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Badiou, Mathematics and Model Theory

With his mathematically based theory of subjectivity, ontology, phenomenology and the advent of the new, Alain Badiou has produced a systematic reflection on forms and formalisms that is dramatic in its consequences and unparalleled in our time. In the contemporary context, one of the most innovative and courageous aspects of this rigorous reflection on forms is Badiou's unyielding insistence on the reality and value of *truth* in structuring the very possibility of novelty and progressive change. This insistence yields Badiou's clearsighted critical recognition of the ultimate untenability of the dominant "postmodern" position that sees behind truth-claims only contingent power struggles and local situations. Simultaneously, it supports a profound formally based ontological thinking of the structure of being itself, as well as its possible interruption by the arrival of what Badiou calls the event, suspended in its worldly appearance from the support of an unyielding and universal truth. As I shall argue here, this appeal to the possibility of a *formally* based and universalist thinking of the real marks Badiou's thought as a rigorous (if unorthodox) Platonism, one which accordingly inherits some of the most remarkable features, as well as many of the internal problems, of Plato's original thinking of forms.

For Badiou as for Plato, the ultimate basis of the structure of thinkable being is to be found in mathematics, which occupies a pre-eminent position in exemplifying as well as making possible the formal thought of what is real in being. Badiou, in particular, goes so far as to make this connection an identity in formulating the meta-ontological axiom of his formal thought as the claim that "mathematics is ontology." Yet if the formalism of mathematics remains for Badiou, as much as for Plato, both the paradigm and structure of thought's possible passage to the real, Badiou's thinking also stands under the condition of the vastly more complex and differentiated mathematics of our time. This condition includes not only Cantor's extension of the theory of sets to the rigorous calculus of infinity and the vast and innumerable domains of multiple transfinite multiplicities, but also the profound developments of mathematical logic and metalogic that have both situated and contested the formal implications of mathematical thinking over the course of the twentieth century, from a position internal to mathematical formalism itself.

As I have argued in more detail elsewhere, in connection with these developments of contemporary mathematics, mathematical logic and metalogic, it is no longer possible simply to consider the manifold implications of formalisms and formalization – from the abstract, structural analysis of linguistic syntax (common, in different ways, to both the French structuralist and the "analytic" traditions in philosophy) to the impact of formally based informational technologies of computation and communication, up to the much-bemoaned "formalization" and instrumentalization of everyday life – from a position that is itself simply *exterior* to the development and pursuit of formal and mathematical results.[1] In particular, if Cantor's definition of a (finite or infinite) set as any "many" that can be thought of as a "one" indeed captures, as Cantor thought it did, something of the nature of what Plato himself ventured to think as *eidos* or *idea*, then it is apparently possible to envision, under the changed conditions of contemporary formal and meta-formal reflection on set theory, a displaced *repetition* of Plato's original inquiry into forms and their relation to the real, at once among the most profound and the most problematic philosophical investigations of the relation of thinking to being itself.[2] In my recent book, *The Politics of Logic*, I have developed a comprehensive taxonomy of ontological and critical positions, including Badiou's own, on the consequences of formalism within contemporary thought, interrogating in each case the ways that the reflective considerationof these consequences orients itself toward the "universal" dimension of a consideration of the unity and totality of the real in being. In the present paper, my goals are necessarily more modest. I wish simply to raise a few questions, both internal and external to Badiou's text, about the application he makes of mathematical formalisms to facilitate a passage to truth and the real. And although I am not at all convinced by the suggestion, prevalent in many contemporary "continental" reactions to Badiou, that his set-theoretical ontology simply represents an untenable attempt to formalize the "unformalizable" or to schematize what is in itself mutely incommensurable to structure, I am, nevertheless, sensitive to the fact that the power of forms both to capture and shape reality has been rigorously thought, as well, at other sites and according to very different methods. In fact, one of the exemplary sites of the twentieth-century thinking of the power of forms is that of the "analytic" inquiry

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[1] Livingston, P. M.: *The Politics of Logic: Badiou, Wittgenstein, and the Consequences of Formalism*. New York: Routledge, 2011 (henceforth: *PofL*).

[2] In 1883, Cantor gave this definition as follows:

> By a 'manifold' or 'set' I understand in general any many [*Viele*] which can be thought of as one [*Eines*], that is, every totality of definite elements which can be united to a whole through a law. By this I believe I have defined something related to the Platonic *eidos* or *idea.*

Cantor, G.: "Über unendliche, lineare Punktmannigfaltigkeiten" *Mathematische Annalen* 21, 545-86 (1883), quoted in Hallett, M.: *Cantorian Set Theory and Limitation of Size*. (Oxford: Clarendon, 1986),.p. 33.

into logic and metalogic, an inquiry which has yielded both powerful new structures and philosophically profound results.

In particular, both for the analytic tradition and for Badiou, one of the most important concrete sites for the internal consideration of the relationship of set-theoretical formalism to the real is the formal or meta-formal field of *model theory*. Broadly speaking, model theory is the intra-mathematical study of the "interpretation" of natural or formal languages in terms of set-theoretic structures, and hence of the ways in which a regular language specified wholly syntactically may be seen as bearing semantically on a structured "reality" in principle separate from it. But despite its origins as a device for making sense of the application to formal languages of certain formal and meta-formal concepts (including, decisively, the concept of truth), model theory has also come to occupy a central position in making possible the very metatheoretical results that clarify the structure and limitations of axiomatic mathematical set theory itself. As such, it plays, as we shall see, a central role in several of the results that Badiou himself interprets as determining the structure of "ontology" as well as what remains foreclosed or subtracted from it, thus marking the very possibility of the advent of the new. This reliance on the level of the *content* of his (meta-)formal results does not prevent Badiou, however, from conducting a highly creative and sophisticated reflection on the structure and implications of model theory itself. In fact, in one of his first substantive contributions to philosophy, *The Concept of Model* from 1968, Badiou undertakes to construct and defend a construction of models that is rigorously defensible and usable within a "materialist" theoretical *praxis* of mathematical science, carefully distinguishing this productive and progressive concept of models from what he takes to be its various ideological and even "bourgeois" rivals.[3] Nevertheless, as we shall see in more detail, Badiou's later applications of set-theoretical results – including, crucially, his consideration of the possibility of the "event" in *Being and Event* – in several ways depends on an employment of models that is substantively quite far removed from their original use in the semantic "interpretation" of syntactically specified formal languages. Although this extended employment is certainly not without precedent in the *praxis* of axiomatic set theory itself, and although there is nothing formally "wrong" with the central and decisive model-theoretic results to which Badiou centrally appeals, it does nevertheless appear, as we shall see, that there remains more room for these results to be interpreted as bearing on different kinds of real structures (and on the same structures in different ways) than Badiou's terse extrapolations sometimes allow.

[3] Badiou, *The Concept of Model: An Introduction to the Materialist Epistemology of Mathematics*. Ed. and Transl. by Zachary Luke Fraser and Tzuchien Tho. (Melbourne, Australia: Re.Press, 2007). (henceforth: *CM*). See also Zachary Luke Fraser's immensely helpful introduction to the work, in the same volume.

How, then, does model theory illuminate the capacity of *formal* thought (whether understood as constutively linked to the thought of the syntactical and regular determination of language, or as the inherently involving the problematic "semantical" dimension of its link to an external referent, or again as the direct grasping of forms in mute and extra-linguistic intuition) to succeed in touching the real? Without minimizing the depth and difficulty of this question, I will aim here simply to register some similarities and differences between what seem to be two distinct ways of thinking about possible answers to it. The first is Badiou's, which takes the formalisms of set theory *including* the "internal" theory of their models as an exemplary domain of ontological and meta-ontological structures, given in themselves quite independently of their syntactic or semantic figuring in language and symbolism. By contrast, the second way of thinking about formalism's access to the real is exhibited by analytic philosophy's ongoing critical consideration of set theory and its implications, a consideration that involves a far-ranging, if somewhat aporetic, investigation into the very nature of signs and their meanings. I shall conclude by suggesting that, although both of these sharply divergent ways of thinking about forms and meanings, models and structures (of both "real" and "symbolic" types and registers) remain open options, in different ways, for contemporary thought, it is also possible to anticipate at least a partial reconciliation of them that offers to bring us back, in ways that are still surprising and new, to the Platonic text itself and to its profound original problematic of the being and transit of forms.

I.

The origins of contemporary model theory lie in Alfred Tarski's penetrating investigation of the possibility and limits of the definition of truth in formal languages. Using methods closely related to those employed in the proof of Kurt Gödel 's notorious incompleteness theorems, Tarski showed by 1933 that it is impossible for a formal language (with a certain minimal structure) consistently to formulate its own truth-predicate.[4] That is, it is impossible for such a language to express a concept capable of capturing the distinction between true and false sentences within that language itself, without

[4] See Tarski, A., "The Concept of Truth in Formalized Languages," trans. J. H. Woodger. In *A. Tarski: Logic, Semantics, Metamathematics*, second edition, ed. J. Corcoran. (Indianapolis: Hackett) and, for what is in some ways a more accessible presentation, Tarski, A., "The semantic conception of truth and the foundations of semantics," *Philosophy and Phenomenological Research* 4 (1944), pp. 341-76.

inconsistency. On the other hand, Tarski showed how such a truth-concept for a given formal language *can* be defined without inconsistency, *provided* that the definition is carried out in a distinct and "stronger" language, a second, so-called "metalanguage" that describes the syntactic functioning of the first from an exterior position. In particular, the definition of truth for a particular object language requires first that classes or sets of objects be assigned to primitive general (non-logical) property terms, and that relations be assigned to primitive relational terms. These objects and relations are said to "satisfy" the basic formulas constructed from these terms, and given the satisfaction relations it is possible recursively to define a truth predicate for the object language (working always, of course, within the "stronger" metalanguage).

Because of this need to assign extra-syntactic "meanings" to the primitive terms by means of a characterization of satisfaction, Tarski described his approach as a "semantic" conception of truth. More generally, what is today called a "semantics" for a given formal system requires the provision of an "interpretation" of that system within what is called a *model*. Intuitively, a model is a domain of objects and relations, finite or infinite in extent. A model is said to *interpret* a given formal system if the axioms of the formal system, given an assignment of primitive terms in the system to objects and relations in the model, in fact hold true for those objects and relations.

As theorists quickly realized, it is then possible to use the construction of models to interrogate the *consistency* of various systems of axioms as well, since a system will have at least one model if and only if it is consistent. This result quickly proved useful in addressing the most significant question left open in the systematic set theory founded by Cantor and developed by Zermelo and Fraenkel to comprise the standard (ZF) system of set-theoretical axioms. This was the question, already formulated by Cantor, of the status of the so-called "continuum hypothesis." In standard, ZF set theory, it is always possible, given any set, to produce the "power set" of the original set. This is the set of all *subsets* of the original set, the set that re-groups all possible combinations of elements of the original set. Cantor proved that the power set is always strictly larger than the original set that is its basis (regardless of whether the original set is finite or infinite). However, the question of the continuum hypothesis is the question of *how much* larger: in the case of an infinite set, by *how much* does the size of the set of all subsets exceed that of the original set? If the continuum hypothesis holds true, then the difference in size between an (infinite) set and its power set is strictly minimal: the power set is a set of the very next "size" or *cardinality* that is possible at all. If, on the other hand, it does not, then the excess of the power set over the original set is

"immeasurable" in that it is possible for sets of many different sizes to exist "between" a given set and its power set.

It is here that model-theoretic methods proved particularly useful. In particular, given that any system of axioms which has a model is consistent, it is possible to show the consistency of such a system by displaying a model of it. For set theory in particular, one can use specifically defined domains of sets as models for various possible combinations of axioms, and so derive results about the consistency of those systems of axioms. In 1940, Gödel himself showed the consistency of combining the continuum hypothesis with the standard ZF axioms. He did so, in particular, by constructing a *model* – a specific domain of sets – in which all of the standard axioms as well as the continuum hypothesis holds true. The proof establishes that, since there is a model in which the ZF axioms *as well as* the continuum hypothesis hold true, it is impossible to *refute* (i.e. to prove the negation of) the continuum hypothesis by means of the standard axioms. However, this leaves open the question whether it is possible to *prove* the continuum hypothesis from the axioms. In fact, in 1963, Paul Cohen proved that it is *not* possible to do so: this result, when combined with Gödel 's, shows that the continuum hypothesis is *independent* of the ZF axioms in the sense that they themselves do not suffice to settle the question either way. Cohen's method was, once again, the construction of a model; this time, however, the necessary model is one in which the CH definitely does *not* hold, and there are a great (actually more or less arbitrary) number of sets of various sizes "between" an arbitrary infinite set and its power set. He accomplished this by means of the technically formidable method of "forcing".

In his masterpiece *Being and Event*, Badiou draws upon the details of both Gödel 's and Cohen's constructions in order to address key questions about ontology, subjectivity, power structures, and the possibility of evental change.[5] In particular, Badiou develops a theory of being as "pure multiplicity" within which the ZF set theory axioms formulate "ontology," or the conditions under which it is possible for anything at all to be presented. The operation of set grouping, what Badiou calls the "count-as-one" accomplishes this presentation in a primitive way, while the arbitrary re-grouping of elements already presented within some set, the "re-count" of what is initially presented, is termed "representation." By way of a metaphorical affinity to politics, Badiou furthermore terms the power set, or the complete re-

[5] Badiou, A. *Being and Event*. Trans. Oliver Feltham. (London: Continuum, 1988/2005). Translation of *L'être et l'événement*. Paris: Éditions du Seuil. (Henceforth: B&E). For further details see also Livingston, P. "Alain Badiou: Being and Event (a Review)" *Inquiry*, 51:2 (2008), pp. 217-238.

count of any existing situation, its "state". What he terms the "event" is, on the other hand, a very peculiar kind of set, actually foreclosed from existence by fundamental axioms of ZF set theory or "ontology," which includes itself as an element.

Given these assumptions, both the possibility of the event and the extent of the power of "state" systems of representation in foreclosing or re-appropriating its radical implications depend in detail on the question of the status of the continuum hypothesis. In particular, Badiou suggests, *if* the continuum hypothesis holds, then the possibility for novelty within any given situation is always strictly controlled, since any addition of the new to the existing situation is already effectively re-appropriated by the "state" level of representation. If, on the other hand, the continuum hypothesis fails to hold, then the event is free to create radically unforeseeable consequences, adding an essentially "immeasurable" amount of novelty to the situation in a way that is not simply already re-appropriated by the state or provided for in advance by its mastery of representation. In the last part of the book, a detailed reconstruction of Cohen's method of forcing and his proof of the consistency of the negation of the continuum hypothesis becomes the key to Badiou's defense of the possibility of a faithful tracing of the consequences of an event by a "subject" whose inquiry into the consequences of the event can indeed bring about a fundamental and unanticipated structural transformation of the existing situation.

In more specific ways, as well, Badiou's theory of the event depends upon the details of both Gödel 's and Cohen's constructions, in ways that illuminate, according to Badiou, the fundamental relations of evental change to the possibilities of *linguistic* representation and expression in any given situation. In particular, in constructing a model of sets in which the continuum hypothesis holds true, Gödel employed the assumption of "constructability," which restricts the sets that can be constructed from any given set to those that can be predicatively *named* in the existing situation. Constant adherence to this assumption yields what is called the "constuctible universe," a hierarchically organized totality or universe of sets in which existence is controlled strictly by the assumption of predicative nameability and, as a result, the continuum hypothesis follows. Because of this connection to the assumption of a regular nameability, Badiou takes Gödel 's construction as a model (this time, in an informal sense of "model") of the general philosophical orientation (occupied, according to Badiou, by philosophers from Foucault to the logical positivists) that Badiou calls "constructivism." This is a critical orientation which, in accordance with the "linguistic turn" sees all existence, as Badiou puts it, as controlled by nameability and the assumption that what is not nameable simply cannot exist. For the constructivist orientation, according to Badiou, the controllability of all existence by naming and representation guarantees the impossibility of any

occurrence of fundamental novelty and confirms that the "state" legislates, always already in advance, over existence. Similarly but inversely, the specific details of the construction employed by Cohen to show the consistency of the *denial* of the continuum hypothesis prove essential to Badiou's positive account of the subject's capacity to "force" the advent of a truth. In particular, Cohen's model of a situation in which the continuum hypothesis does not hold involves the construction of an indiscernible or "generic" set which is *not* constructible and (in Badiou's terms) thus escapes the state's control over names. By means of the infinite procedure of constructing, element by element, the generic set, it is possible, as Cohen showed, to "force" the existence of arbitrarily many cardinal sizes between a set and its power set. For Badiou, the procedure of "forcing" which creates the generic set is the direct paradigm of the "generic procedures" or truth procedures of art, science, politics, and love, which achieve, in each of these domains, by way of the subject's faithful action, the advent of the genuinely new.

In this way, Badiou makes essential use of the model-theoretic results of Gödel and Cohen in demonstrating the very coherence of his concrete theory of the event. This use involves not only the analogy that Badiou draws between the issue of the cardinality of various sets and the power of various state and non-state agencies, but also the more determinate analogy between the technical property of constructability and the possibility of linguistic expression in various, situation-specific languages. Both analogies are in fact questionable; as I argue in detail in *The Politics of Logic*, the latter in particular involves a series of highly problematic assumptions about the nature and plurality of empirical languages, assumptions which can readily be questioned from the position of a more classical type of critical consideration of the structure of language as such and its limits.[6] In the present context, however, what is perhaps more significant is the specific position that Badiou effectively occupies in using the model-theoretic results in just this way. As we have seen, the results of Gödel and Cohen jointly establish the *independence* of the continuum hypothesis from the standard axioms of ZF set theory, which Badiou understands as the underlying structure of "ontology," the theory of whatever exists. That is, as far as the standard axioms go, it is apparently perfectly possible either to affirm *or* to deny the continuum hypothesis; the axioms appear, in light of the model-theoretic results, to be simply mute about the "real" structure of the transfinite hierarchy of sets. This implies not only (as I argue in the book) that Badiou cannot claim that Cohen's result establishes the positive being of events, in his sense – at best, Cohen's construction establishes only that this is a *possibility* that is not completely *foreclosed* by ontology – but also that it is difficult to say that these results actually do establish anything at all about the "real" universe of sets.

[6] *PofL*, chapter 9.

This universe, as a totality, is sometimes referred to as "V", and the "constructivist universe" of sets restricted to the assumption of constructability as "L". The "possibility" that the actual universe is the constructivist universe can then be put as the "hypothesis" that V=L, or that the total universe of sets contains nothing other than the sets regulated by the assumption of constructability and its associated assumption of rigorously limitative linguistic control. We know from Gödel 's work that it is possible – but by no means required – to add this hypothesis as a further axiom to the standard ZF axioms. On the other hand, it is also possible (as we know from Cohen's work) to assume hypothesis that V does *not* equal L, and this is the possibility upon which Badiou depends most directly to verify the possibility of a formal forcing of truth.

This dependence (just as much as any kind of philosophical dependence on the opposite hypothesis, that V=L, which Badiou criticizes under the label of "constructivism") raises what might be seen as problems for Badiou's approach in at least two respects. First, it demands that Badiou's own theoretical work effectively take place from a position *outside* the total set-theoretical universe V, a position from which it is possible to speculate and formulate positive hypotheses about its nature and extent as a whole. In one sense this is just the position of a meta-language which, given Tarski's assumptions and results, must apparently be accessible if a *consistent* access to truth itself is. However, Badiou's apparent occupancy of this position, in the context of his identification of ontology with the axiom system of ZF set theory, raises the specter of a problematic element of exteriority or transcendence in his approach, an element which clearly becomes significantly more problematic in the context of Badiou's attempt to use the formal position of model theory to comprehend thought's very relation to the totality of being itself. Second, and relatedly, in the use he makes of Cohen's results, Badiou effectively appeals to models, not as reduplicative or secondary structures with which one can *evaluate* claims about the structure of sets, but rather as completely *substantive* set-theoretical existences in their own right, capable (through their own structure) of establishing a direct access to truth and the real on the part of the faithful subject. Here, models are no longer, as they were for Tarski, the mere guarantors of a "semantics" for a formal/syntactical systematicity of reasoning coherent in itself but seen as lacking the "external" dimension of reference. Rather, Badiou's realism about set-theoretical structures *including* models means that model-theoretic results work, for him, directly as formal illustrations of possible (or even actual) configurations of being in itself, quite apart from the secondary, largely reduplicative role that they have in the more familiar Tarskian theory. This means that the construction and invocation of particular models, far from simply illustrating or demonstrating results about the large-scale contours of the set theoretical universe (as determined by the standard axioms) itself, takes on for Badiou the significance of

a substantial domain of mathematical existence in its own right, one in which he must suppose it is meaningful for not only theoretical access to the character of being, but even its determinate transformation by means of subjective action, to take place.

II

We have seen, then, that Badiou's elaborate defense of the possibility of the event (and hence of a possibly transformative subjective access to the real) in *Being and Event* depends in detail on a robustly realist consideration of models and model-theoretic results. This is an uncompromising adherence to the assumption that the model-theoretic results of (for instance) Gödel and Cohen illustrate total possibilities, not only for understanding the structure of the universe in itself, but even for modeling in a privileged way the kinds of transformation and fundamental change of which the faithful subject is capable. This realist and substantialist motif is, moreover, essential to Badiou's defense of the possibility of the event and of (what he calls) truth in *Being and Event*, for it is this formal realism about models that guarantees the *applicability* of the model-theoretic results to the specific claims about evental change and its capacity to effectuate the new on which the argument of the book is staked. We can better understand the basis for this position by turning to one of Badiou's first substantial philosophical works, his construction and articulation of a "scientific" and "materialist" conception of the nature models and the fields of their application in 1968's *The Concept of Model*. Here, Badiou aims to extract and defend a "materialist" conception of mathematics in line with a strong distinction, inherited from Althusser, between the "scientificity" of materialist mathematics and its various "ideological" rivals. This distinction itself may in fact be seen as somewhat problematic from the position of the later Badiou, particularly in that it essentially ties or "sutures" the mathematics of models to what is for the later Badiou only one of four "generic procedures" (viz. that of science), but it is clear that the defense here of a mathematical *praxis* that achieves, in part through its deployment of models, a direct and transformative access to the real of being remains a constant throughout Badiou's subsequent work, including *Being and Event.* We can therefore already locate in the 1968 work the key contours of the epistemology of formalism and its application that will remain in place for Badiou through all of the subsequent uses he makes of model theory, up to and including Being and Event's complex but decisive appeals to the results of Gödel and Cohen.

Interestingly, the first rival conception that Badiou criticizes in the *Concept of Model* is the one employed by Rudolf Carnap in a 1938 article, "The Logical Foundations of Science." In particular, Badiou suggests, Carnap's conception of models is determined and regulated by the latter's assumption of a constitutive and essential *distinction* between the formal and the empirical.[7] This distinction, on one hand, licenses Carnap's characteristically reductionist attempt to locate all of the facts of the various empirical sciences within a single formal framework, the framework of so-called "unified" science, and on the other, allows Carnap to treat the entire relation between empirical facts and their formal structuration in terms of the familiar distinction between semantics and syntax. This fundamental distinction between the empirical and the formal, according to Badiou, not only subsumes Carnap's whole theoretical project, but even remains at work in a variant form in the work of Quine, who notoriously criticizes Carnap's whole approach by denying the possibility of an intelligible and rigorous formal/empirical distinction. This is because, according to Badiou, although "Quine, in effect, defines the particularity of his enterprise … by justifying the negation of a distinction that Carnap, for his part, aims to reduce," nevertheless "the distinction in question – between 'fact' and logical form – is the common motor of the two discourses."[8] Thus, when Quine appears to settle the question of being in favor of a mutual co-implication of the formal and the empirical with his famous declaration that "to be is to be the value of a variable", he remains, according to Badiou, within the range of the closed "variations" of the ideological regulation of the formal/empirical distinction. This is so, as Badiou suggests, not only in that Quine's ontology remains determined by the application of formal and logical methods of regimentation and analysis to capture the contact of language and the world, but also in that Quine's discourse continues to situate itself within the ambit of a programmatic "empiricism" that aims to maintain the ontological support of the real by means of its appeal to the givenness of facts.[9]

While there are, Badiou readily admits, various illustrative or presentational uses of (what may be called) "models" in science that are not ideologically problematic in this way, Carnap and Quine's common presupposition of a regular or at least theoretically productive distinction between the formal and the empirical points to a much broader question about the scientific use of models, whenever this use is seen as not merely illustrative or reduplicative, but as part of the positive theoretical work of science itself. In particular, Badiou suggests,

[7] *CM*, pp. 5-6.
[8] *CM*, pp. 6-7.
[9] *CM*, pp. 6-7.

> An epistemological problem surges up against every proposition struggling to describe the difference and the relation [*rapport*] between model and empirical reality: against every enterprise knotting together ways of thinking that which, in the model, speaks of its object; and against every placement, outside the model, of the thing whose model it is.
>
> There is an epistemological problem if we pretend that the invention of models constitutes the very activity of science. That is, if we present scientific *knowledge* as knowledge via models.[10]

On this basis, Badiou criticizes a conception of models evident in the work of Lévi-Strauss and employed in his project of structuralist anthropology. According to this conception, formal and structural models are to be constructed "after" empirical reality in such a way as to "account for" the empirical facts, themselves here conceived as simply given as such.[11] Here, the supposed givenness of facts antecedently to the activity of modeling protects an uncritical empiricism, but the claim that models themselves are productive of knowledge lands the theorist who follows Lévi-Strauss, according to Badiou, in an obvious circle. For this empiricist conception, in particular, models are first invoked to "explain" the facts antecedently given, but the true or accurate model can, in turn, only be defined in circular fashion as being the one that best succeeds in this work of explanation, for instance by resembling or bearing similarties to factual "reality". This circular definition has the effect, Badiou says, of "obscuring" the distinction between the production of knowledge through models and the mere regulation by their use of concrete processes of discovery, and accordingly effaces the very possibility of thinking of science as a "process of production of knowledge," one that no longer simply replicates, in secondary fashion, what is seen as a given order of facts, but rather yields "demonstrations and proofs *internal to* a historically specified materiality."[12]

It is to the articulation and defense of such a "materialist" conception of the scientific effectivity of models that Badiou now turns. As instances of the demonstrations of which such a conception is capable, Badiou mentions some of the key results of model-theoretic arguments in set theory, including the model-theoretic results of both Gödel and Cohen about the continuum hypothesis, as well as the Löwenheim-Skolem theorem (which shows that every system with *any* infinite model also has a model of *countable* cardinality). These are all, Badiou says, rigorously "unambiguous theoretical statements" inscribed as the

[10] CM, p. 14.
[11] CM, p. 14.
[12] CM, p. 17.

result of a determinate and theoretically productive process of proof.[13] The aim of a materialist and productive concept of models should be to validate their status as genuine and basic contributions to knowledge, owing nothing to their resemblance to or reduplication of an exterior, previously existing realm of facts. However, Badiou's defense of this conception demands that he dispute not only the exterior and reduplicative concept of models that he finds in Lévi-Strauss, but also the logical positivist "semantic" conception of models that is also the actual historical setting of the derivation of all of these specific model-theoretical results. This is none other than the "Tarskian" conception, which sees the development of models as necessary in order to provide semantical reference, and hence the possibility of access to meaning and truth, for syntactic systems conceived as in themselves meaningless, mere "formal games."[14]

Badiou here repeats the observation already made with respect to Carnap and Quine, that this conception of models, as it is applied within logical positivism at least, effectively presupposes and presumes a theoretical distinction between the formal and the empirical which cannot be sustained as part of a materialist conception of the productivity of model-theoretical results. However, the way in which this distinction is here maintained is effectively reverses Lévi-Strauss's picture: here, rather than the model serving as a formal, theoretical object simply reduplicative of the empirical, given facts, the model, by providing a semantics *for* a formal language, becomes instead the "material" domain of its real force.[15]

Even more important than this reversal for Badiou, though, is the fact that, despite first appearances, the distinction between syntax and semantics actually functions to demonstrate key results such as those of Gödel and Cohen *internally* to a particular science, the "science" of mathematical logic. Thus, according to Badiou, the demonstration of key properties of formal systems – for instance Gödel 's later demonstration of the incompleteness of all formal systems of a certain type – should be understood as rigorously *objective* results about formal structures with a robust level of reality all their own, not to be understood simply in terms of their reduplication of, or secondary relation to, previously existing structures of any kind (whether empirical or formal). This is enough to demonstrate the inadequacy of any conception of models as "explaining" what they are models of. For:

[13] CM, p. 18
[14] In referring to this as the "Tarskian" position, I do not wish to endorse the claim that it corresponds in all details to what was in fact Tarski's own conception.
[15] *CM*, pp. 21-22.

> If we say that the model should 'explain' ["*rendre raison*"] all the facts, our assertion does nothing but redouble – *vary* [*varier*] – the fundamental couple of vulgar epistemology. If, however, we speak of the completeness of a formal system, then we designate a property that can be demonstrated or refuted. This is the object of Gödel 's most famous *theorems*, which establish the incompleteness of the formal system of arithmetic, being a formal system that admits recursive or 'classical' arithmetic as a *model*. The criteria of the pertinent syntax relative to a given model are not left to the arbitration of resemblances. They are theoretical properties.[16]

This suggests, as Badiou goes on to emphasize, that the role of models in establishing positive knowledge cannot ultimately be thought as a matter of their exteriority -- whether as similitude or as "semantic" illustration – to the formal structures that they model. Rather, since "the construction of the concept of model is strictly dependent, in all of its successive stages, on the (mathematical) theory of sets," the syntax/semantics distinction must itself be re-appropriated within the total and unitary field of a mathematically determined formalism.[17] In particular, Badiou says, it is possible to see from this perspective that:

> …it is already inexact to say that the concept [of model] connects formal thought to its outside. In truth, the marks 'outside the system' can only deploy a domain of interpretation for those of the system within a mathematical envelopment, which preordains the former to the latter.[18]

In particular, semantics itself should be seen as "an intramathematical relation between certain refined experimental apparatuses (formal systems) and certain 'cruder' mathematical products, which is to say, products accepted, taken to be demonstrated…"[19]

This makes it possible, according to Badiou, to understand semantics as a kind of "experimental protocol" for mathematical theory and its results as wholly internal productions of mathematical thought, no longer depending on the "ideological" double of the formal and the empirical in any of its versions. By contrast with the logical-positivist thinking of the model, which "pretends to outfit empiricist ideology in *words* which designate moments of a mathematical process," thus ensuring that, in the logical positivist

[16] *CM*, p. 22.
[17] *CM*, p. 42.
[18] *CM*, p. 42.
[19] *CM*, p. 42.

discourse, " 'formal languages' and 'empirical facts' effectively confront one another as two heterogenous regions," Badiou's concept understands models as the "places" at which "a mathematical region … finds itself transformed, tested, and experimented upon…"[20] In this respect, the formal apparatus of model theory is already itself "a materialized theory," one that "can enter into the process of the production of mathematical knowledge" not as an "outside to be formalized," but rather as "a mathematical material to be tested."[21] Only this "materialized" conception of the model, Badiou suggests, can adequately capture the radical historical productivity of formal thinking, a kind of historically situated process in which the knowledge of the real that mathematics produces undergoes "experimental transformation" by means of its own apparatus.[22] Such transformation, as for instance when model-theoretic reasoning illuminates the possibility of new axiomatizations and new, unanticipated possibilities of structural configuration, amounts even to a kind of "retroactive causality of formalism on its own scientific history," a radically productive *praxis* of the effectiveness of forms which not only *illuminates* but in fact directly *produces* the "history of formalization" itself.[23]

In summary, then, Badiou's resistance in *The Concept of Model* to what he sees as the "bourgeois epistemology" that the discourses of both Carnap and Lévi-Strauss represent allows him to re-appropriate the "materialist" concept of model as one that is wholly intra-mathematical and thus wholly internal to the wholly real domain of mathematical formalism. This domain is, moreover, conceived neither purely "syntactic" nor "semantic" but rather as a direct domain of the effectiveness of forms in producing knowledge and determining the history of its progress. Looking ahead, it is easy to see how an essentially similar conception of models underlies the broader application that Badiou makes in *Being and Event* of the model-theoretic results of Gödel and Cohen. For here, too, it is the application of model-theoretic structures licenses Badiou in re-appropriating the question of linguistic significance within (what Badiou claims to be) a purely formal structure of mathematical reasoning, and here too, the actual effectiveness and productivity of real actors is licensed primarily (if not exclusively) by the direct application of model-theoretic structures, in particular Cohen's methods of forcing and the construction of generic sets.

Thus, the privileged connection between the development of models and the production of knowledge that Badiou forges, already in the *Concept of Model*, becomes in *Being and Event* the wholesale basis for

[20] *CM*, p. 46.
[21] *CM*, p. 47.
[22] *CM*, p. 54.
[23] *CM*, p. 54.

the subject's potentially transformative access to the Real of extra-structural truth. Although this is certainly not "Platonism" as it is usually conceived (i.e. as the claim of the "reality of mathematical objects" and our effective intuition of them) it nevertheless echoes in multiple ways Plato's own radical thought of the effectiveness of forms. For Badiou as for (a certain) Plato, the mathematical remains the privileged domain of the ultimate determination of the real, up to the Pythagorean motif, which Badiou echoes, of an ultimate determination of the structure of being by mathematical theory. Moreover, the mathematical is, for Badiou as much as for Plato, the exemplary domain of forms, within which both the empirical instance and its "ideal" model must be thought *unitarily* as distinct instances of a single formal determination which passes common judgment over their existence.

III

Through his immanent reflection on the concept of a model, Badiou thus attempts to develop a conception of mathematical *praxis* that, despite its "materialism," is in fact remarkably equivalent to a certain Platonism, one that accords, at any rate, to the pure *forms* of mathematical configuration a privileged relationship to the real and the true. Without yet venturing to evaluate the merits of this conception, it is helpful to juxtapose it with another recent immanent reflection on models and their uses, this time one that is certainly historically continuous with (if not, as I shall claim, simply another variant of) the "ideological" logical positivist discourse of Carnap and Tarski, with which Badiou aims to break.

In 1977, Hilary Putnam delivered to the Association for Symbolic Logic an address entitled "Models and Reality."[24] In it, he considers the status of models and model-theoretic reasoning in order to illuminate the larger metaphysical question of the bearing of rational thought and language on the world. Putnam begins by considering a familiar result in model theory, the Löwenheim-Skolem theorem, which establishes that every abstract theory which has any infinite model (of *any* cardinality, no matter how large) also has an infinite model of the very first (or smallest) cardinal size, the size of the "countable" set of natural numbers, which is symbolized as "$\aleph_0$". This result leads directly to a somewhat counter-intuitive implication that has sometimes been termed "Skolem's paradox": the paradox (or seeming one)

[24] Reprinted as Putnam, H. "Models and Reality," *The Journal of Symbolic Logic*, Vol. 45, No. 3 (Sep., 1980), pp. 464-482

is that *any* statement about transfinite sets and their cardinalities (no matter how large) can be re-interpreted in a countable model, and so can be modeled by structures of (only) countable size. It follows that any arbitrary statement about transfinite cardinalities – for instance the statement that there is at least one non-countable set – can be re-interpreted in a model with *only* countable sets and so can apparently hold true in a model that "actually" falsifies it. The usual way of handling this paradox within mathematical set theory, suggested already by Skolem himself, is to point out that the plurality of possible models means that cardinality is itself a "relative" notion. In particular, since the cardinality of a set is defined by the possibilities of its being put (or not) into one-to-one correspondence with other sets *within the same model*, the apparent "collapse" of cardinality in the countable models guaranteed to exist by the Löwenheim-Skolem theorem is simply a consequence of the availability, in certain models, of relations that are not available in others. A statement involving the existence of a nondenumerable set is then seen as true "in reality" even though it can be verified by a wholly countable model, owing simply to the relative lack of relations in that model.[25] Moreover, it is clear (as the usual gloss on Skolem's paradox emphasizes) that the countable model cannot, here, be the "intended" one, which is after all supposed to be the *whole* universe of sets, not just some limited, countable *ersatz.*

In this way the standard response to Skolem's paradox defuses the air of paradox by insisting upon the distinction between the "intended" interpretation of set-theoretic statements in terms of the "real" universe of sets rather than any of the non-standard and obviously unintended models whose existence the Löwenheim-Skolem theorem demonstrates. However, it is just here that a deeper and more properly philosophical paradox arises, according to Putnam. For given this standard response, which depends on the distinction between "intended" and "unintended" interpretations to foreclose the paradoxical collapse of cardinalities, we must ask what can possibly determine *any* interpretation of the axioms as the (unique) "intended" one. In particular, if, as Skolem in fact showed, *no* axiomatic system of set theory *uniquely* determines the "intended" interpretation (or corresponds uniquely to what is sometimes called the

[25] "Models and Reality," pp. 464-66. More technically, we can construct a set, S, which is *provably* (by Cantor's theorem) non-denumerable in the sense that we can prove that it cannot be put into a one-to-one relation with any countable set. However, by the Löwenheim-Skolem theorem, this statement itself has an interpretation in a wholly countable model. The apparent paradox that this appears to pose is resolved by noting that *in* this model, the statement only says that there is no one-to-one relationship available *in the model*, and thus only that S is of non-denumerable cardinality in a *relative* sense of "cardinality".

"intuitive" notion of a set), then what, Putnam asks, *could* possibly determine which interpretation this is?[26]

To illustrate how this deeper question of interpretation bears on standard set-theoretical results, Putnam considers the assertion "V=L", which (recall) asserts that all sets in the general set-theoretical universe are "constructible," that is that they are definable in a particular way from previously existing and simpler sets. Gödel , as we have seen, showed that *if* V=L is true in a particular model, so is the continuum hypothesis (as well as the axiom of choice) in that model. But what is relevant to Putnam's discussion is not simply this result, but the question of the status of the assertion itself. As Putnam notes, Gödel himself briefly thought that V=L might be "really" true in the sense of "true of the actual universe of sets" and should accordingly be added to the basic set-theoretical axioms; later on, he reversed this opinion and held the belief that V=L is "really" false.[27] Gödel 's robustly realist intuitions about mathematical reality apparently demanded that he hold one or the other of these views, but Putnam poses the important skeptical question of whether holding either view of the "true" nature of the set-theoretical universe in fact (so much as) makes sense at all. In fact, Putnam argues, there is every reason to think that it does not. In particular, Putnam uses a strong version of the Löwenheim-Skolem theorem (the so-called "downward" version, which requires the axiom of choice to prove) to argue that *if* (as we may conclude from Gödel and Cohen's results) the ZF axioms do not settle the question of the truth of V=L, then *no* additional set of constraints that we could plausibly determine (either by empirical observation *or* by the sorts of "mathematical intuition" that it is reasonable to assume we might have) can do so either. This extended application of Skolem's argument suffices to show that, for any actual language such as English, "*the total use of the language* (operational plus theoretical constraints) does not "fix" a unique "intended interpretation" any more than axiomatic set theory by itself does." [28] This suggests, as Putnam goes on to argue, that is it an important sense not even so much as intelligible to suppose that the hypothesis that V=L is true or false *in reality*. In particular, if we were to encounter an alien civilization that regarded it as false (assuming that *we* regard it as true), there would be an important sense in which that civilization had simply made a different "decision" than we have.[29] For this and a variety of important questions, even an idealized complete theory of sets will still leave ample room for such a free decision, ungrounded by anything that is reasonably present in set-theoretical "reality" itself.

[26] "Models and Reality," pp. 465-66.
[27] "Models and Reality," pp. 467-68.
[28] "Models and Reality," p. 466.
[29] "Models and Reality," p. 471.

In fact this kind of problem has important and dramatic questions, as Putnam argues, for the much more general question of the metaphysical "relationship" of language and reality itself. For a variant of the Skolem argument can be employed to suggest that the question of the "right" interpretation of our total theory (or total operational use of language) will always be left open, *no matter how fully* we try to specify this interpretation in terms of the theory itself. This is because *any* specification of the "actual" relationship between our terms and objects in the world will amount to "just more theory," and will, as a consequence of the Löwenheim-Skolem theorem, be open to interpretation in (infinitely) *many* different possible models.[30] This broader application of the theorem suggests, in fact, that the theoretical attempt to specify a unique relationship between language and the world must be doomed, in all of its versions. Specifically, Putnam considers a series of instances of this attempt: the search for a uniquely determinate basis of perceptual knowledge in given experience, the attempt to tie down the intentionality of mental states by means of a self-interpreting "language of thought," and the provision of "causal" theories of reference are all attempts to specify the *unique* relationship between language and the world in virtue of which meanings are fixed.[31] But all of these attempts, Putnam argues, succumb to the generalized Skolem argument, which establishes that it is impossible for any theory, no matter how complex or complete, to fix its *own* reference in this sense.

This suggests that there is, in a very important sense, no story to be told from a metalinguistic perspective "about" the way that our words and thoughts "reach out" to contact reality at all. As Putnam notes, if this is right there remain only two possible positions, each one "extreme" in its own way. Either we can renounce any claim to establish a distinctive relationship between theory and world at all, settling only for what can be proven or established *internally* to our theories or, if we are prepared to accept that we somehow *do* have access to the contours of reality in themselves in a way that is somehow independent of our total theories and uses of language, we can hold that this access itself establishes a unique interpretation for these theories and guarantees their unique application to reality. The first position, in denying in principle the coherence of the supposition of a reality wholly independent of our theories and uses of language, can be considered a (possibly sophisticated) kind of verificationism; the second, with its requirement of "mysterious", trans-linguistic capabilities of the mind to grasp truth, corresponds to a kind of Platonism. But what is not left open given the Skolem arguments is the "middle ground" realist

[30] "Models and Reality," p. 477.
[31] "Models and Reality," pp. 475-477. For a further development of these arguments with respect to intentionality in particular, see Putnam's book *Reality and Representation*

position according to which linguistically (or operationally) determined syntactical structures face their "semantic" interpretation from a position of mutual exteriority:

> This observation [viz., that the Skolem argument can be extended in the way that Putnam suggests] can push a philosopher of mathematics in two different ways. If he is inclined to Platonism, he will take this as evidence that the mind has mysterious faculties of "grasping concepts" (or "perceiving mathematical objects") which the naturalistically minded philosopher will never succeed in giving an account of. But if he is inclined to some species of verificationism (i.e., to indentifying truth with verifiability, rather than with some classical "correspondence with reality") he will say, "Nonsense! All the 'paradox' shows is that our understanding of 'The real numbers are nondenumerable' consists in our knowing what it is for this to be proved, and not in our 'grasp' of a 'model'." In short, the extreme positions -- Platonism and verificationism -- seem to receive comfort from the Löwenheim-Skolem Parodox; it is only the "moderate" position (which tries to avoid mysterious "perceptions" of "mathematical objects" while retaining a classical notion of truth) which is in deep trouble.[32]

Much the same can be said, Putnam suggests, with respect to the more general situation of any scientific theorizing whatsoever. The apparent possibility of "Skolemizing" any totality of theory – whether mathematical *or* empirical – appears to leave the theorist with two starkly divergent choices, neither one of which is particularly attractive. Either we can adopt the *strongly* "metaphysically realist" position that we somehow have a form of access (for instance by means of "mysterious powers of the mind" to the contours of "universals" and the constitutive categories of reality that is completely independent of our actual activities of theorizing (this is the analogue to Platonism), or we must drop the assumption that our "access" to reality must or even can go significantly beyond what is established internally to our best current theories. This amounts to dropping the "metaphysical realist" assumption that reality is fixed "out there" in a way that is completely independent of theory, and in denying the very coherence of such a verification-transcendent realm of objects suggests, Putnam says, at least a mild form of "verificationism." What is in any case rendered inadmissible, though, is the middle ground of a

[32] "Models and Reality", p. 466.

"moderate" realism, the position that "would like to hold on to metaphysical realism *without* postulating mysterious powers of the mind."[33]

Though neither of these two remaining options is particularly attractive, Putnam's reasonable doubt that there *can* exist such "mysterious powers" leads him to reject metaphysical realism and to express at least a guarded preference for the other remaining option, the "verificationism" that refuses to see reality as fixed completely independently of our best current theories of it. (This is essentially the position that was also sometimes called, by Putnam and others, "internal realism"). This position is akin to classical intuitionism in that it denies the meaningfulness of claims of the existence of structures and objects going essentially beyond our procedures of verification and proof (and thus is "anti-realist" in Dummett's sense), but it can, Putnam suggests, be significantly more nuanced and subtle than the doctrine that classically went under that heading. In particular, there is no need for the doctrine here contemplated to comply with classical intuitionism (or, for that matter, with the similar position that Badiou calls "constructivism") in its rejection of "impredicative" sets; nor does it necessarily require any revision of classical logic. What *does* have to be sacrificed, though, is the assumption of a "ready-made world," a metaphysically "real" domain of objects and relations fixed in themselves and thus capable of determining the "right" interpretation of our theories and claims, quite independently of the internal content of those theories and claims themselves.

According to Putnam, the problem posed by the "Skolemization" of theory can thus only be resolved through the adoption of either a robust Platonism or a (sophisticated kind of) anti-realist position. It is the second (anti-realist) option that Putnam himself recommends, arguing that it despite the challenges it poses for realism it preferable to the Platonistic invocation of "mysterious" extra-linguistic mental powers. However, what may be more important than opting for one resolution or the other is to *diagnose* the underlying problem, and in the last section of the paper Putnam offers some suggestions as to how and where we might locate its ultimate roots. Specifically, Skolem has essentially demonstrated that "no interesting theory … can, in and of itself, determine its own objects up to isomorphism".[34] This appears to leave us in the by-now-familiar predicament that the "actual reference" of our terms and thoughts must either be given by way of supernatural, mysterious powers of mind, or it cannot be given at all. However, Putnam suggests, there is a deeper source of this (apparent) predicament in the very assumptions underlying the classical use of model theory itself:

[33] "Models and Reality," pp. 474-75.
[34] "Models and Reality", p. 481.

> The problem, however, lies with the predicament itself. The predicament only *is* a predicament because we did two things: first, we gave an account of understanding the language in terms of programs and procedures for using the language (what else?); then, secondly, we asked what the possible "models" for the language were, thinking of the models as existing "out there" *independent of any description.* At this point, something really weird had already happened, had we stopped to notice. On any view, the understanding of the language must determine the reference of the terms, or, rather, must determine the reference given the context of use. If the use, even in a fixed context, does not determine reference, then use is not understanding. The language, on the perspective we talked ourselves into, has a full program of use; but it still lacks an *interpretation*.
>
> This is the fatal step. To adopt a theory of meaning according to which a language whose whole use is specified still lacks something -- viz. its "interpretation" -- is to accept a problem which *can* only have crazy solutions. To speak as if *this* were my problem, "I know how to use my language, but, now, how shall I single out an interpretation?" is to speak nonsense. Either the use *already* fixes the "interpretation" or *nothing* can.[35]

In other words, what really lies at the root of the dilemma between the "Platonist" and the anti-realist positions, according to Putnam, is the original "Tarskian" assumption of a distinction between syntax and semantics itself. According to this "Tarskian" position, as we've seen, languages and their patterns of use amount to nothing more than uninterpreted formal calculi, unless and until they are provided with a semantic *interpretation* by means of a coordination of terms to objects (or, equivalently, a specification of the satisfaction relations for objects and terms). But to think in this way is essentially to make the assumption that Putnam here calls "fatal", the assumption that it is coherent to assume that a language could be fully specified in terms of its ("syntactic") rules and total patterns of use but *still* lack an interpretation. As long as we make this assumption of the need to supplement syntax with semantical, model-theoretical interpretation from outside, Putnam suggests, we will face the question of the specification of the "unique" relationship between language and reality; and as long as we face this question, due to the Skolem problem, we will not be able to solve it. In this way, far from establishing the robust and even potentially transformative relationship to the real, the original metaphysical assumptions

[35] "Models and Reality," pp. 481-82.

underlying model theory are here undermined from within by means of one of its most important substantive results.

The critical position that Putnam thereby defends is in many ways continuous with the "linguistic turn" at the historical basis of the tradition of analytic philosophy. In particular, in defending the position, Putnam here develops (as many "analytic" philosophers have done, in different ways) the implications of the thought that there can be no access to truth or to the effective transformation of situations that is not conditioned, in a broad sense, by the formal and syntactical features of language and its structure. But to depend upon this thought in sketching a critical position that, like the present one, emphasizes the formal and structural features of language in the determination of (anything that can appear as a truth) is *not* (as Badiou himself sometimes seems to assume it must be) simply to subject the effects of truth to contingent, local, and historically situated language games. For what is at stake in the model-theoretical arguments that Putnam draws upon is not the contingency of any particular historical language or the accidental contours of its cultural conditioning, but rather a rigorous universalism of sets and structures that owes nothing to any empirical or historicist referent. And the application of this "mathematical" structure to a fundamental consideration of meaning and reference shows that what is at issue is not the contingent limits of this or that language but, plausibly, the very expressive limits of language (*any* language) as such.

Additionally, although Putnam's position here thus involves an essential and constitutive appeal to reasoning about language and its limits, it is clearly far from the "constructivism" which Badiou describes and criticizes in *Being and Event*. For as Putnam argues, the sophisticated variant of intuitionism that may still be adopted in the wake of the Skolem arguments need not adopt the linguistic restrictions characteristic of early forms of that doctrine. Rather, "language," as it is treated here, is plausibly much richer and more powerful than anything that is captured by Gödel 's assumption of constructability, since it includes from the beginning the *full* resources of the ordinary language in which we state our theories (including our theories of models!).[36] In *The Politics of Logic*, I argue that metalogical reflection in the analytic tradition has produced a series of important critical results bearing on the structure and limits of *any* symbolic language, as such, results that are only very poorly captured (if at all) by anything that comes under the heading of what Badiou criticizes as the "constructivist" orientation, which he sees as representative of the linguistic turn as a whole. In fact, for analytic philosophers from Quine to

[36] Although it may still be the case, as Putnam argues, that it does not *settle* the question of whether the "real" universe is identical with the constructible one.

Wittgenstein, these results, I argued, formulate an interrelated series of profound structural aporias that result directly from a strictly formal consideration of the powers of language *as such* in its attempt to capture the whole of what is. If this is correct, then Putnam's own somewhat aporetic argument might be seen as another instance of this *genre*, directly continuous with such results as Quine's thesis of the indeterminacy of radical translation and the late Wittgenstein's far-reaching critique of rules and rule-following in demonstrating the permanent structural paradoxes and aporias of our lived relationship to language itself, but going somewhat further than even these results in formally demonstrating the critical aporias to which the attempt to regulate the "meaning" of language by means of an unincritical syntax/semantics distinction is inevitably prone. These results, as well as various parallel projects and results within the French structuralist and broader "continental" tradition, may be taken to represent (as I argued in the book) a profound theoretical alternative to Badiou's own "generic" orientation (what I there call, by contrast, the "paradoxico-critical" orientation), and so should certainly be taken into account at their own level of theoretical specificity in any comprehensive discussion of the implications of formal thinking in our time.

Despite Putnam's close historical linkages to both Carnap and Quine, moreover, it is clear that the position he occupies here is not simply the "ideological" position about models that Badiou targets in *The Concept of Model*. For, far from affirming or even tacitly presupposing the coherence of the "ideological" distinction between formal and empirical sciences, Putnam works from the beginning entirely within the formal matrix of model theory, coming to reject on this basis the coherence of *any* attempt to coordinate the relationship between language and reality from an empirical (or indeed *any* extra-formal) position. In fact, in arguing that this is the only reasonable conclusion to draw from the model-theoretic phenomena evidenced by the Löwenheim-Skolem theorem and the associated Skolem arguments, Putnam is in remarkably close agreement with Badiou's own criticism of the "ideology" of the formal/empirical distinction and its usual employment to license the distinction between syntax and semantics. Of course, Badiou does not explicitly employ the category of "ideology" to discuss the rivals to his own view, and although he suggests at several places in the article that his own view may be the one most amenable to a "naturalist" methodology and epistemology, he obviously does not work within the context of the programmatic Althusserian "materialism" that orients Badiou's project in 1968. Nevertheless, in finding grounds internal to model theory itself for rejecting the "Tarskian" view of the coordination of syntax and semantics along with any of the "middle ground" realist positions that aim to

coordinate the formal and the material from a position exterior to both, Putnam (far from simply echoing the assumption of a formal/material distinction characteristic, according to Badiou, of "logical positivism" as such) in many ways essentially repeats the result of Badiou's earlier analysis.

IV

As we have seen, then, both Badiou and Putnam employ reflection about the nature of models and their mathematical status to consider the (apparently much broader) metaphysical questions of realism and the very possibility of our access to reality and truth. Both philosophers, moreover, appeal not only to broad and general thinking about the nature of models, but also to the use of specific model-theoretic results – Gödel and Cohen's result in the case of Badiou, and the Löwenheim-Skolem theorem in the case of Putnam – to derive what can only be seen as "metaphysical" or "meta-ontological" results. And although Putnam and Badiou defend what are on their face very different results (Putnam defending a sophisticated variant of intuitionism that denies the ultimate coherence of the "metaphysical realist" picture of a completely theory-independent reality, while Badiou defends a robustly realist Platonism that sees formal modeling as capable not only of revealing "metaphysical" truth but even as positively producing it) the two investigations nevertheless display a quite surprising degree of convergence. In particular, both philosophers, in taking up the question of how results internal to model theory promise to illuminate the relationship between our linguistic and cognitive systems and the real itself, find good grounds for questioning the standard "Tarskian" picture of the functioning of models, as well as the whole familiar picture of the "syntax"/"semantics" distinction which it formulates and upon which it relies.

Denying the Tarskian position, what are left open are precisely the two positions that Putnam sketches: on the one hand, an "internalist" position that, seeing no possibility of coordinating our words and theories to an "external" metaphysical reality, conceives of the entirety of our substantial "reference" to the world as taking place internal to theory, and on the other, a Platonism that allows for direct access to the universal categories that assure the connection of language and thought to the real. The first is the position that Putnam goes on to defend; the second (albeit with some important modifications) is essentially Badiou's. In occupying these two "extreme" positions, though, what the two philosophers disagree about is *not* the coherence of a total distinction between formal structures and their external, "material" or empirical

correlates (such as the one that figures at the basis of the Tarskian conception of semantics); both philosophers agree, on both model-theoretic and more general philosophical grounds, that *this* kind of distinction cannot be made to work. What they disagree on is rather the status of the unified field of formal work (or "theoretical production") itself. For Putnam, since there is no way to escape from the interpretations and theories determined by our use of words and language (even by the "provision" of "semantic" relationships between words and objects), we remain in a position wherein *all* theoretical language and activity is essentially "syntax": there is no possible escape (of the kind the standard Tarskian picture fantasizes) into a semantical "true reality" independent of linguistic use. By contrast, for Badiou, the realm of formal/theoretical work is always already irreducibly "semantic" in its direct relevance to structures and phenomena in the world, bearing within itself not only the inherently productive power to "test" the reality and consistency of forms but even the dramatic ability to transform lived reality itself on this basis.

What kinds of considerations might help to resolve this difference between the two "extreme" positions staked out by these somewhat differently motivated pictures? When Putnam sketches the two positions that remain open in the wake of the Skolem argument, he rejects the "metaphysically realist" position of Platonism largely because, as Putnam assumes, *this* position requires "mysterious powers of the mind" to grasp universals, powers whose epistemology and even existence seem to Putnam to strain credulity. Putnam does not deny that respectable philosophers have held the view that we *do* have such powers – Gödel 's own views on our access to "mathematical reality" offer an obvious example, and Putnam mentions Chisholm as a philosopher who has held something similar with respect to "natural" reality – but their existence is on its face difficult to accept without a very compelling argument, and in any cases offers profound challenges for any (even broadly) "naturalist" position.[37] On the other hand, the rather unorthodox character of Badiou's variety of Platonism here potentially gives him resources to defend his position against this kind of epistemologically based objection. For as Badiou emphasizes early on in Being and Event, his Platonism is not primarily a doctrine of *access*, but rather one of *decision*: for Badiou, it is not a kind of superlative and essentially mysterious access of the mind to transcendent being that ultimately produces formal thought's contact with the real, but rather the free choice by means of which fundamental axioms are adopted or refused. It is through this kind of choice and only by means of it, Badiou suggests, that mathematical *praxis* ultimately interpolates the Real that it captures, rigorously testing the consequences of its variant hypotheses up to the very point of internal contradiction. By emphasizing this element of choice and decision, Badiou avoids (and this is a large part of the basis of his

[37] "Models and Reality," p. 467, 474.

claim to be able to maintain a distinctively "materialist" kind of Platonism at all) the need to invoke mysterious *epistemic* powers of the mind; the outcome of axiomatic decision is here not to guarantee the contact of forms with the real but rather directly to ensure their effectiveness and productivity by rigorously working out the consequences of their adoption.

But even if this element of decisionism in Badiou gives him some resources for avoiding the direct challenges to an epistemological form of Platonism that Putnam suggests, there nevertheless remain important questions that could be asked, from Putnam's position or one like it, about even the more deicisionist form that Badiou does in fact advocate. To begin with, there is the broad-based challenge to conventionalism that is in fact already voiced in Putnam's article itself: if, we may echo Putnam in asking, it is simply a matter of convention or axiomatic decision that V=L (or not), then how are we justified in viewing the consequences of *either* of these "decisions" as the (unique) truth? As we have seen, the results of Gödel and Cohen about the continuum hypothesis jointly establish that the CH is simply *undecidable* from the standard axioms: as far as these axioms go, we are just as free to assume that the CH is true as to assume that it is false (or, for that matter, simply to adopt an "agnostic" attitude about its truth). The price of an honest acknowledgment of these results is apparently that we must renounce the hope of uniquely describing the *whole* reality of the "set-theoretical universe," or at any rate must accept the possibility that in an important sense this reality is itself not determinate. But if this is so, there is something at least misleading about Badiou's celebration, in *Being and Event*, of one side of this result over the other. In defending the possibility of the event and the subject's faithful action in drawing out its consequences, Badiou recurrently speaks as if it is *Cohen*'s model-theoretic result *rather* than Gödel's converse one that establishes the real possibilities of subjective intervention and change that he is effectively defending. That is, for anything like the event and the subject's forcing of truth actually to take place, it must be the case not only that the falsehood of the continuum hypothesis and the correlative availability of Cohen's "forcing" constructions is not only conceivable or thinkable, but actually in fact realizable in the *actual* universe, as it in fact is. But if Cohen has shown, through the apparatus of forcing, that it is possible (i.e., completely consistent with the ZF axioms to assume) that the CH is false and what Badiou terms an event *can* take place, it must be admitted that it is just as much possible, as Gödel showed, that the CH is true and accordingly that (given Badiou's other interpretative assumptions) *no* radical transformation of existing situations is possible.

More broadly, there are questions to be raised about the very integrity and possibility of the position that Badiou's meta-ontological claims about the effective forcing of the real essentially require him to occupy.

As we have seen, Badiou identifies the ZF axiom system with ontology, or the "theory of being *qua* being." This puts the model theorist, or anyone who attempts to reason about the consistency of various hypotheses with the ZF axioms, in the position of a transcendent arbiter of the very boundaries of what can be captured by ontological theory, or what can be thought in being itself. This is a position outside all of the multiplicities regulated by the ZF axioms, a position, as it were, from outside the *whole* of the set-theoretical universe (the universe of "ontology") that it theoretically contemplates. To a certain extent, this is admittedly the position presupposed by *any* systematic model-theoretical reasoning about the totality of what is or is not prescribed by the axioms. But as we have also seen, Putnam's use of the Skolem arguments poses a dramatic challenge to the claim that it is possible for us to occupy this position at all. For as Putnam suggests, this model-theoretic reasoning must *itself* take place within the total field of the theory that specifies what we know or derive about sets, and can thus hardly be the locus of a fundamental operation of theory conducted from a transcendent "outside" position. This challenge to existing set-theoretical results is, if anything, intensified by Badiou's identification of ZF with ontology and his claim to trace the elusive structure of the event at its very boundaries. For these identifications suggest that the theorist must be speaking not only from outside the total set-theoretical universe V, but also and correlatively from outside what is *ontologically* thinkable as well. If, on the other hand, there is (as Putnam suggests) no reason to think that we can attain (or even really entertain the possibility of) such an "outside" perspective, then it is doubtful whether Badiou's theoretical defense of the event and the possibilities of radical change that it represents can advance beyond its very first stages.

In defending his doctrine of the faithful action of the subject and its link to the four generic "truth-proceudres" in *Being and Event*, Badiou maintains, as we have seen, a formidable doctrine of mathematical and (especially) model-theoretical reasoning as an exemplary realm of production and effect whose thought and *praxis* succeeds in directly capturing and even bringing about the Real of effective and radical change. This can certainly be contrasted with the *motivations* of Putnam's view, which sees model-theoretic activity and *praxis* as simply a specialized part of the broadly "syntactical" network of our total theory, emphasizing the significant problems involved in supposing it might be possible to get outside this network to gain some kind of access to the real in itself. And it is true that Putnam's ontologically noncommittal position yields few, if any, of the transformative implications that Badiou claims for his own. If there is, as Putnam suggests, in an important sense "only" syntax, then it is probably impossible (but why should we have thought anything different?) to intervene upon the total field of our theory from a coherent "outside" position to produce, at the limit-point of syntax, the radical possibility of dramatic change. Of course, from the perspective of certain determined programs of action,

this result may be unattractive, but there is no clear argument from the mere desire to think the possibility of effectuating change to the actual existence of the formal basis for it. If, on the basis of his positive argument, Badiou can indeed keep *open* the formal possibility, consistent with everything actually established on the level of "ontology" by ZF set theory itself, of the constructions upon which (what he models as) the subject's post-evental forcing of a truth relies, it is nevertheless worth noting how little support this yields for the claims that this kind of intervention is in fact actual or, indeed, that the extra-linguistic subjective position from which it would have to be effectuated is coherent at all.

In *The Politics of Logic*, I suggested that the divide that we have effectively seen here between Putnam and Badiou has its roots in broader issues in the history of twentieth-century philosophy, and specifically in very different ways of thinking about the possible access of thought to the totality of the world (or universe) in which it takes place. Here, it is essential to the heady and dramatic consequences apparently entailed by Badiou's doctrine of the event that mathematics *itself* be capable of acting as a radical region or *organon* of positive thought, indeed as the privileged domain from which the real itself shows forth. Thinking of mathematics in this way requires that it be prioritized over logic as the primary and basic realm from which ontological truth derives, and thus that any attempt to *derive* mathematics from logic (whether in classicly logicist, formalist, or intuitionist terms) must be steadfastly resisted (a priority for which Badiou has indeed argued in detail elsewhere).[38] On the other hand, Putnam's consideration of model theory, with its emphasis on the impossibility of escaping from syntax and syntactical derivations of truth, is in many ways continuous with the classical logicist project and can easily be seen as continuing, in broad terms, its original approach. What is really at issue here, as I argued in the book, is the question whether the forms of mathematics can be seen as immediately prescribing the truth of being as well as what transcends it from a position integral in itself and having no need of external interpretation, or whether, quite to the contrary, it is necessary to see mathematics as a technique among others involved in the complex form of a human life determined by its access to language, and so to interrogate the results (including the model-theoretic ones) of mathematical investigation always only from within the total system of language (or syntax) itself.[39]

[38] See, e.g., Badiou, A. *Briefings on Existence: A Short Treatise on Transitory Ontology.* Trans. Norman Madarasz. (SUNY Press, 1998/2006), especially chapter 8.

[39] See *The Politics of Logic*, especially chapters 7, 8, and 9.

It is not possible here to resolve or even really begin to adjudicate this dispute, but it may be useful simply to note that the issue that is effectively in dispute between Putnam and Badiou poses a fascinating and perhaps unprecedented challenge for the future of thought about formalism and the effectiveness of forms. For if both philosophers, working within different traditions and according to different methods and motivations, ultimately deny the coherence of an empiricist or substantialist metaphysics that would locate the fixed point of the real outside formal reasoning, both effectively echo a thought that is in a profound way Platonist, even if both disagree (in different ways) with the letter of "Platonism" itself. This is the thought that the question of the real is to be settled, if at all, *only* on the level of forms, and this is a thought with which Plato could certainly agree. In the 2007 interview with Tzuchien Tho that accompanies the English translation of *The Concept of Model*, Badiou mentions this genuinely "Platonic" inherent element of our access to the real through sets and models:

> Thus, what we can see here is that the model is that which permits the study, on one hand, of the power of formalization, but also on the other hand, at its limits, that it becomes something that permits us to arrive at a dialectical point, the most concentrated point. These are specific points or certain times, at which the infinite power of formalization and its limitations are irreducible and persent a point of undecidability distinct from the others. Thus, what interests me in particular is something that in fact supports my peculiar Platonism. I should say that Platonism, in the end, is the knowledge of ideality. But this is also the knowledge that we have access to ideality only through that which participates in ideality. The great problem of Platonism is not really the distinction between the intelligible and the sensible, but the understanding that sensible things participate in the intelligible. … The model is thus that which allows us to conceive formalization; conceived after the fact, given mathematical inventions are not simply formal inventions but rather an invention of models. It is that which permits us to access formalization or to access the universality of things, at the same time it permits us to determine the particular point of limitations. To put it more directly, the model is that which allows us to think through participation.[40]

As Badiou here suggests, the biggest problem facing formal thought today, and which comes to the fore in a dramatic way through reflection on both Badiou's and Putnam's considerations of models, is the problem of *participation* that already posed the occasion for Plato's deepest and most penetrating thought

[40] "The Concept of Model, Forty Years Later," in *The Concept of Model* (op. cit), p. 92.

about forms in their relationship to being itself. For if the thought of the twentieth century recurrently and obsessively traverses its formal determination with the aim of locating a fixed point of the real from which to anchor its radical and transformative programmatic visions, the unanswered question around which this thought circulates is always that of the realization of form in the world.[41]

This Platonic question of participation is an essential problem not only for socio-political reflection about abstract reasoning and the institutional formalization of intersubjective life, but also, and perhaps even more profoundly, for the characteristic ("analytic" as well as "continental") twentieth-century philosophical inquiry into linguistic meaning, including among others the questions of the nature of linguistic generality and the "logical" form of the sentence itself.[42] (In this connection, it is, of course, essential to recall the *many* passages and moments in Plato's text that connect the problem of forms to the question of the revelatory power of the *logos*, and of the correctness and effectiveness of names).[43] But this question of participation, as Badiou says, is again the question that is really raised investigating the nature of models and the level of their force. And if, as Badiou and Putnam both suggest, there can no longer be any viable answer to this question which assumes (as, for instance, the "Tarskian" view still does) a stable confrontation of the "ideal" level of forms with the "real" level of objects in an unproblematic relationship of instantiation or predication, then the very formal results that in the twentieth century decompose and deconstruct this assumption from within plausibly provide the only possible basis on which the twenty-first century can answer to this problematic twentieth-century experience of forms. It is thus and in similar ways that a certain problematic Plato, no longer tied to "Platonism" in the received ontological (or "metaphysical") sense, might come once again to preoccupy a reflective critical awareness of what most calls for thought in our time.

[41] For this conception of the leading projects of the twentieth century as obsessed with the pursuit of a rigorous formalization adequate to achieve a kind of impossible capture of the Real, see Badiou's *The Century*, trans. Alberto Toscano (Cambridge: Polity Press, 2007). For some discussion of this text, see *The Politics of Logic*, chapter 10.

[42] For a suggestive recent "analytic" inquiry into these questions in which the close relationship between Tarskian model theory, the Platonic problematic, and the underlying question of truth is very much in evidence, see Donald Davidson, *Truth and Predication* (Cambridge, MA: Harvard U. Press, 2005)

[43] Just to scratch the surface, see, for instance, *Cratylus* 384c-390c, *Meno* 75a, *Phaedo* 78e-79c and 89d, *Phaedrus* 249b-c, *Sophist* 244c-e and 260e, and *Parmenides* 135e and 147d-148a.