2	5.8	4.3	7.4
10.3	5.8	5.0	7.7
10.3	6.2	5.5	7.9
10.3	6.5	5.8	8.6
10.8	7.7	6.0	8.9
11.0	7.7	6.2	9.4
13.0	7.9	7.2	9.6
13.9	8.9	7.4	10.6
14.6	9.6	7.7	10.8

We now know how to characterize data visually and numerically. The next step is to learn one important theoretical distribution, the normal distribution. Knowledge of the characteristics of the normal distribution allows us to draw conclusions about real distributions that take a similar form. The normal distribution is the subject of Chapter 5.

Figure 4.1. Two distributions with identical means and sample sizes but different dispersion patterns.




16
14
12
10
8
6
4
2
0
N =
12
12
12
12
Basalt
Chert
Obsidian
Quartzite