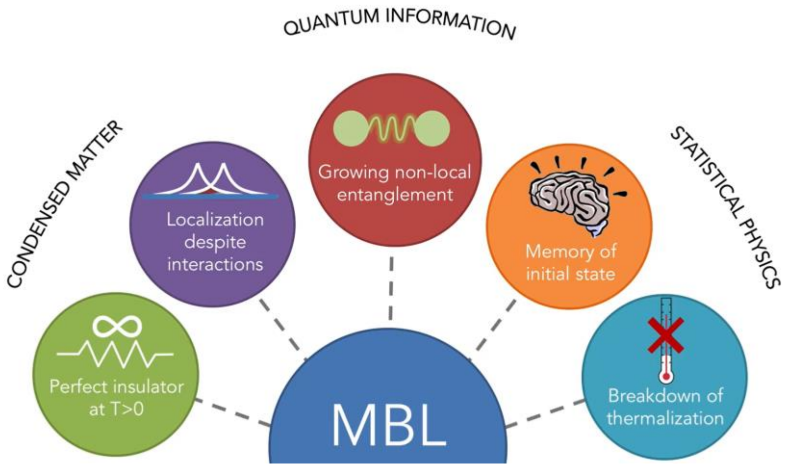
Many-body Localization(MBL) and Quantum Scars

Karthik Chinni and Anupam Mitra



Review of Eigenstate Thermalization

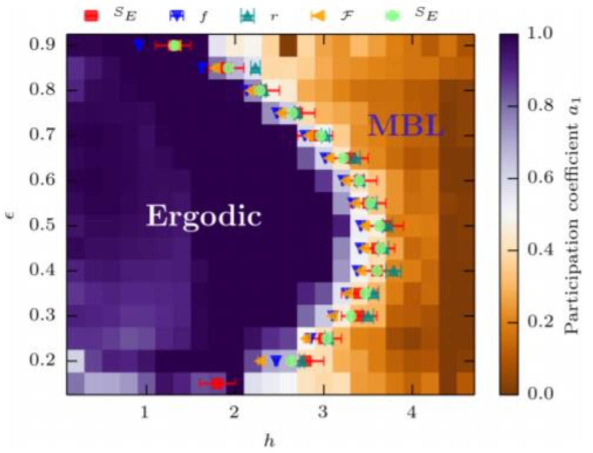
$$H = J_{ij} \sum_i (\sigma_i . \sigma_j) + \sum_i h_i^z \sigma_i^z$$

with disorder term $h_i \epsilon [-h, h]$

- Thermal phase and MBL phase are opposite dynamical phases.

- Interacting many-body systems associated with highly excited states.

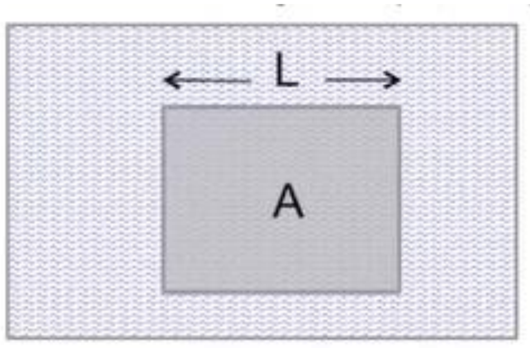
- Pure states in isolated quantum systems reach thermal equilibrium values: memory of the initial state is lost



Numerics from Luitz et al. PRB 2015

$$A_{\alpha,\beta} = A(\bar{E})\delta_{\alpha,\beta} + e^{-\frac{S(\bar{E})}{2}} f(E,\omega) R_{\alpha,\beta}$$

Where $\bar{E} = \frac{E_\alpha + E_\beta}{2}$ and $\omega = E_\alpha - E_\beta$



Serbyn, Maksym. "Many-body localization, thermalization, and entanglement."

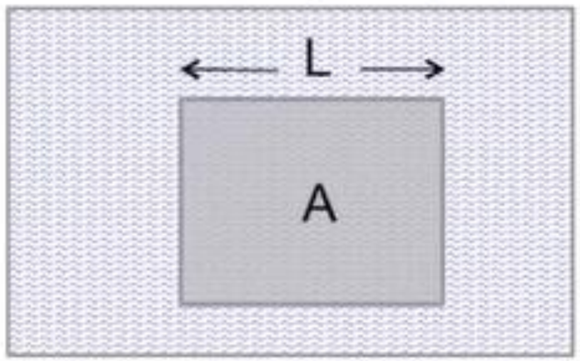
Review of Eigenstate Thermalization

- For an eigenstate $|\alpha\rangle$ obeying ETH, all observables within A will have thermal expectation values. This implies that the reduced density matrix $\rho_A = \mathrm{Tr}_B(|\alpha\rangle\langle\alpha|)$ $(B = \bar{A}$) is thermal.

- Entanglement entropy is equal to the thermodynamic entropy.

$$S_{\mathrm{ent}}(A) = -\mathrm{tr}(\rho_A \log \rho_A) = S_{\mathrm{th}}(A)$$

Thermodynamic entropy is extensive, so entanglement entropy of the subsystem follows volume law, $S_{\mathrm{ent}}(A) \propto \mathrm{vol}(A)$.



- Sensitivity of eigenstates to perturbations and Wigner-Dyson statistics.

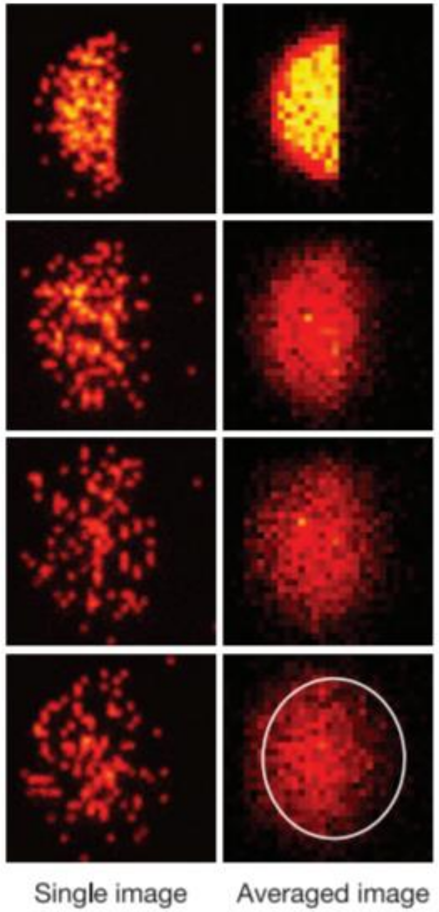
Deviations from ETH

- Systems that fail to thermalize:

  1. **Traditional integrable systems**: extensive sum of local operators, equilibrate to generalized Gibbs ensemble, isolated point in the family of Hamiltonians, Poisson statistics.

  2. **Many-body localization (MBL)**: complete set of localized conserved operators, stable phase, KAM type integrability, Poisson statistics.

  3. **Quantum many-body scars**: scarred systems that are thermal in weak sense, isolated point in the phase space of Hamiltonians.



Choi et al. Science(2016)

Many-body Localization

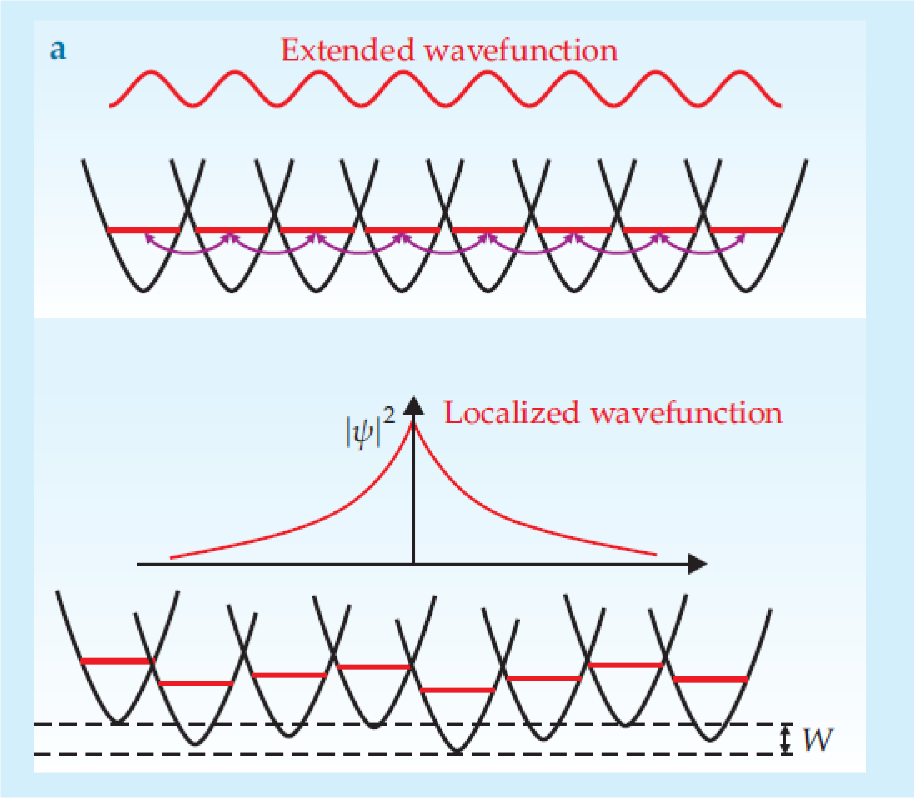
- **Anderson localization**: localization of a single particle

$$H = \sum_n (\varepsilon_n a_n^\dagger a_n + t_n (a_n^\dagger a_{n+1} + \mathrm{h.}c.))$$

where $\varepsilon_n \in [-W, W]$.

- For $t = 0$, all the sites are unconnected and the eigenfunctions are totally localized. For $W = 0$, eigenfunctions are Bloch functions, which are not spatially localized. There exists $(W/t)_{\text{critical}}$.

- For W/t > $(W/t)_{\text{critical}}$, $[H, n_\alpha] = [n_\alpha, n_\beta] = 0$, complete set of localized operators.



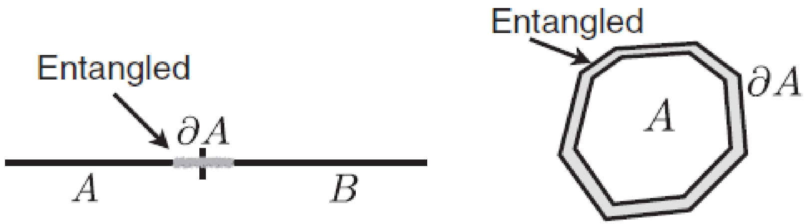
Aspect and Inguscio, Physics Today (2009)

Area-law Entanglement of MBL Eigenstates

- Interacting many-particle systems: effect of local perturbations remains local.

- **Area-law entanglement**: entanglement of

eigenstates proportional to the area of the subsystem.



- Heuristic argument: $H = H_A + H_B + V_{AB}$
  - With coupling turned off, $|I\rangle_{AB} = |\alpha\rangle_A \otimes |\beta\rangle_B$
  - Introduction of local coupling will only affect degrees of freedom within localization length from the boundary.*

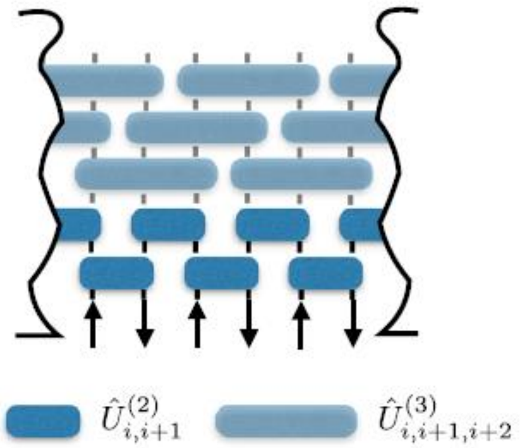
* Follows volume-law entanglement following a quantum quench.

Quasilocal Integrals of Motion

- Area law implies that MBL eigenstates are connected to product states by a sequence of quasilocal unitary transformations*.

- Quasilocal unitary: $U = \Pi_i \; \ldots U^{(3)}_{i,i+1,i+2} U^{(2)}_{i,i+1}$

with $\left\| 1 - U_{i,i+1,\ldots,i+n} \right\|_F^2 < e^{-\frac{n}{\xi}}$.



- Such unitary transformations diagonalize the Hamiltonian in a given product state basis.

- Example: $H_{XXZ} = \frac{J_\perp}{2} \sum_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y \right) + \underbrace{\sum_i \left(\frac{J_z}{2} \sigma_i^z \sigma_{i+1}^z + h_i^z \sigma_i^z \right)}_{H_0}$

Quasilocal Integrals of Motion

- Example: $H_{XXZ} = \frac{J_\perp}{2}\sum_i\left(\sigma_i^x\sigma_{i+1}^x + \sigma_i^y\sigma_{i+1}^y\right) + \underbrace{\sum_i\left(\frac{J_Z}{2}\sigma_i^z\sigma_{i+1}^z + h_i^z\sigma_i^z\right)}_{H_0}$

- New integrals of motion: $\tau_i^z = U\sigma_i^z U^\dagger$ where

$$\tau_i^z = Z\sigma_i^z + \sum_n V_i^{(n)} O_i^{(n)}$$

Complete set of independent quasilocal integrals of motion (LIOMs): localized bits or l-bits.

$$V_i^{(n)} \sim e^{-n/\xi}$$

- Complete basis of operators $\left\{\tau_i^{x,y,z}\right\}$ and MBL Hamiltonian:

$$H_{MBL} = \sum_i h_i\tau_i^z + \sum_{i>j} J_{ij}\tau_i^z\tau_j^z + \sum_{i>j>k} J_{ijk}\,\tau_i^z\tau_j^z\tau_k^z + \cdots$$

Dynamical Properties of the MBL Phase

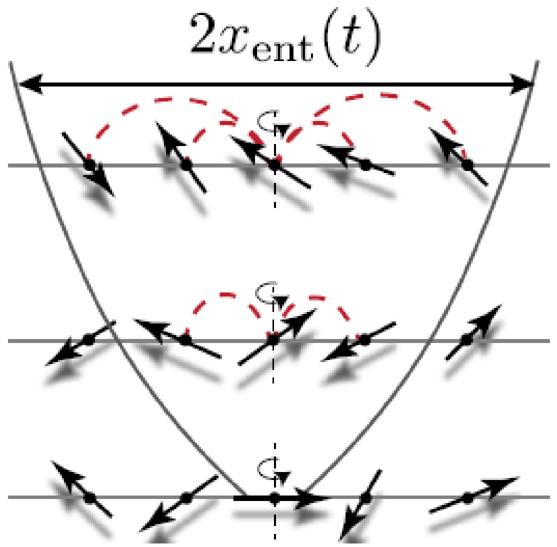
1. Logarithmic growth of entanglement following a quench:

   - A given spin acquires a phase dependent on another spin at x away after a time $t(x)$ set by the condition $\tilde{h}_{i,i+x} t \sim 1$.

   - The effective magnetic field is exponentially suppressed $\tilde{h}_{i,i+x} \sim J_0 e^{-x/\xi'}$ leading to $x_{ent}(t) = \xi' \log(J_o t)$ and

$$S_{\text{ent}} \propto \xi' \log(J_o t)$$

   - In a finite system, $s_{\text{ent}}(\infty) \propto L$



Dynamical Properties of the MBL Phase

- Logarithmic propagation of entanglement is different growth in ergodic, integrable models and Anderson systems.

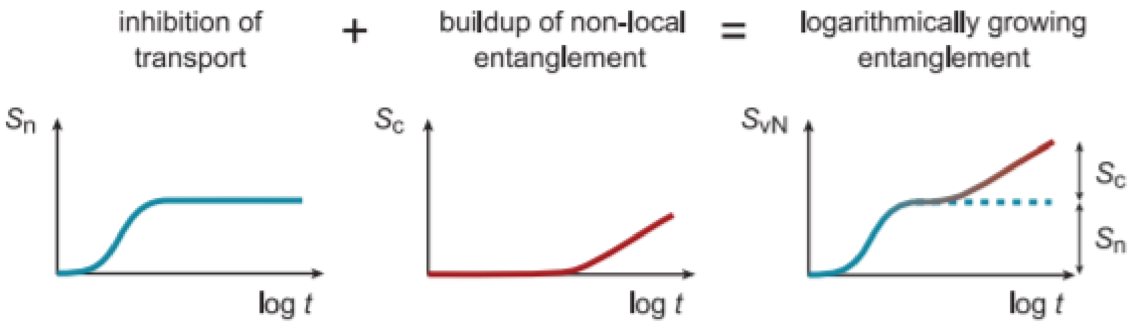
2. Generic local observables equilibrate to equilibrium value in a power-law fashion.

Probing entanglement in a many-body-localized system

Alexander Lukin, Matthew Rispoli, Robert Schittko, M. Eric Tai, Adam M. Kaufman*, Soonwon Choi†, Vedika Khemani, Julian Léonard, Markus Greiner‡

$$\hat{\mathcal{H}} = -J\sum_i(\hat{a}_i^\dagger \hat{a}_{i+1} + h.c.) + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) + W\sum_i h_i \hat{n}_i$$

$$h_i = \cos(2\pi\beta i + \phi)$$



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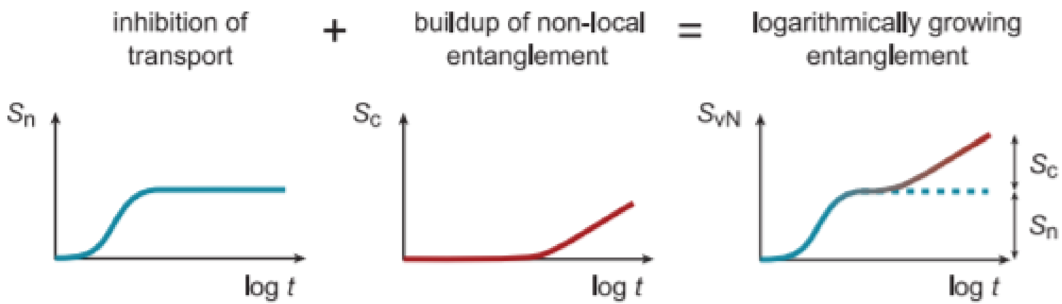
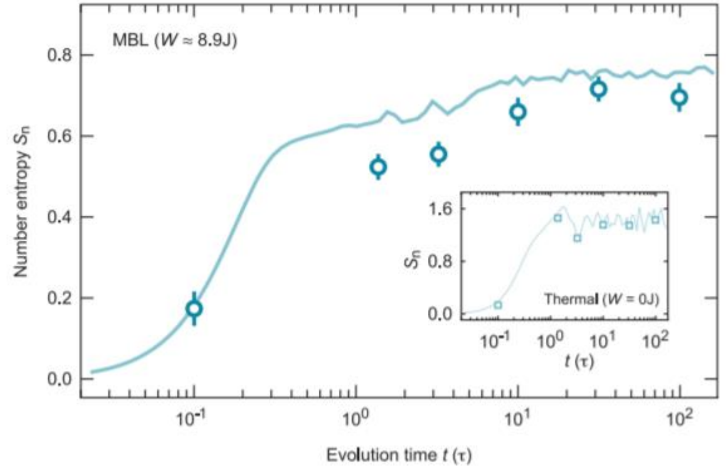
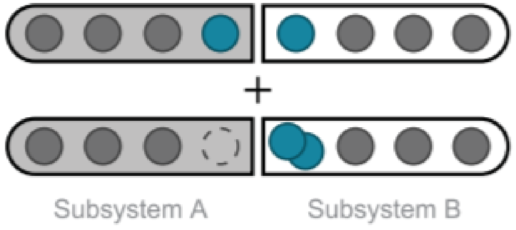


Number entanglement stems from a superposition of states with different particle numbers in the subsystems and is generated through particle motion across the boundary.

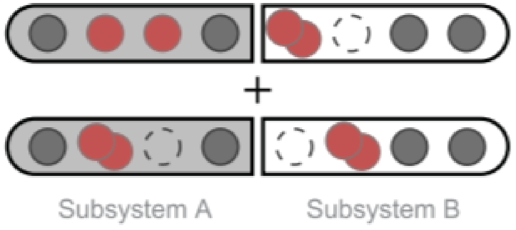
Configurational entanglement stems from a superposition of states with different particle arrangement in the subsystems and requires both particle motion and interactions.

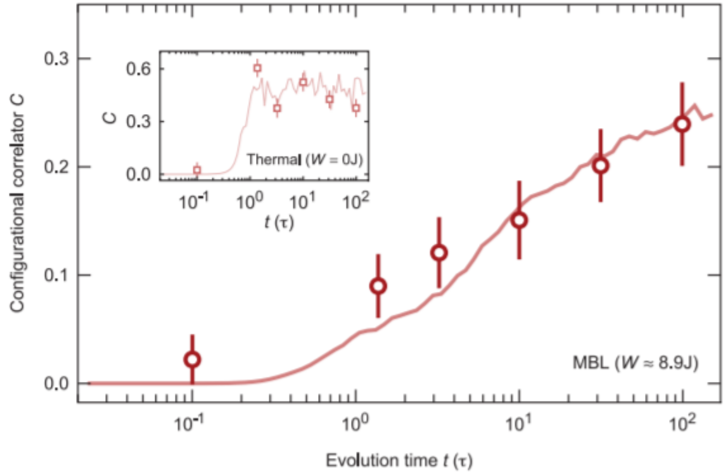


Number entanglement



Configurational entanglement





Number entropy,

$$S_n = -\sum p_n \log(p_n)$$

$$C = \sum_{n=0}^{N} p_n \sum_{\{A_n\}\{B_n\}} |p(A_n \otimes B_n) - p(A_n)p(B_n)|$$

Direct measurement of non-local interactions in the many-body localized phase

B. Chiaro*[1], C. Neill*[2], A. Bohrdt*[3,4], M. Filippone*[5], F. Arute[2], K. Arya[2], R. Babbush[2], D. Bacon[2], J. Bardin[2], R. Barends[2], S. Boixo[2], D. Buell[2], B. Burkett[2], Y. Chen[2], Z. Chen[2], R. Collins[2], A. Dunsworth[2], E. Farhi[2], A. Fowler[2], B. Foxen[2], C. Gidney[2], M. Giustina[2], M. Harrigan[2], T. Huang[2], S. Isakov[2], E. Jeffrey[2], Z. Jiang[2], D. Kafri[2], K. Kechedzhi[2], J. Kelly[2], P. Klimov[2], A. Korotkov[2], F. Kostritsa[2], D. Landhuis[2], E. Lucero[2], J. McClean[2], X. Mi[2], A. Megrant[2], M. Mohseni[2], J. Mutus[2], M. McEwen[2], O. Naaman[2], M. Neeley[2], M. Niu[2], A. Petukhov[2], C. Quintana[2], N. Rubin[2] D. Sank[2], K. Satzinger[2], A. Vainsencher[2], T. White[2], Z. Yao[2], P. Yeh[2], A. Zalcman[2], V. Smelyanskiy[2], H. Neven[2], S. Gopalakrishnan[6], D. Abanin[7], M. Knap[3,4], J. Martinis[1,2], and P. Roushan[2]

$$H_{\rm BH} = \underbrace{\sum_i^{n_Q} h_i a_i^\dagger a_i}_{\text{on-site detuning}} + \underbrace{\frac{U}{2}\sum_i^{n_Q} a_i^\dagger a_i (a_i^\dagger a_i - 1)}_{\text{Hubbard interaction}}$$

$$+ \underbrace{J \sum_{\langle i,j \rangle} \left(a_i^\dagger a_j + \text{h.c.} \right)}_{\text{NN coupling / hopping}},$$

$$h_i \in [-w, w]$$

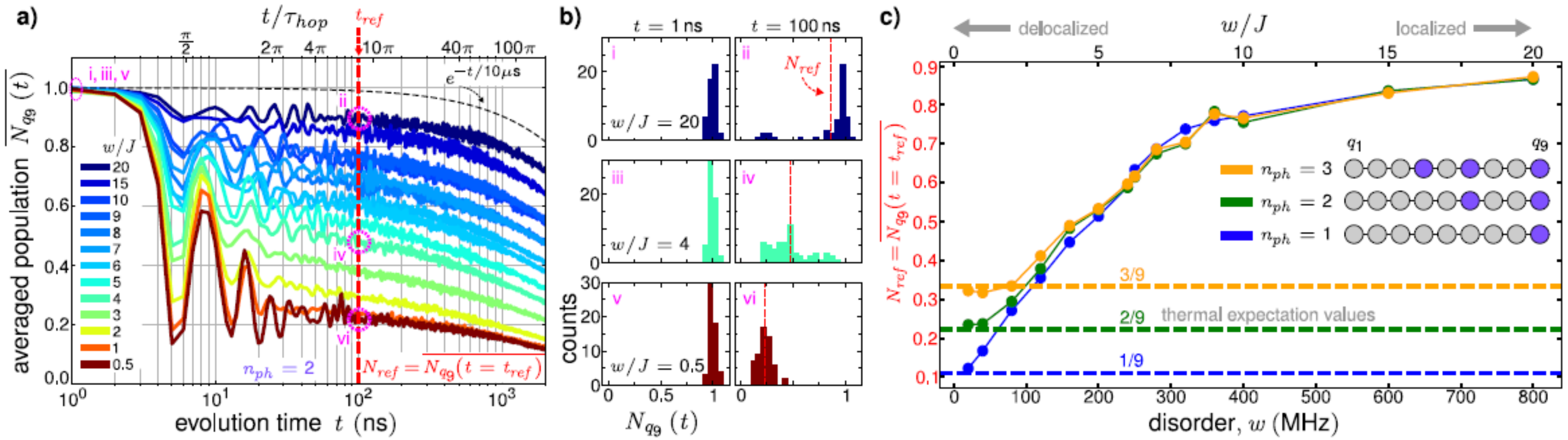


Figure 2. **Ergodicity breakdown at strong disorder.** **(a)** Disorder averaged on-site population vs. time for $n_{ph} = 2$. In a chain of 9 qubits, two qubits were excited (q6, q9). The on-site population of q9 was measured for various magnitudes of disorder w/J, with $J = 40\,\text{MHz}$ (averaged over 50 realizations). The parameter $\tau_{hop} = (2\pi J)^{-1}$ has been introduced to connect the laboratory time t with the hopping energy. N_{ref} is defined to be the average on-site population across instances of disorder at the reference time $t_{ref} = 100\,\text{ns}$, after initial transients have been damped. The dashed black line indicates average photon loss for a single qubit measured in isolation. **(b)** Histograms of $N_{q9}(t)$ at the times and disorders indicated in (a) by numerals i - vi. **(c)** N_{ref} vs. disorder for $n_{ph} = 1, 2, 3$. Inset shows which qubits were initially excited.

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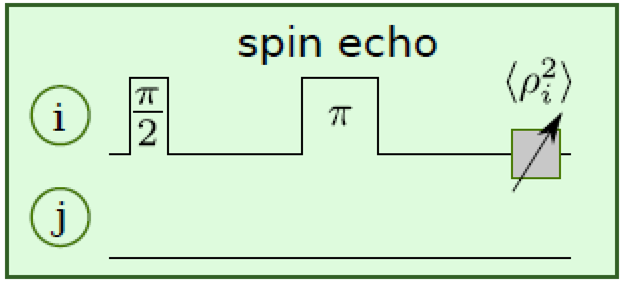
$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |...0_j...\rangle$$

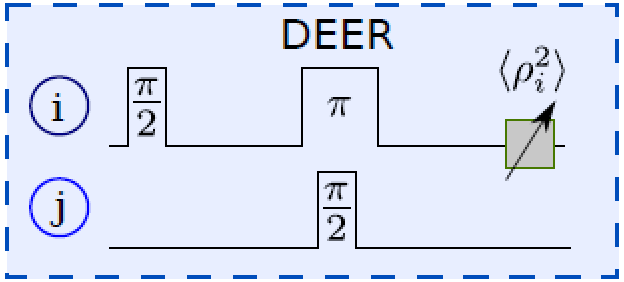
time evolution $\longrightarrow$ $\frac{1}{\sqrt{2}}(e^{i\Delta_i t}|0\rangle + e^{-i\Delta_i t}|1\rangle) \otimes |...0_j...\rangle$

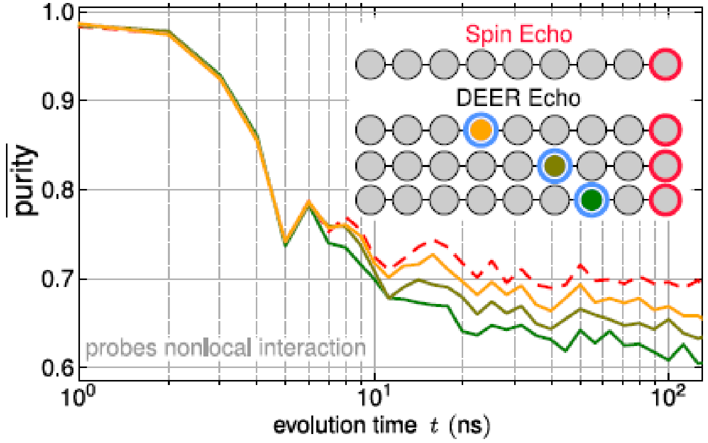
π pulse $\longrightarrow$ $\frac{1}{\sqrt{2}}(e^{i\Delta_i t}|1\rangle - e^{-i\Delta_i t}|0\rangle) \otimes |...0_j...\rangle$

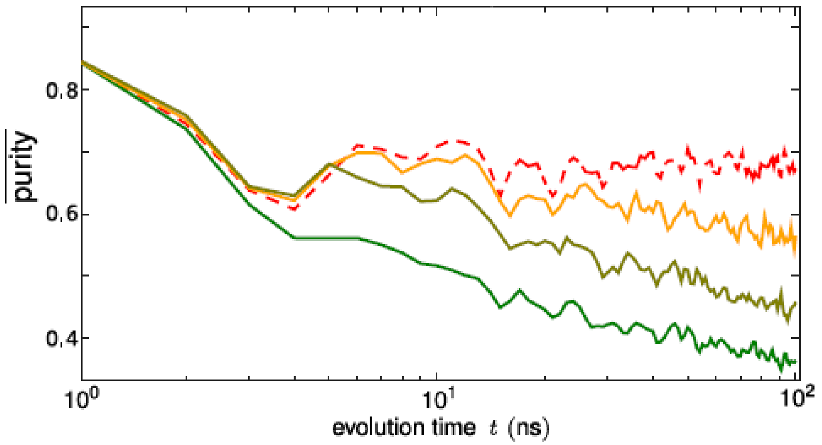
time evolution $\longrightarrow$ $\frac{1}{\sqrt{2}}(|1\rangle - |0\rangle) \otimes |...0_j...\rangle$

$$\mathrm{tr}\left(\rho^2\right) = \cos^2\left[\left(\Delta_i - \tilde{\Delta}_i\right) t\right]$$









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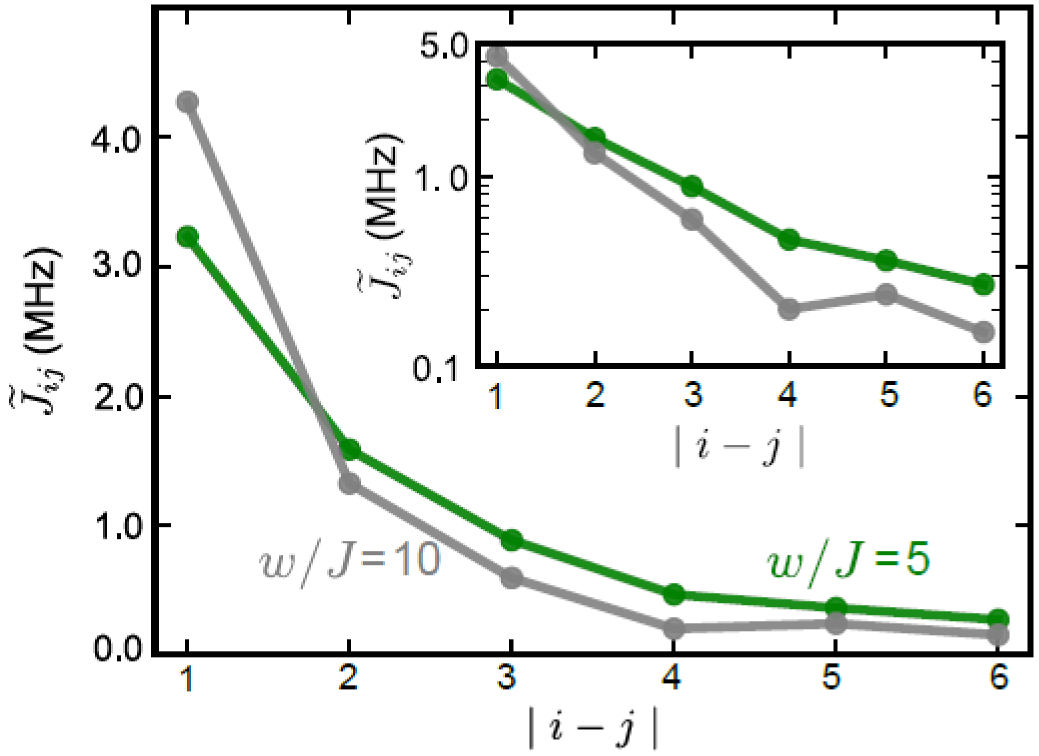

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Video References:

- Annabelle Bohrdt: https://www.youtube.com/watch?v=yyZOi1BVPZI&t=2134s
- David Huse: https://www.youtube.com/watch?v=-Ou702pChUo
- Julian Leonard: https://www.youtube.com/watch?v=47KG1D_qQKQ

Extra Slide:

The von Neumann entropy for the reduced density matrix ρ_A of subsystem A is defined in the Schmidt basis as:

$$S_{\text{vN}} = -\sum_i \rho_{ii} \log(\rho_{ii}), \tag{S10}$$

where the sum runs over all diagonal entries ρ_{ii}. In the Fock basis, ρ_A can be written as the sum of diagonalized blocks due to the block diagonal structure:

$$S_{\text{vN}} = -\sum_{n=0}^{N} \sum_i p_n \rho_{ii}^{(n)} \log\left(p_n \rho_{ii}^{(n)}\right) \tag{S11}$$

Here, p_n refers to the probability of populating states with n atoms in subsystem A, and the total atom number is N. Each block in ρ_A that consists of n atoms is denoted as $\rho^{(n)}$ and is normalized, such that

$$\sum_i \rho_{ii}^{(n)} = 1. \tag{S12}$$

The normalized blocks $\rho^{(n)}$ are multiplied by their relative particle number probability p_n in the reduced density matrix. The expression for the von Neumann entropy can then be reduced to a sum of separate entropy contributions S_{n} and S_{c} in the following way:

$$\begin{aligned}
S_{\text{vN}} &= -\sum_{n=0}^{N} \sum_i p_n \rho_{ii}^{(n)} \left(\log(p_n) + \log\left(\rho_{ii}^{(n)}\right)\right) \\
&= -\sum_{n=0}^{N} p_n \log(p_n) \sum_i \rho_{ii}^{(n)} - \sum_{n=0}^{N} p_n \sum_i \rho_{ii}^{(n)} \log\left(\rho_{ii}^{(n)}\right) \\
&= -\sum_{n=0}^{N} p_n \log(p_n) - \sum_{n=0}^{N} p_n \sum_i \rho_{ii}^{(n)} \log\left(\rho_{ii}^{(n)}\right) \\
&= S_{\text{n}} + S_{\text{c}}.
\end{aligned} \tag{S13}$$